

Complexity and vagueness in the capability approach: strengths or weaknesses?

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So far as laws of mathematics refer to reality, they are not certain.
And so far as they are certain, they do not refer to reality (Albert Einstein, 1922)

All traditional logic habitually assumes that precise symbols are being employed.
It is therefore not applicable to this terrestrial life
but only to an imagined celestial existence [...] logic takes us nearer to heaven than most other studies” (Russell, 1923, p. 88-9).

All economists know the economy is complex – very complex.
That is one of the reasons why society needs economists –
to try to make that complexity somewhat simpler
and more understandable (Colander, 2000: 1).

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Abstract

This paper discusses the meanings of complexity and vagueness in relation to the capability approach and the implications of operationalizing intrinsically complex and vague concepts such as poverty and well-being. Special attention is paid to the fuzzy sets methodology, which seems to be well able to capture these concepts, preserving the richness of the theoretical framework behind them.

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1. Introduction

Concepts of poverty and well-being, like many – probably most – issues and phenomena relating to the human sciences are intrinsically complex and vague, though no more so than crucial economic concepts such as utility, rationality, or recession. This is largely due to the fact that they involve a plurality of interrelated variables, dimensions, and spaces with no clear-cut boundaries within and between them. While both common sense and some of the social sciences (sociology and psychology in particular) plainly acknowledge the intrinsically complex and vague nature of poverty and well-being, the same cannot be said for economics. Only infrequently mentioned, rarely acknowledged at the foundational level, almost always feared and avoided from an empirical point of view, complexity and vagueness are often perceived in economic analysis as elements of weakness within a theoretical framework, and potential obstacles to its operationalization.

Generally speaking, it is widely believed that overly complex concepts are or can be vague, and that when a concept is not delineated by clear boundaries, it is not precise. Moreover, theoretical frameworks characterized by an excessive degree of complexity can be difficult to implement at the empirical level, which makes them less appealing. Authors such as Sugden (1993), Srinivasan (1994) and Roemer (1996) have critiqued Sen's capability approach based upon such arguments. The multidimensional and context-dependent nature of this approach, its lack of specificity as to how these dimensions should be selected and assessed, the absence of a rigorous formalization, definite metric, algorithm or index that provide a full ranking and make it possible to carry out interpersonal comparisons, are all seen as elements of weakness that can seriously preclude the practical application of the approach¹.

It is interesting to note that the great interest in and significant support for the capability approach by scholars from numerous disciplines are often based on a reverse interpretation of the same arguments. Indeed, it is precisely the rich and unrestricted nature of the capability approach that makes it so appealing to many researchers – its usefulness in addressing a plurality of issues within a multiplicity of contexts, its interpretative richness regarding what a good life is, and the attention it focuses on human diversity. Multidimensionality and the lack of narrow, predefined boundaries are here seen as major strengths rather than potential constraints or weaknesses.

The aim of this paper is to discuss if and to what extent concepts of well-being and poverty are intrinsically complex and vague, what degree of complexity and vagueness is admitted and acknowledged at foundational level, and how this complexity is preserved, captured, simplified, or simply ignored, in the move from the theoretical foundation to its practical application. Special

¹ For an excellent and detailed discussion on whether the capability approach should be considered “an unworkable idea or a promising alternative”, see Robeyns (2000).

attention will be focused on the capability approach, currently, among the richest and most multifaceted frameworks of thought for analysing poverty and well-being. What I will attempt to argue here is that the intrinsic complexity and vagueness of these concepts, which are captured and described in such a clear and holistic manner by the capability approach, should be viewed not as elements of weakness or obstacles, but as strengths; the real challenge is to find measurement tools able to capture and preserve this richness adequately.

Since its formulation in the mid-1980s, a fair – though not vast – amount of empirical applications based on the capability approach have been produced; various statistical tools have been tested, including the fuzzy sets methodology, which has been applied with interesting results². In earlier papers I have tried to describe the advantages and potentials of this technique; in the present paper my aim is to clarify in what sense this tool may be considered adequate for capturing the intrinsic complexity and vagueness that characterize the capability approach and the ideas of poverty and well-being that the approach generates.

The paper is structured as follows: Sections 2 and 3 are devoted to clarifying and qualifying the meanings of complexity and vagueness; Section 4 discusses in what sense the capability approach is a complex and vague theoretical framework; Section 5 focuses on measurement issues related to vagueness and complexity, and compares traditional tools based on classical bivalent logic with tools founded on many-valued logic, with special attention to fuzzy set analysis. The last section draws some conclusions.

2. *Complexity*

The meaning of complexity used in this paper is quite restricted, as it refers merely to the nature of economic concepts such as well-being or poverty, standard of living or quality of life. More precisely, I will reserve the term *complexity* to describe multifaceted, multidimensional concepts consisting of many interrelated elements and patterns for which, generally speaking, the whole cannot be fully understood by separately analyzing its components. From this point of view, what determines complexity is not only the existence of many parts and how they are related or connected to one another, but also the necessity of considering them jointly³. This means that the whole can be greater than the sum of its parts, so that by looking at the overall picture we are able to capture linkages and details that cannot be fully perceived through a partial and fragmented vision.

² See, in particular, Chiappero-Martinetti (1994, 1996, 2000), Lelli (2001), Clark and Qizilbash (2002) and Balamoune-Lutz (2004).

³ This definition is similar to that expressed by Alkire (2002: 182): “By dimension I mean [...] any of the component aspects of a particular situation. The key features of dimensions are that they are *component aspects* of something – in this case human development – that *coexist* with other components”

For our purposes, it may be useful to distinguish between two different levels of analysis⁴:

- i) *intrinsic (or conceptual) complexity*, which pertains to the nature of a given phenomenon, and can be partially or fully reflected in the way in which the phenomenon has been conceived and conceptualized;
- ii) *complexity in measurement*, which arises as one moves from the elaboration phase of theoretical concepts to their operationalization, and mainly refers to the choice of technical tools adopted for empirical investigations.

With reference to the notion of intrinsic complexity, a further distinction can be made by looking at the degrees of complexity or heterogeneity entailed, which can *be related to*:

- i) *the object of analysis*, which refers to the number and the nature of evaluative spaces involved. From this point of view, the degree of complexity increases as the number of dimensions, and the level of specificities of each dimension, grow. This usually happens when, for instance, many dimensions are taken into consideration or a plurality of variables is required for describing a given dimension, or if the spaces involved are characterized by a high level of specificity;
- ii) *the unit of analysis*, which relates to the way in which human diversity is accounted for. The degree of complexity goes from the simplest assumption based on the idea of a representative agent to subsequent levels of complexity where group-based analysis and personal and household heterogeneities are taken into account;
- iii) *the context of analysis*, which refers to the degree to which structural elements, spatial factors, and environmental diversities are taken into account. Also in this case, the atomistic view of a representative agent living in an undefined and indeterminate place is the simplest and common assumption. Complexity can progressively increase when a plurality of socioeconomic, institutional and environmental contexts, but also heterogeneous social constraints and cultural norms affecting individual well-being, are taken into account;
- iv) *the relationships and linkages among each level and element involved*. Complex concepts should be characterized not only by the existence of a multidimensional nature, but also by the attention reserved to the connections and linkages existing

⁴ This distinction can be partially referred to Sen's distinction between the "foundational level" and the "practical level" (see Sen, 1999), which is discussed in detail by Comim (2001a).

among the three previous levels of analysis⁵. Intrinsic complexity requires that these linkages be highlighted, the causes and effects of each investigated, their joint effect in affecting, determining, and explaining inequality and deprivation in living conditions understood.

Reviewing the broad range of well-being and poverty approaches currently available in the literature, one quickly verifies that each of them copes with these degrees of complexity in different ways, ranging from highly simplified frameworks to substantial and multifaceted, yet highly differentiated, views of what well-being is. On the one hand, we can find the most basic and common conceptualization of human well-being, which avoids any kind of complexity, restricting the evaluative space to a single dimension as regards both the object and the unit of the analysis. The underlying hypothesis of this purely individualistic and univariate characterization of welfare is that nothing other than income distinguishes individuals, and the amount of income of an individual is all that is required to identify deprivation, evaluate inequalities,⁶ or depict quality of life and human well-being in line with the neoclassical perspective. On the other hand, there are more complex and rich schemes that consider multidimensionality from different perspectives and among these, the capability approach is certainly one of the most complete.

The degree of complexity associated to the capability approach will be discussed in detail in Section 4. What I want to emphasize here is that whatever degree of complexity is involved in the theoretical formulation of a concept (*intrinsic complexity*), this should be distinguished from the *technical* issues involved when that concept is translated into a quantitative or qualitative measure of well-being (*complexity in measurement*). Keeping this distinction in mind can be useful for several reasons. First, the two levels of complexity can (though not necessarily must) be closely linked. Relatively simple concepts can be hard to measure when imperfect information, lack of knowledge, or imprecise tools are all that is available; on the other hand, concepts and theories with a high degree of complexity can be operationalized with a relatively high degree of accuracy⁷.

⁵ The idea of intrinsic complexity is something that can easily be explained referring to different fields of analysis that carry out their investigations in multidimensional spaces. In medicine, for example, a diagnosis is normally based on a broad number of elements: subjective symptoms expressed by the patient, objective health conditions checked through medical examinations and clinical tests, quantitative data and qualitative information on the health and life-style of the patient according to his or her personal features, such as sex, age, and living conditions, as well as the relationships among these different aspects and how they can reciprocally affect each other. A good practitioner knows that he or she cannot make an accurate diagnosis based on a single element or symptom. What this example makes quite plain is, however, not widely accepted in well-being analysis, where the diagnosis of deprivation is often, if not always, based on a single symptom – namely, the lack of income.

⁶ Traditionally this assumption is also included in the axiomatic structure of poverty and inequality index by the symmetry axiom, which assumes homogeneity as regards each characteristic different from income.

⁷ Some Authors rightly point out that “complexity” is not the same thing as “complicated”. “A system is complicated if it can be given a complete and accurate description in terms of its individual constituents, no matter how many” (Cilliers, 1998). “Complication is a quantitative escalation of that which is theoretically reducible” (Chapman, 1985, p. 370). A complex system, on the contrary, cannot be reduced without losing its true nature (Reitsma, 2003).

Second, well-being approaches formulated using accurate algorithms and advanced analytical tools are usually based on a unique evaluative space, and thus on a relatively restricted and rough idea of human well-being; so that it can be difficult to distinguish between concepts that are genuinely complex and concepts that are instrumentally simple⁸. Third, strong simplifications, which are usually introduced with the empirical implementation of complex concepts, are typically justified by resorting to various arguments: the scarcity of statistical data, the impossibility of carrying out expensive *ad hoc* surveys, the fact that some dimensions are unobservable or only partially and indirectly measurable, or the lack of adequate measurement tools. I believe that the intrinsic complexity of a concept can be preserved, even if not necessarily in its entirety, even if a price must be paid with some technical compromises. Furthermore, sometimes complex approaches can be simplified intentionally in order to focus the attention on specific aspects that are relevant to the aims of the empirical investigation, ignoring others that are derivative. But in this case, too, if the richness of the theoretical framework and its spirit is preserved, the empirical assessment will still reflect this complexity to a certain extent⁹.

A final remark: it is evident that the meaning of complexity discussed here is very simple and restricted if compared to the idea of complexity found in the economic thought of Authors such as Smith, Marshall, Kaldor, and Hayek, among others, or in more recent complex, non-linear, adaptive systems developed, for instance, by the Santa Fe approach¹⁰. However, it is worth noting that even if aims, context, and motivations are entirely different, conclusions are quite similar. Complexity characterizes many (most?) economic phenomena; and neoclassical models and standard mathematical tools do not seem to be able to fully reflect and measure this complexity.

⁸ On this point, see also Robeyns, 2000.

⁹ As argued by Sen: “Even when precisely capturing an ambiguity proves to be a difficult exercise, that is not an argument for forgetting the complex nature of the concept and seeking a spuriously narrow exactness. In social investigation and measurement, it is undoubtedly more important to be vaguely right than to be precisely wrong” (1989). I am in full agreement with this opinion and it is exactly on this premise that the present work is based. However, as I will argue in Section 5, vagueness and ambiguities are similar but not exactly identical concepts. In the case of the capability approach. It seems to be more appropriate to refer to vagueness rather than to ambiguity.

¹⁰ See Arthur, Durlauf, Lane (1997). Complexity has been a recurrent topic in economic theory, from Smith’s emphasis on increasing returns and his interest in the economic growth process that involves an increasingly complex pattern of specialization, to Marshall’s appeal for substituting biological analogies with mechanical analogies, to more recent and in-depth analyses expressed by Hayek in “*Theory of complex phenomena*” (1967), his critique of a reductionist approach in economics and his idea of “spontaneous and organized complexity”. But as David Colander points out (2000: p. 34) “Many earlier economists understood the implications of complexity, of which increasing returns and multiple equilibria are important elements, but they also recognized that they did not have the tools to deal with it formally, and therefore did not work on them”. Complexity theory, today, has and makes use of these analytical and computational tools that allows it to introduce hypotheses of non-linearity, to scrutinize an individual’s actions and interactions for understanding aggregate behaviour, to stress the adaptive, evolutionary, inductive processes by which agents in the real economy actually learn, to emphasize path dependence and the role of institutional structures. For an interesting attempt to bridge notions of complexity in economic thought and today’s complexity theory, see Colander (2000) and, in particular, Comim’s and Montgomery’s contributions therein.

3. Vagueness

Vagueness is another distinctive and persistent feature of many concepts and phenomena. Many propositions in ordinary language are inherently vague, and their meanings almost invariably context-dependent¹¹. For instance, “young” or “big” or “beautiful” are vague linguistic concepts in the sense that they do not have exact and universal definitions (various meanings are possible, and clear cut-off points between a given concept and its opposite – such as “old”, “small” or “ugly” - do not exist); moreover, their connotations can change according to the situations to which they are applied (i.e. “young” or “beautiful” take on different meanings when they are used to describe a human being or an animal or vegetable species).

Again, it can be useful to keep the foundational level separate from the measurement level, and to distinguish between:

- i) *intrinsic vagueness*, which refers to the nature of a given concept or phenomenon; and
- ii) *vagueness in measurement*, which relates to the way in which vagueness can be accounted for.

In this section attention is focused on the conceptual level of the discourse, whereas discussion on the measurement issue will be postponed to the second part of this paper.

The historical roots of vagueness go back to the ancient Greeks, who introduced vagueness into philosophy with the sorites paradox,¹² which has fascinated and troubled philosophers for centuries¹³. However, it was only around the 1970s that attention to this topic began to grow significantly, and formal treatment of vagueness became common.

¹¹ Russell argued that all natural language expressions are vague, including symbols and logical connectives (Russell, 1923, p. 88-9). However, as outlined by Wright, “Ordinary language is always more or less vague, but a logically perfect language would not be vague at all: so the degree of vagueness of a natural language is a direct measure of its distance from being everything which it “logically” ought to be” (Wright, 1976, reprinted in Keefe and Smith, 1997, p.151). The intrinsic vagueness of many economic concepts was often noted and remarked upon by Keynes: “Net-income, being based on an equivocal criterion which different authorities might interpret differently, is not perfectly clear-cut” (Collected Writings, vol. 7, p. 60). Finally, in distinguishing between consumption and investment goods, and finished and unfinished goods, he remarked: “All these arbitrary distinctions, however, are only arbitrary in a highly a priori sense. If we had reason to expect that a different set of lines of division would be more appropriate to our psychology of behaviour and decision, we should have to draw them differently” (Collected Writings, vol. 13, p. 433).

¹² Greek paradoxes were usually formulated in terms of a sequence of questions. “Does one grain of wheat make a heap? Do two grains of wheat make a heap? Do three grains of wheat make a heap? Do ten thousand grains of wheat make a heap? It is to be understood that the grains are properly piled up, and that a heap must contain reasonably many grains. If you admit that one grain does not make a heap, and are unwilling to make a fuss about the addition of any single grain, you are eventually forced to admit that ten thousand grains do not make a heap” (Williamson, 1994; 8)

¹³ Galen, in the late second century A.D., wrote: “According to what is demanded by the analogy, there must not be such a thing in the world as a heap of grain, a mass or satiety, neither a mountain nor strong love, nor a row, nor strong wind, nor city, nor anything else which is known from its name and idea to have a measure of extent or multitude, such as the wave, the open sea, a flock of sheep and herd of cattle, the nation and the crowd. And the doubt and confusion introduced by the analogy leads to contradiction of fact in the transition of man from one stage of his life to another, and in the changes of time, and the changes of seasons. For in the case of the boy one is uncertain and doubtful as to when the actual moment arrives for his transition from boyhood to adolescence, and in the case of the

The starting point is quite straightforward. Classical logic is bivalent and doesn't admit vague predicates: a predicate can only be true or false. As Sainsbury writes, "*According to this classical picture, the job of classificatory concepts is to sort or segregate things into classes by providing a system of pigeon-holes, by placing a grid over reality, by demarcating areas of logical space. Boundaries are what count, for a concept must use a boundary to segregate the things which fall under it from the things which do not*" (1997:251-2).

There is no doubt that the principle of bivalence is applicable and necessary in many cases and contexts: for example, laws must certainly demarcate a boundary between legitimate and illegitimate acts. However, at times it may seem irrational or unfair to draw substantial distinctions in terms of rights and duties based on differences that are often marginal: in this field, too, predicates can be coined intentionally in order to leave their application indefinite in some cases¹⁴.

In all fields of science as well as in ordinary language, there are plenty of concepts and predicates that seem to escape the law of the excluded middle. Keefe and Smith in their excellent book on vagueness (1997) offer numerous examples discussed in philosophy and logic: bald, tall, or heap, but also red or child are usually mentioned as examples of vague propositions. These propositions are vague because, generally speaking, it can be difficult and sometimes impossible to identify exactly when a rapidly-receding hairline turns into baldness, or how tall a person must be to be defined "tall", or how many grains of sand make a heap: we cannot do so because sharp boundaries between bald and not bald, tall and not tall, a heap and not a heap, do not exist and because there is a fringe or penumbra of borderline cases regarding which decisive judgments are impossible¹⁵.

As Mozaffar Qizilbash has pointed out (2000, 2003) in his brilliant and novel papers on vagueness and poverty, the predicate "poor" can also be considered *per se* a vague predicate since i) there can be cases where it is not clear whether or not the predicate applies (when, for example, a person is not clearly poor and not clearly not poor); ii) along a hypothetical scale of well-being, an exact point at which a poor person ceases to be a poor person does not really exist; iii) if X is a rich person, with an income of, say, one billion dollars, and Y, with an income that is only one dollar

man in his prime when he begins to be an old man. And so it is with the seasons of the year when winter begins to change and merges into spring, and spring into summer, and summer into autumn. By the same reasoning, doubt and confusion enter into many other things which relate to the doings of men in spite of the fact that knowledge of these things is obvious and plain" (Galen, On medical experience). Similarly, Cicero noted "*Nature has permitted us no knowledge of limits such as would enable us to determine, in any case, how far to go. Nor is it so just with a heap of corn, from which the name (sorites) is derived: there is no matter whatever concerning which, if questioned by gradual progression, we can tell how much must be added or subtracted before we can give a definite answer – rich or poor, famous or unknown, many or few, large or small, long or short, broad or narrow*" (Cicero, *Academica*). Both are quoted in Keefe, Smith (1997, p. 58-60).

¹⁴ For instance, Tappenden (1995; quoted in Keefe and Smith, 1997, p. 21) describes a case where the US Supreme Court coins the phrase "brownrate" to mean "with all deliberate speed" in order to avoid specifying a precise speed.

¹⁵ It should be noted that not only predicates, but also adverbs such as "very" or "many", are intrinsically vague: from this point of view, the use of categorization such as "very poor" or "extreme poverty" actually increases the degree of vagueness.

less, is also a rich person, then if we carry on, dollar by dollar, along a continuum line of people ranked by amounts of income that are always one dollar less than the previous one, this yields the conclusion that a person with no income is also a rich person; and this is clearly false¹⁶.

What is wrong is to ignore the intrinsic vagueness inherent in the idea of poverty and to assume that a clear cut-off point exists, and can be drawn, between “poor” and “not poor”, as the standard approach for measuring poverty has traditionally done, by defining a poverty threshold in terms of income or consumption expenditure.

The problem is more substantial than formal: in drawing a line, sharp boundaries are immediately established and borderline cases are by definition excluded. However, this is not a way to solve vagueness but simply an attempt to ignore it. Moreover, an infinite number of lines might be drawn differently by different researchers; yet this means that the word “poor” can assume a potentially infinite number of representations (as, in fact, does occur). In any case, the sorites paradox emerges in all its strength: a person with an income y is poor, but a person with an income a hundredth greater is definitively not.

It should be clear from the above definition that the meaning of vagueness used here is not dependent on the nature of the evaluative space that we have chosen for analyzing poverty; the idea of poverty is intrinsically vague, no matter whether it is measured in terms of income or nutrition, physical health or any other relevant –single or multiple – dimension of human well-being. The above three features that identify a vague predicate are fully applicable in all these cases. The crucial point here concerns the meaning and the existence of a cut-off point – that is, the possibility of drawing a line between poor and not poor without coming up against the sorites paradox. Moreover, vagueness is not context-dependent: by and large, phenomena that are intrinsically vague will maintain their nature under any circumstances. Poverty can and should be defined in a different way according to different contexts, but in any space and in any time it will always maintain its intrinsic vagueness. Finally, the “amount” of vagueness does not depend on the number of dimensions or variables taken into consideration. Vagueness exists even when poverty is identified in a unique space – typically, that of income – or in the case of a plurality of focal variables are taken into account, being related to the indeterminacy about how and whether it is possible to identify in each space thresholds below which a person will be characterized as poor. In multidimensional analysis, complexity in terms of a multiplicity of dimensions will simply increase vagueness in each of these dimensions.

¹⁶ Wright (1976) has defined predicates that lack sharp boundaries as “tolerant”, because their application is insensitive to small changes. Tolerance is due neither to laziness on the part of the speaker nor to the lack of information about the objects (“a notion of degree of chance too small to make any difference to their applicability”, p.156); no new evidence would clear up the matter.

Susan Haack (1996) distinguishes respectively between unidimensional and multidimensional vagueness, and points out that in the second case, each single variable may suffer, in addition, from the first kind of vagueness.

This distinction between unidimensional and multidimensional vagueness partially corresponds to the distinction between vertical and horizontal vagueness introduced by Qizilbash (2003). In this paper, Qizilbash traditionally refers the former to the dilemma of the existence of a clear cut-off point below which one must fall to classify as poor, while he identifies the latter in terms of the range of dimensions of poverty¹⁷.

What is important to stress here is that, as already discussed in the previous section, multidimensionality affects complexity and requires analysis into components and weighting; yet it is not, *per se*, a source of vagueness (which, as a matter of fact, also arises when unidimensional, apparently simple, concepts are involved). In contrast, if the adoption of a multidimensional perspective allows us to draw clearer boundaries around an intrinsically complex concept, then vagueness could be reduced instead of increased¹⁸. In any case, a clear distinction between complexity and vagueness is important not only at a semantic level but also in terms of the choice of the most appropriate tools to be used for representing and capturing these two distinct though interrelated issues.

The meaning of vagueness discussed here is basically related to the nature of a given concept and seems to adapt quite well to ideas of poverty and well-being, which can substantially be conceived as intrinsically vague concepts. As will be discussed in next sections, this view would suggest a move away from the classic, bivalent logic, in order to substitute the dichotomy of truth *versus* falsity with a many-valued logic based on *degrees* of truth. This perspective determines what is usually called degree theories of vagueness¹⁹.

¹⁷ While the meaning of vertical vagueness is clear and refers to the difficulty of drawing a poverty line, some doubt can arise regarding the meaning of horizontal vagueness. With reference to the notion of “basic needs” and “basic capabilities”, Qizilbash writes: “In making these notions applicable in any context researchers have to settle on the set of needs or capabilities which count as “basic”. Vagueness about this set of needs or capabilities is horizontal vagueness (2003:50)”. However, if the problem is to choose among any possible dimensions a set of explicative variables or spaces potentially useful for representing a given concept (poverty in terms of basic needs or basic capabilities) this seems to be more a problem of complexity rather than of vagueness.

¹⁸ John Coates, outlining that the vagueness literature offers a sort of philosophical defence of common sense practice, identifies a different relationship between vagueness and complexity: “a complex term is defined as a conjunction of simple terms and vagueness arises when we are in doubt as to whether the complete conjunction of elements is required for the application of the term or something less than this complete conjunction” (1996, p. 157). Again, I think that it seems more useful to conceive complexity and vagueness as distinct features, taking into account the fact that the latter can be present even in the absence of the former.

¹⁹ Lukasiewicz (1930) was a pioneer of modern three-valued logic, which represents the first step toward the multi-valued logics that were subsequently developed by, among others, Goguen (1969), Zadeh (1975), Machina (1976) and, more recently, Zadeh and Kacprzyk (1992), Klir and Yuan (1995), Edgington (1997), Hajek (1998), Nguyen and Walker (1999). Degree theories, epistemic views as well as supervaluationist approaches are all discussed in Keefe and Smith (1997). This anthology also includes some of the most renowned essays on vagueness.

However, this is not the only way to formulate the idea of vagueness. There are theories of vagueness that preserve both classical logic and semantics, such as Williamson's epistemic approach, which conceives vagueness basically as a matter of ignorance of the clear-cut boundaries of a given concept. According to this epistemic view, a clear-cut boundary, for instance, between tall people and all the others, or between poor and not poor people, exists; but we are simply ignorant of where such boundaries lie. From this point of view, vagueness is nothing more than a consequence of our inexact knowledge²⁰. Williamson gives an explanation of this source of ignorance in terms of margin for error: when our knowledge is inexact, a belief is reliable only if we leave a margin of error. Reliability increases with the width of the margin and the more accurate our cognitive capacities are, the narrower is the margin needed to achieve a given level of reliability (Williamson, 1994, p. 226).

Another theory of vagueness that preserves the simplicity of classical logic is supervaluationism²¹: the central idea here is that vague predicates can be made more precise by taking into account any admissible valuations or precisification criteria. A statement is true (or super-true) if and only if it remains true under every admissible valuation; a statement is false (or super-false) if and only if it remains false under every admissible valuation; otherwise, it is neither true nor false. In this case there is no truth-value at all²².

Each of these different ideas of vagueness presents advantages and elements of strength as well as weaknesses and theoretical limits²³, and each of these theories requires different measurement tools in order to be adequately represented. The choice among them largely depends on the purpose of the analysis and the contexts in which the vagueness is being considered.

3.1 Vagueness and other related concepts

²⁰ Williamson (1994, p. 216-17) clarifies the meaning of inexact knowledge with several examples: "vision gives knowledge about the height of a tree, hearing about the loudness of a noise, touch about the temperature of a surface, [...] In each case, the knowledge is inexact. One sees roughly but not exactly how many books a room contains, for example: it is certainly more than two hundred and less than twenty thousand, but one does not know the exact number. Yet there need be no relevant vagueness in the number. The inexactness was in the knowledge, not in the object about which it was acquired". On the epistemic view see also Cargile (1969, reprinted in Keefe and Smith, 1997) and Jackman (1994).

²¹ See Fine (1975), Tye (1989) and for a synthetic description Sorensen (2002). Clark and Qizilbash (2002) apply the supervaluationist approach for measuring poverty in South Africa.

²² A typical, non-scientific example is the following: "Toronto is in Canada" is true on each admissible interpretation, so this sentence is true (or super-true); "Toronto is in Europe" is false on each admissible interpretation, so it is false or super-false. The sentence "the number of trees in Toronto is even" could be true on some admissible interpretations and false on others, so it is neither true nor false. Of course, the sentence "the number of trees in Toronto is either odd or even" is true on each admissible interpretation and therefore super-true. See, among others, Williamson (1994).

²³ It has not been clarified, for example, why we are ignorant about where the boundaries of vague predicates lie and whether such knowledge is possible or not. On this, see Keefe and Smith (1997).

Having described in brief how vagueness is conceived and conceptualized from different theoretical perspectives, it might be useful to spend a few words now on clarifying the similarities and the differences between vagueness and other related concepts such as inaccuracy, generality, ambiguity, and unspecificity. This distinction is not only semantically important, but also instrumentally significant in terms of the choice of which measurement tools are most appropriate for capturing and representing different meanings of vagueness.

- i) *Vagueness should be distinguished from inaccuracy.* Russell (1923) defines accurate representation as isomorphism: maps, charts, or photographs all draw closer to this definition in so far as they are accurate. In contrast, a representation is vague when the relation of the representing system to the represented system is not one-to-one but one-to-many. Vagueness is a matter of degree while accuracy represents a sort of ideal limit (Williamson 1994).
- ii) *Vagueness should be distinguished from imprecision (or inexactness):* here the distinction between measurement imprecision and intrinsic imprecision is important. The former is associated with the degree of exactness with which we can measure a given quantity, and depends on the quantity of available information and the quality of measurement tools as well as the statistical correctness of our measurements. The latter is related to the properties of a phenomenon and not to the measurement of these properties (Cox, 1994). While the first type of imprecision is negatively related to the degree of exactness of a given measurement tool, the second type does not diminish with an increase in the precision of a given metric.
- iii) *Vagueness should be distinguished from unspecificity or underspecificity:* the sentence “someone is poor” is obviously vague, but only in the sense that is less than adequately informative for understanding in what sense one is poor and how poverty can be identified. This has nothing to do with borderline cases or with the lack of sharp boundaries and thus with vagueness, which will remain even if the amount of available information increases, reducing the lack of specificity²⁴.
- iv) *Vagueness should be distinguished from semantic ambiguity,* which stems from the existence of multiple meanings attached to one word. A typical example might be the word “bank”, which can be understood as a financial institution but also as a rising ground bordering a lake or a river; or the sentence “the food is

²⁴ Russell also defines unspecificity as generality.

hot”, which is ambiguous because hot could refer to the food’s temperature but also to the presence of hot spices. It could be, as in these examples, that terms and predicates are both ambiguous and vague; however, while semantic ambiguity may be eliminated by distinguishing between these meanings or by paraphrase or by a plausible interpretation, the same does not hold for vagueness. What is the temperature or the amount of spices that qualify a food as hot or not hot? Is there a precise point at which the food is not hot? These remain vague and intrinsically ambiguous issues.

In contrast, the idea of vagueness as a lack of clear-cut borderlines fits very well with the concepts of indeterminacy (Haack, 1996) and fuzziness (Zadeh, 1972). From a linguistic point of view, fuzziness has also been defined as referential vagueness (see Kempson, 1977 and Zhang, 1998): the meaning of a lexical item is in principle clear, but it may be hard to decide whether or not the item can be applied to a certain objects. In the case of referential vagueness or fuzziness we do not have clear-cut criteria for distinguishing the referential boundaries of sets of poor, tall, or bald people. In line with this distinction, vagueness, fuzziness, and indeterminacy will be used interchangeably in this paper and kept conceptually and semantically distinct from other apparently similar but different concepts such as inaccuracy or imprecision. As I will discuss in the next section, intrinsic vagueness and indeterminacy are fundamental properties of the fuzzy set theory, which is a precise, well-specified, accurate tool that appears to be particularly promising for dealing with intrinsically vague and fuzzy predicates.

4. *Intrinsic complexity and intrinsic vagueness in the capability approach*

Is the capability approach a complex scheme? According to the taxonomy suggested in this paper and to common opinion in this field of research, yes, there is no doubt that the capability approach is intentionally and intrinsically complex²⁵. But the prescription to refer to a multiplicity of evaluative spaces for assessing well-being represents only one - probably not the most crucial one - of the qualifying elements of this approach. Many other approaches offer a broad-ranging view of human well-being²⁶. But what is worth noting is that with the capability perspective, the plurality goes well beyond the prescription of multivariate spaces for assessing well-being, involving other

²⁵ Bourguignon, Chakravarty (2003, p. 26): “[...] well-being is *intrinsically* multidimensional from the view point of ‘capabilities’ and ‘functionings’ [...]”

²⁶ See, among others, the contribution of the so-called Scandinavian School with the Swedish approach suggested by Erikson and Aberg (1987) and Erikson (1993) and the Swedish Scandinavian Welfare Study formulated by Allardt. For the Basic Needs school, see Stewart (1985) and Doyal and Gough (1991). For a comparison among different multidimensional approaches to human well-being, see Alkire (2002).

fundamental aspects and levels of analysis that are often neglected. First of all, in this multivariate evaluative perspective two distinct levels or spaces of analysis - the capability space and the functioning space - are brought into focus, distinguishing what individuals *can* do from what they *do*. Secondly, in most cases - as the empirical applications show - both evaluative spaces are generally interpreted as multidimensional spaces: recurrent functionings such as education, health, participation, and housing are assessed by using a plurality of qualitative and quantitative variables. Thirdly, the way in which the capability approach conceives human diversity is intrinsically pluralistic: it acknowledges how the high degree of heterogeneity in personal features such as sex, age, and physical and psychological conditions makes each person substantially different from the others, generating deep interpersonal variations in the conversion of resources into functionings and capabilities. Fourthly, two hypothetically identical people living in different households and facing different socioeconomic contexts and different cultural and natural environments will generally have different sets of opportunities and different living conditions²⁷. Finally, what makes the capability approach an intrinsically complex (and not only a multidimensional) approach is the attention it focuses on investigating the linkages among the aforementioned layers of analysis: what are the differences and the relationships between capability and functioning sets, what kind of practical advantages and technical difficulties are associated with the decision to focus attention on one space or another²⁸, how can pervasive human diversities and the environment (*latu sensu*) positively or negatively influence our achievements, and our possibility to achieve our overall freedom, via the conversion factors. Thus, the capability approach is much more than a mere multidimensional framework for assessing poverty and well-being; it offers a broader, richer, and intrinsically more complex perspective for describing the multifaceted nature of poverty, understanding its causes and effects, and investigating interrelated layers of analysis that have traditionally been neglected or not adequately debated. However, while this intrinsic complexity is often (though not always) considered a strength at the conceptual level, it is also generally perceived as a potential drawback due to the indisputable challenges it entails at the methodological level²⁹.

²⁷ A different interpretation of layers of analysis and interconnections between individual and social context is offered by Comim (2001b), who distinguishes between poverty as a property of an individual and system-level poverty that “is due to causes that transcend individual characteristics or circumstances” (p.2). In another paper, Comim and Kuklys (2002) explore how these two conceptually different levels of poverty, their determinants and interactions can be empirically measured.

²⁸ Elsewhere I have argued why it is important to include both spaces in a well-being assessment, rather than to choose between them. See Chiappero-Martinetti (1996).

²⁹ Srinivasan (1994) and Sugden (1993), for example, do not hide their scepticism towards the possibility of making this approach truly operative, while Ysander (1993) points out the fact that “capabilities are often rather elusive things to catch” and concludes that “economists may be confused here because they are used to dealing with one of the few aspects of life where the opportunity set can be reasonably well defined and measured” (p. 84). Responses to these criticisms can be found in Sen (1999) and Robeyns (2000, 2003).

Is the capability approach a vague scheme? Are concepts of poverty and well-being formulated within this framework vague concepts? With reference to the meaning of vagueness developed in the literature and discussed in Section 3, where a vague predicate is defined as a predicate i) with borderline cases, ii) no sharp boundaries and iii) susceptible to sorites paradoxes, the answer appears obvious. Yes, the capability approach is undeniably vague, *yet no more and no less so than any other concepts of well-being or poverty usually are*. The existence of vagueness refers to these predicates *per se* and does not seem to be a consequence of the way they are formulated. The excessive rigidity of this tool, the scarce lack of realism in the assumption that a clear-cut demarcation exists between poor and not poor, is simply reproduced and reinforced when many dimensions of well-being are involved.

As Sen himself writes (2003, p. 5, 6), “There are many ambiguities in the conceptual framework of the capability approach. Indeed the nature of human life and the content of human freedom are themselves far from unproblematic concepts. [.....] In so far as there are genuine ambiguities in the underlying objects of value, these will be reflected in corresponding ambiguities in the characterization of capabilities. [...] if an underlying idea has an essential ambiguity a precise formulation of that idea must try to capture that ambiguity rather than attempt to lose it. Even when precisely capturing an ambiguity proves to be a difficult exercise, that is not an argument for forgetting the complex nature of the concept and seeking a spuriously narrow exactness. In social investigation and measurement, it is undoubtedly more important to be vaguely right than to be precisely wrong”. The next section will be devoted to a discussion of these methodological issues.

As a matter of fact, even while there is a growing empirical poverty and well-being literature based on or inspired by the capability approach, it is very difficult to find applied exercises that fully and perfectly reflect the richness and complexity of this approach. In most cases functionings instead of capabilities are estimated and few words are spent for justified this kind of choice (but for an interesting exception see Burchardt 2002); there is not a clear distinction between means and ends and often income is included as a dimension of well-being; linkages among functionings are not always investigated; human diversity and environmental heterogeneity are generally described *ex-post* more than included *ex-ante* in the empirical exercise; conversion factors are rarely mentioned or taken into account. From this point of view, much work must still be done.

5. *From theory to practice: how complex and vague concepts can be measured*

In Sections 2 and 3, I investigated intrinsic complexity as multidimensionality, and vagueness as difficulty or appropriateness, at a conceptual level, in “drawing a line”. In this section, I will discuss complexity and vagueness from a measurement point of view. The main aim is to review how these aspects have been dealt with in multidimensional poverty analyses, with special attention to empirical applications based on the capability approach.

As far as the first aspect is concerned, multidimensionality requires managing a variety of variables (qualitative and quantitative, dichotomous and polytomous, cardinal and ordinal) as well as an analysis into components and weighting. In particular, the methodological issues that must be dealt with are related to layers of complexity associated with:

- i) the number and the nature of a plurality of symptoms of deprivation and dimensions of well-being;
- ii) the forms of heterogeneity among individuals and circumstances which can affect or generate poverty;
- iii) the correlation among variables, dimensions and levels of analysis;
- iv) the aggregation across attributes, across units of analysis (i.e. individuals, households, groups) and between attributes and units for establishing an overall deprivation index³⁰.

Two different strategies seem to be available for dealing with the first two of above issues: either to bring multidimensionality back to a univariate approach, or to make use of multivariate techniques. Some examples of the first strategy are the conversion of needs into a money metric by needs-adjusted disposable income, the equivalence scales methodology or the aggregation across various attributes into a single index, followed by the identification of a unique poverty line³¹.

Factor analysis, structural equation modelling, cluster analysis, log-linear models, principal component analysis, and fuzzy sets theory are examples of the multivariate techniques most commonly used in poverty analysis to reduce, measure, and aggregate deprivation dimensions, as well as to analyze causal relationships among them. Again, if poverty were identified as a shortfall from a threshold on *each* dimension of an individual’s or household well-being, a plurality of poverty lines would need to be established. In this case, an additional problem could arise regarding whether or not a given threshold can be defined independently from the others. Moreover, even if both absolute and relative poverty lines could potentially be applied when evaluative spaces

³⁰ On the substantive difference of these different forms and level of aggregation see, among others, Atkinson 2002.

³¹ Bourguignon and Chakravarty (2003, p. 27) outline that in this way the concept of poverty essentially remains a one-dimensional concept. They also argue that “the issue of the multidimensionality of poverty arises because individuals, social observers or policy makers want to define a poverty limit on each individual attribute: income, health, education, etc.” (p. 27-8). In this paper the Authors also formulate a methodology for combining these different poverty lines into a multidimensional index.

ther than income are being considered, some Authors point out that absolute poverty lines seem more viable compared to relative thresholds, which could be unclear or debatable in some evaluative spaces³².

Regarding the issues of aggregation and comparison, the most recent literature on multidimensional poverty measurement is moving essentially in two different research directions, focused respectively on the formulation of multidimensional decomposable poverty indexes and on the extension of dominance stochastic conditions in a multidimensional framework³³.

There is no doubt that both multidimensional poverty measurement and poverty orderings, compared to the single dimensional approach case, generate some challenging issues. To mention some of the most important: the aggregation order (should we aggregate first across individuals, and then across attributes, or vice versa?)³⁴ and the procedure for combining deprivation dimensions (how should we choose among union, intersection or other aggregative procedures, and what are the technical and normative implications of this choice?)³⁵; the existence and extension of overlapping among these dimensions³⁶; the necessity of reformulating the axiomatic structure of multidimensional poverty indexes³⁷; and the complexity of extending the stochastic dominance conditions beyond the bi-dimensional case³⁸.

Even so, broad agreement as to the multidimensional nature of poverty bears out the need to solve the measurement problems that it poses, by readapting the methodological tools used in the unidimensional case as well as by experimenting with innovative tools able to preserve the intrinsic complexity of a given phenomenon. There is undoubtedly a trade-off between the complexity and richness of a theoretical framework on the one hand, and, on the other hand, the degree to which a complex theoretical scheme can be formalised, measured, and employed for the purpose of comparison. However, the attempt to simplify complexity by operating (in the most extreme instance) within a single unidimensional space does not really seem to be the right solution³⁹.

As far as the second issue is concerned – i.e. how vagueness is measured or dealt with – just as often occurs in the univariate case, in a multivariate setting, too, vagueness is in most cases simply ignored: the assumption that a threshold or a cut-off point can be established for every dimension,

³² In the case of health, for instance, it seems more reasonable to identify a minimally acceptable level rather than a relative one. See Bourguignon and Chakravarty (2003) and Garcia Diaz (2003).

³³ For the former, see Chakravarty et al. (1998), Bourguignon and Chakravarty (2002,2003) Tsui (2002); for the latter see Atkinson, 2002 and Atkinson et al. 2002. On multidimensional poverty measurement, see also Maasoumi (1999), Dagum and Costa (2002), Duclos et al (2002), Bibi (2003) and Garcia Diaz (2003).

³⁴ As Atkinson (2002) points out, there are substantive differences between these two forms of aggregation.

³⁵ See Atkinson et al. 2002; Duclos et al. 2002.

³⁶ This refers to the case in which attributes can be substitutes, complements, or have no relation at all.

³⁷ See Tsui (2002), Bourguignon and Chakravarty (2002, 2003).

³⁸ See Atkinson, 2002; Atkinson et al. 2002.

³⁹ For instance, Kuklys (2003), in measuring well-being in accordance with Sen's approach, makes interesting use of the hierarchical structure of goals for reducing the dimensionality of well-being.

without any justification offered as to how realistic or reasonable this assumption is, is still a recurrent approach. If it seems meaningless to affirm, as often occurs, that an individual should be identified as poor if his or her disposable income is equal or lower to a given threshold – say 1 dollar a day – but *not* poor if this income is equal to 1.001 dollars, then it seems even more controversial to assume that an abrupt, sharp cut-off point exists between sick and healthy men, happy or unhappy women, fully integrated or totally excluded people. It might be more reasonable to assume human well-being, in its various dimensions, as being distributed along a continuum that ranges from a condition of complete deprivation to one of full well-being, whatever the reference space – income, health, knowledge, participation, social life, and so forth – being considered.

Of course, there are understandable descriptive reasons that justify the choice of a single, conventional poverty threshold – for example, the need to identify, count, rank, and compare those who are poor. However, there is also a substantive issue related to the normative and prescriptive implications of this choice, and this issue is often neglected or not sufficiently taken into consideration. What, for instance, is the standard of living associated with a poverty line conventionally set at half of the mean or median income, or corresponding to 1 dollar a day? Do such thresholds reflect an average standard of living or a “decent” level of life? Do they represent a minimum or optimal goal for a given society? Or are they related to the amount of economic resources that could be, or should be, allocated for reducing poverty?⁴⁰

Furthermore, the assumption that a single, identical poverty line should be used to identify a sort of homogeneous category – i.e. “poor people” – seems to be equally questionable. As I pointed out earlier, the capability perspective envisions the relationship between income or resources and capability as something that is mediated and affected by conversion factors, which seriously calls into question the concept of a *single* threshold for *all* individuals, independent of their personal characteristics and circumstances. As Sen himself points out (1992, 1997), in this case what should be considered is not the absolute lowness of income, but the inadequacy of a given amount of income for achieving a minimum threshold of capabilities, according to the personal characteristics and circumstances that affect the conversion factors⁴¹.

⁴⁰ The linkage between technical criteria and normative issues is more clearly defined for the so-called *statutory*, official or political threshold, which corresponds to the minimum amount of income adopted in social or fiscal legislation – for instance, the subsistence income in Belgium (see Atkinson et al. 2002). An advantage of this method is precisely that it reflects the political view of poverty; a potential disadvantage is that a statutory threshold also reflects budget constraints and other political factors, meaning that the amount of money in question does not necessarily guarantee that individuals or households will be able to satisfy their basic needs or achieve a decent standard of living.

⁴¹ Bottiroli-Civardi and Chiappero-Martinetti (2001) have suggested an approach based on a plurality of (absolute or relative) poverty lines differentiated by subgroup of populations, and proposed a reformulation of the FGT class of poverty measures for capturing the between and within poverty components.

As we move from the theoretical to the empirical level, the question that arises is whether the standard tools used in poverty analysis are adequate for representing the complexity and vagueness that characterize the capability approach. What kind of multivariate techniques have most frequently been used for operationalizing the capability approach? Are they coherent with the main features of this approach?

Table 1 lists some of the empirical applications of the capability approach⁴². A review of this table evidences some interesting features:

- i) Both micro and macro dataset are used, and a large amount of variables and a variety of dimensions are taken into account: from a minimum of 3 to a maximum of over fifty quantitative and qualitative variables, and from 3 to 20 dimensions. None of these empirical applications is based on *ad hoc* surveys and all of them adapt secondary data according to their respective purposes;
- ii) Human diversity is more frequently taken into account when empirical applications are based on micro-data. However, few studies consider human diversity *ex-ante*, for instance by focusing attention on homogenous population sub-groups. In most cases, empirical applications refer to the overall population and construct only *ex-post* a poverty profiles for describing personal or social characteristics common to poor people⁴³;
- iii) Linkages between dimensions of well-being and levels of analysis are more likely to be investigated when empirical applications are based on a micro-data set;
- iv) Accurate descriptions are generally made of how well-being is distributed in its multiplicity of dimensions and across individuals; however, the “temptation” to aggregate in a single index in order to make comparisons seems to prevail, even though some Authors (e.g. Schokkaert and Van Ootogem, 1990) expressly reject this

⁴² This list is of course neither complete nor exhaustive. The empirical applications included have been selected because they are among the best-known and most frequently cited, and because of the variety of methodological tools used. We have excluded the empirical application of the capability approach, which takes us back to the univariate case: see, for instance Lelli (2003) and Kuklys (2004). We have also excluded some brilliant empirical analyses that use the capability approach for purposes other than the assessment of well-being. See, among others, Burchardt 2002; this paper, for the first time, measures capabilities rather than functionings, with particular reference to women’s employment in Britain. For a broad, updated survey on the empirical applications of the capability approach, see Robeyns (2003).

⁴³ With the *ex-ante* approach, homogeneity among individuals or households is postulated implicitly, a comparison with a common reference point (the poverty line) is carried out, and finally a poverty profile is constructed with the aim of discovering, *ex-post*, which individual or household explicative factors (sex, race, age, geography, or other characteristics) can generate poverty conditions. However, this procedure does not seem to be fully coherent with the theoretical underpinning of the capability approach, since it neglects the fact that *ex-ante* these differences affect human needs and generate differentiated conversion rates. It would seem more reasonable to analyze poverty by focusing attention on homogeneous population sub-groups, or making use of a plurality of poverty lines (or adjusted equivalence scales when a money metric is used) that attempt to capture the heterogeneity of personal features and circumstances. On this, see again Bottiroli-Civardi and Chiappero-Martinetti (2001).

temptation. On the whole, a large number of interesting multidimensional well-being and deprivation indexes have been formulated;

- v) A wide range of multivariate and innovative techniques has been tested for selecting dimensions, describing variables, and deriving weighting and aggregate dimensions, including, in particular, factor analysis (Balestrino and Sciclone, 2001; Schokkaert and Van Ootegem, 1990; Lelli, 2001); principal-component analysis (Klasen, 2000); structural equation models (Kuklys, 2003); and fuzzy set theory (Balioune-Lutz, 2004; Chiappero-Martinetti, 2000; Lelli, 2001). Brandolini and D'Alessio (1998) also attempt to apply the sequential dominance for well-being comparisons between population subgroups;
- vi) In most of the empirical applications of Sen's approach, vagueness is acknowledged either implicitly or explicitly: rarely are poverty thresholds drawn, but when they are, it is usually for aggregative purposes. When innovative methodological tools such as fuzzy set theory or supervaluationism approach (Clark and Qizilbash, 2002) are applied, these choices are motivated precisely by the need to handle the aspect of vagueness.

The above-mentioned empirical applications show that what are often considered "weaknesses" of the capability approach (too demanding in terms of statistical information, too complex to be operationalized) can be overcome without great difficulty. Most importantly, these applications attempt to preserve and indeed emphasize, as much as possible, the "strengths" of the approach – its consideration of the heterogeneity of individuals and their contexts and of the intrinsic vagueness of well-being and poverty concepts – by adopting a wide range of methodological solutions.

I do not believe that an "optimal technique" should be identified among those that have been tested until now, nor that operationalization of the capability approach demands a single measurement tool or "formula". The broad nature of the approach, its being "a framework of thought" (Robeyns, 2000, 2003), is absolutely compatible with a plurality of techniques and methodological solutions, which must be selected from according to the specific purposes of the empirical investigation in question, and the availability of statistical data. Much more work has yet to be done: new and innovative tools need to be developed and tested, and contributions from other disciplines and fields of research could be very useful in this respect.

Even so, I would like to conclude this paper by explaining why I consider fuzzy measurement techniques to be particularly promising and powerful for the operationalization of Sen's approach.

There are two main reasons for this, one more specific and pragmatic and the other broader and founded on methodological grounds.

From a practical point of view, fuzzy sets theory is more than just a simple generalization of the crisp or classical sets theory⁴⁴. It is a flexible and rigorous mathematical tool that makes it possible to represent quantitative (continuous or discrete) and qualitative variables, to quantify linguistic attributes as well as the so-called hedges or qualifiers⁴⁵ making use of a plurality of functional forms. Compared to crisp sets theory, it offers a broader class of aggregation functions for combining elementary sets – complement, union, intersection, and averaging operators - where each class of functions is characterized by a properly justified set of axioms and the appropriate aggregation operator can be chosen according to different contexts and purposes⁴⁶. Finally, fuzzy methodology is intuitively appealing, theoretically accessible and also computationally simple. All these features are important for the operationalization of Sen's approach. First of all, almost all of the empirical analyses cited in Table 1 make use of both qualitative and quantitative variables, and the latter are often categorical. Secondly, multidimensionality makes the aggregation process more challenging, adding to the intensity of deprivation in one space and the extension of deprivation in more than one space. The availability of a plurality of aggregation operators makes it possible to capture both the depth and the breadth of dimensions of deprivation, and to make explicit the normative judgements underlying each process of synthesis⁴⁷.

From a methodological point of view, the fuzzy sets theory captures a dimension of uncertainty that traditional statistical tools, based on classical logic and crisp sets, are unable to grasp. From a semantic point of view, there are two categories of uncertainty. The first type is associated with events or statements that are well-defined, yet the lack of information, elements of indecision associated with time, the degree of precision of the measurement tools used, or our ability to use these tools in a proper way, make the choice between two or more alternatives unspecified. This type of stochastic uncertainty corresponds to the notion of ambiguity (and broadly to the related

⁴⁴ A crisp set divides the elements of a given universe into two groups: those that certainly belong to the set (membership value equal to 1) and those that certainly do not (membership value equal to 0). A sharp, unambiguous distinction between these two opposite cases certainly exists. A fuzzy set allows for gradual, continuous values of membership between the whole interval [0,1] assigning to each element of a given universe a value representing its grade of membership to the fuzzy set. Because full membership and full non-membership to the fuzzy set can still be indicated, respectively, by values 1 and 0, a crisp set can be considered as a restricted case of the more general fuzzy set.

⁴⁵ Linguistic variables are words or sentences expressed in natural language. Age, for instance, is a quantitative variable when is expressed in terms of years. It becomes a linguistic variable if we refer to it with a (fuzzy) predicate such as, for instance, old or young. A linguistic hedge or fuzzy quantifier modifies the meaning of a predicate or, more generally, of a fuzzy set: *very*, *close to*, *quite*, *fairly*, are all examples of hedge. In the same fashion as adverbs and adjectives in language, these qualifiers change the shape of fuzzy sets. For instance, applying the hedge *very* to the linguistic variable or fuzzy set of young people, we obtain a different fuzzy set, and thus a different representation of the corresponding membership function.

⁴⁶ See Zimmerman (1991). For a discussion on the meaning of fuzzy aggregations and their application to multidimensional well-being based on the capability approach, see Chiappero-Martinetti (1994, 2000).

concepts of inaccuracy, imprecision, unspecificity, or underspecificity, as described in Section 3.1) and has been largely and appropriately dealt with by probability theory and statistics⁴⁸.

The second type of uncertainty is related to the notion of vagueness as the difficulty in defining sharp boundaries and precise distinctions: fuzzy methodology provides a mathematical tool for dealing with this kind of uncertainty. Thus we have uncertainty as ambiguity and probability theory on the one hand, and uncertainty as vagueness and fuzzy sets theory on the other. Probability and fuzziness are not competing, alternative tools for measuring uncertainty: they are appropriate tools for measuring two different types of uncertainty, and they can complement each other⁴⁹.

This distinction is crucial and should be emphasized: even if membership grades and probabilities can take on similar values, they are not the same thing; and substantial differences at the conceptual level exist⁵⁰. As already pointed out, probability is *negatively* related to the amount of information available. As information increases, probability disappears: more information reduces uncertainty, and thus probability values will be smaller as well. In a context of perfect and total information there is no uncertainty, and probability values will be equal to zero. Furthermore, probability is "time-dependent": if a given event (for example, winning a lottery) has a probability of 0.02%, then it is sufficient to wait and see if the event will occur or not.

⁴⁸ However, in literature fuzzy measures have been formulated with the aim of dealing with ambiguity and generalizing probability as well as other classical measures such as plausibility measures or belief measures. A fuzzy measure assigns a value to each crisp set of the universal set X according to the degree of evidence or belief that a particular element belongs in the set. Klir and Yuan (1995) provide an intuitive example regarding the difference between fuzzy measures and the degree of membership to a given fuzzy set. The example refers to jury members who are uncertain about the guilt or innocence of the defendant. The set of guilty people and the set of innocent people are crisp sets with distinct boundaries and very different results: sentence and prison for one and acquittal and freedom for the other. Perfect evidence could assign full membership (the defendant is totally guilty) to one and only one of these two sets (the set of guilty people). However, if we exclude the cases of people who have confessed to their crime or been caught in a criminal act, generally speaking evidence is not perfect and some uncertainty prevails. A fuzzy measure indicates the degree to which the evidence provides proof of the defendant's guilt or innocence, that is, the extent to which it represents the uncertainty associated with two well-defined alternatives. This type of uncertainty, which derives from lack of information (i.e. ambiguity), is totally different from fuzziness, which results from the lack of sharp boundaries (i.e. vagueness). In the former case, a fuzzy measure assigns a value to each crisp set of the universal set (in our example, the two sets of guilty and innocent people); in the latter, a value (a membership degree) is assigned to each element of the universal set (in our example, to each defendant to be judged by jury members). After having expressed their guilty sentence, the jury members must also define the degree to which the defendant can be considered guilty, and this can be included in the interval between 0 and 1, according to different degrees of responsibility for a given crime.

⁴⁹ Both the fuzzy sets and probabilities theories belong to a wider and generalized theory of uncertainty that includes the possibility theory and the Demster-Shafer theory of evidence, among others. Within this large set of theory, possibility theory is closer to the fuzzy sets theory than is probability theory. Zadeh (1978) views fuzziness as a possibilistic, rather than probabilistic, type of vagueness. Klir and Yuan (1995, p. 205) write: "Probability theory is an ideal tool for formalizing uncertainty in situations where class frequencies are known or where evidence is based on outcomes of a sufficiently long series of independent random experiments. Possibility theory, on the other hand, is ideal for formalizing incomplete information expressed in terms of fuzzy propositions".

⁵⁰ A possible reason for confusion is due to the fact that membership functions μ_A are usually defined with reference to the interval of real numbers from 0 to 1 inclusive. This choice is convenient for normalization and makes the comparison with crisp sets theory immediately evident. However, it is a choice, not a constraint, and any other specification is equally possible. A further distinction between membership degrees and probabilities that is immediately apparent is that the sum of the latter must be equal to one, while there is no such requirement for membership degrees.

In contrast, fuzziness can be *positively* related to the amount of information (the more information available, the greater the vagueness may be, thus weakening the possibility of defining sharp boundaries for a given event or phenomenon) and it does not dissipate with time, since it is an intrinsic property of an event or a given object. A typical example used to clarify the distinction between probability and fuzziness refers to statements such as “there is a 30% chance of a light rain tomorrow”. A greater amount of statistical data and more sophisticated measurement tools could make this statement more precise; in any case, it is sufficient to wait until tomorrow and the probabilistic uncertainty will be solved. However, the fuzziness or vagueness regarding whether or not the rain will be light remains.

The way in which we should interpret membership degrees and probabilities is also different: if a given individual *i* belongs to the fuzzy subset of poor people with a membership degree equal to 0.7, this does not mean that he or she has a 70% probability of being poor, but that his or her condition of being poor is vague or fuzzy in a measure of 70%. In other words, if probability affirms that through time individual *i* is poor 70 counts out of 100, fuzzy analysis assigns this membership degree on the basis of the conditions of individual *i*, and the value does not change if the individual’s condition does not change – no matter what the number of observations.

Is the fuzzy sets theory the only tool for measuring vagueness, or are other options also available? More generally, how can vagueness be solved? Formal treatments of vagueness, which have become common in the last few decades, can be classified according to two main approaches: those that seek to preserve almost all of classical logic and those that replace the dichotomy of truth and falsity with a manifold classification. As already discussed in Section 3, the supervaluationist approach and Williamson’s epistemic approach, which conceive vagueness basically as a matter of ignorance, are examples of the first group, while multivalued logic and fuzzy logic refer to the second group.

The juxtaposition here is evident: in the former case, the basic assumption is that vague arguments must be “regimented” (Haack, 1996) so that classical logical can be applied; in the latter, vagueness must be accepted and dealt with through the use of many-valued or fuzzy logics. The suggestion of a fuzzy logician is to “fuzzify” classical logic to obtain a new logic, which is directly applicable to “unregimented” arguments⁵¹.

⁵¹Susan Haack (1996), in her critique of fuzzy logic, affirms that by assigning membership degrees, for example, to the predicate *tall* rather than settling on an arbitrary cut-off point avoids the need to regiment unidimensional vagueness (or vertical vagueness, to use Qizilbash’s definition); yet it does not avoid the need to regiment multidimensional (or horizontal) vagueness and, at this level, introduces further complexities. From her point of view, fuzzy logic simply postpones, rather than eliminates, the need to introduce arbitrary boundaries. The conceptual distinction between complexity and vagueness suggested here, however, should clarify that as they are different issues, they require different methodological solutions.

Qizilbash (2003) has recently discussed and compared these three different philosophical accounts of vague predicates - i.e. epistemic approaches, supervaluationist theory, and degree theories, which include fuzzy sets theory - and applied a supervaluationist approach for measuring poverty and vulnerability in South Africa (Clark and Qizilbash, 2002)⁵². He underlines the arguments in favor of supervaluationism, contrasting it to the fuzzy approach, which he criticizes for a series of reasons. Among them, he points out that it is not fully clear how membership degrees should be interpreted when they are applied for measuring “vertical vagueness” – that is, vagueness about the critical level below which one should be identified as poor; fuzzy sets theory does not focus on what he calls “horizontal vagueness” – i.e., vagueness about the dimensions of poverty – nor does it offer criteria for identifying a “core” dimension of poverty. I believe that a clearer distinction regarding the nature and purpose of the fuzzy sets theory *vis-à-vis* the supervaluationist theories and its appropriateness for measuring vertical and horizontal vagueness might be sufficient for responding to these critiques.

On the one hand, as I have tried to show, the aim of the fuzzy methodology is not to “make more precise” concepts that are intrinsically vague, but to evidence, preserve, and account for this vagueness. The supervaluationist theories recognize the vague nature of many predicates, but their principal aim is to solve vagueness, to get back to the Aristotelian law of true or false, A or not-A, poor or not poor⁵³. They are two different alternatives – and significantly different tools – for dealing with “vertical vagueness”; and the choice between them depends on how predicates and concepts are formulated, and thus how the idea of vagueness is conceived⁵⁴. I believe that vagueness plays a very significant role in poverty and well-being analysis, taking on a precise meaning that must be preserved, particularly when these phenomena are viewed through the lens of the capability approach. The fuzzy sets theory currently seems to represent the best solution for filling the gap between the theoretical formulation of intrinsically vague concepts and their measurement, even while much more work can and should be done in order to adapt this tool to the specific context of multidimensional poverty analysis.

⁵² The application of this approach for measuring poverty necessitates a series of assumption: as Qizilbash (2003) points out, “there are certain ‘admissible’ ways of making the predicate ‘poor’ more precise [...] If all admissible sharpening of ‘poor’ include some particular dimension of poverty, ..., it is a ‘core dimension’ (p. 50-1). Then, he assumes that “there is a range of minimal critical levels, which are involved in admissible sharpenings of ‘poor’ and a set of ‘admissible dimensions’ each of which has a range of admissible critical levels associated with it . Finally, “someone is poor (or core poor) if she falls at or below the lowest admissible minimal critical level on at least one core dimension” (Qizilbash 2003, p. 51)

⁵³ Qizilbash writes (2003, p.50): “... supervaluationism can be easily interpreted in the context of poverty measurement. The central concept is that of a specification space, and it is in this space that the notion of *making things more precise*, or ‘*sharpening*’ them is developed” [italics added].

⁵⁴ As also Keefe and Smith, 1997 recognize, “Perhaps different ways to model vagueness are useful for different purposes and none is ideal for all” (p. 49).

On the other hand, the usefulness of a supervaluationist approach emerges more clearly in what Qizilbash defines as “horizontal vagueness” (which, however, I think would be better defined as “intrinsic complexity”: see note 17). In a multidimensional setting, it certainly represents an alternative to other multivariate techniques (for example, the factor analysis) for reducing complexity, selecting dimensions, and identifying core poverty⁵⁵.

I feel certain, however, that issues such as the choice of what are the relevant dimensions of well-being or what it means to belong to the “core poor” involve inescapable normative judgements; and I am convinced that the responsibility for such choices should be left to public debate, or discussed within a prescriptive theoretical framework, rather than being left up to mathematical and statistical tools.

6. Conclusion

Complexity and vagueness are intrinsic and inescapable features of many concepts and phenomena. The aim of science is to remove or to reduce descriptive inaccuracy, imprecision, and inexactness as far as the availability of data, experimental error, and measurement tools permit. However, there is a vagueness and indeterminacy that affects many concepts and phenomena in human as well as in hard sciences and that cannot be removed, since they are aspects that are intrinsic to these concepts and phenomena.

The picture that derives from a multidimensional approach to well-being and poverty analysis is not a simple sum of the parts and neither is it – nor can it be – a clear picture. Some details, in particular, call for our attention; others superimpose, while still others remain in the shadows. Some aspects present sharply-defined boundaries, while others are vague and indeterminate. Our minds and our eyes, by their very nature, are able to grasp details and shades of meaning – more generally speaking, to recognize the complexity and richness of what is in front of us – far better than can the conceptual schemes and methodological restrictions that exist in most fields of research, economics *in primis*.

The thesis that I have tried to support in this paper is that the blurred nature of this picture should be seen neither as constraint nor deficiency⁵⁶; it is rather the *inescapable consequence* that arises when *intrinsically* complex concepts such as those of well-being and poverty are being considered. The value of a theoretical framework should be judged by its ability to acknowledge such complexity, represent multifaceted phenomena, and investigate in-depth relationships, causes, and effects

⁵⁵ As shown by Qizilbash and Clark (2002) who, using survey responses for a sample of South African people, select admissible critical levels and core dimensions of deprivation recurring to the supervaluationist approach

⁵⁶ “Sometimes a blurred picture may communicate more meaning than a sharp one” (Coates, 1996, p.8).

among the plurality of dimensions involved. This is precisely what the capability approach has been able to do.

In this paper, I have suggested that intrinsic complexity (i.e., multidimensionality) and intrinsic vagueness (i.e., difficulty in “drawing a line”) on the one hand, and complexity and vagueness in measurement on the other, should be separated, at least conceptually. In differentiating the two layers of discourse, it becomes possible to clarify why the intrinsic complexity and intrinsic vagueness of this theoretical framework should not be considered a constraint or hindrance that compromises its value. It is the inadequacy of traditional statistical tools in capturing these distinct features that should instead be called into question.

In terms of the measurement issue, the range of multivalued techniques for dealing with multidimensionality is quite wide, as the empirical applications of the capability approach also show, while the choice of tools to account for vagueness is more limited. In this paper I have emphasized arguments in favor of a fuzzy methodology (i.e. fuzzy logic and fuzzy sets) for operationalizing the capability approach, in an attempt to demonstrate how it seems to be able to account for vagueness as well as complexity better than other techniques which are based on classical, bivalent logic and probability theory.

Table 1 – Different methodological solutions tested/applied for operationalizing Sen’s capability approach in well-being and poverty analysis

Reference	Data	Complexity					Vagueness	Aggregation			Technique
		No. of variables	No. of dimensions	Human diversity	Context	Linkages between dimensions or levels	Poverty line?	across dimensions	across units	overall index	
Sen (1985)	macro	3	3	no	Set of countries	no	no	no	no	no	Descriptive statistics Partial ranking
UNDP (1990) HDI	macro	4	3	no	almost all countries	no	no	yes	-	yes	Normalization through scaling (linear functions) Log transformation for GDP Aggregation by arithmetic mean Complete ranking (human development index) ⁽¹⁾
UNDP (1995) GDI	macro	4	3	yes (ex-ante) gender	almost all countries	no	no	yes	-	yes	Normalization through scaling (linear functions) Log transformation for GDP Equally distributed index Arithmetic mean Complete ranking (gender development index) ⁽¹⁾
UNDP (1997) HPI -1 HPI-2	macro	4	3 (4 in HPI-2)	no	a set of developing vs. industrialized countries	no	in some dimensions (i.e long and healthy life, decent standard of living in HPI-2)	yes	no	yes	Generalized weighted mean with $\alpha=3$ Complete ranking (human poverty index) ⁽¹⁾
Balestrino, Sciclone (2001)	macro	26	6	no	Italian regions	yes	no	yes	-	yes	Factor analysis Complete ranking (functioning-based well-being index) ⁽¹⁾
Balioune (2003)	macro	11	7	no	almost all countries	no	no	yes	-	yes	Fuzzy sets theory Complete ranking (human well-being index) ⁽¹⁾
Schockkoert, van Ootegem (1990)	micro	46	6	yes (ex-ante) unemployed	Belgium	yes	yes	no	no	no	Factor analysis Regression analysis
Brandolini, D’Alessio (1998)	micro	20	6	yes (ex-post)	Italy	yes	yes	yes	yes	yes	Descriptive statistics Sequential stochastic dominance + multidimensional poverty index Partial and complete ranking (functioning deprivation index) ⁽¹⁾
Chiappero- Martinetti (2000)	micro	34	5	yes (ex post): subgroups of population	Italy	yes	no	yes	yes	yes	Fuzzy sets theory Complete ranking (overall well-being index)
Lelli (2001)	micro	54	7	yes (ex-post)	Belgium	yes	no	no	yes	no	Factor analysis vs. fuzzy analysis
Klasen (2000)	micro	-	14	yes (ex-post)	South Africa	yes	yes	yes	yes	yes	Principal component analysis Complete ranking (multi-component deprivation index) ⁽¹⁾
Clark, Qizilbash (2002)	micro	30	20 12 (core dim.)	yes (ex-post)	South Africa	no	no	no	yes	no	Fuzzy set theory; supervaluationist approach
Robeyns (2002)	micro	-	14	yes (ex-post) age and gender	UK	between income and functioning	no	only between var. in each dimension	no	no	Descriptive statistics
Kuklys (2003)	micro	15	4	yes (ex-ante)	UK	yes	no	no	yes	no	MIMIC model
Anand et al. (2004)	micro			yes (ex-ante) gender	UK	yes	no	yes	yes	yes	Regression analysis

Note: (1) denomination of the index as indicated by the Authors

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