Measurement of linewidth enhancement factor of different semiconductor lasers in operating conditions

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ABSTRACT

We apply the self–mixing method for the measurement of the linewidth enhancement factor of several types of semiconductor lasers. The $\alpha$–factor value above threshold is determined by analysing the small perturbations that occur to the laser when it is subjected to moderate optical feedback, relying on the well–known Lang–Kobayashi equations. The method is applied to Fabry–Perot, VCSEL, External Cavity Laser (ECL), DFB, Quantum Cascade Laser. It is found that for some lasers the $\alpha$–factor varies with the emitted power, and these variations can be correlated with variations in the laser linewidth.

Keywords: Semiconductor laser, linewidth enhancement factor, laser linewidth, laser dynamics, optical feedback.

1. INTRODUCTION

The linewidth enhancement factor ($\alpha$–factor)\textsuperscript{1,2}, has a great importance for semiconductor lasers (SLs), as it is one of the main features that distinguishes the behaviour of SLs with respect to other types of lasers. The $\alpha$–factor influences several fundamental aspects of SLs, such as the linewidth, the chirp under current modulation, the mode stability, the occurrence of filamentation in broad–area devices. In synthesis, the dynamics of SLs is greatly influenced by the $\alpha$–factor, which is of particular interest for the study of injection phenomena, optical feedback effects, and mode coupling as occurring, for example, in VCSELs.

The $\alpha$–factor is defined as the ratio of the partial derivatives of the real and complex parts of the susceptibility $\chi = \chi_r + i\chi_i$ with respect to carrier density $N$:

$$\alpha = -\frac{\partial \chi_r}{\partial N} = -\frac{\Delta \chi_i}{\lambda} \frac{dn}{dg}$$

where $dn$ and $dg$ are the small index and optical gain variations that occur for a carrier density variation $dN$.

In the scientific literature, several methods have been proposed for the determination of the $\alpha$–factor, which can be broadly be classified as: 1) techniques capable to measure the “material” linewidth enhancement factor; 2) methods capable to measure the “device” $\alpha$–factor. The above classification relies on the fact that methods of class 1 are based on sub–threshold gain/refractive index measurements, and their results might not be closely matched to the behaviour of lasers in operating conditions. Conversely, class 2 methods perform the measurement above threshold, and can account for more complex effects. The most commonly used technique (class 1) is the Hakki–Paoli method\textsuperscript{3}, that relies on direct measurement of $dg$ and $dn$ as the carrier density is varied by an unknown amount $dN$ by slightly changing the current of a SL in sub–threshold operation. Among class 2 methods, the FM/AM modulation technique\textsuperscript{4} can give accurate results, but it requires high–speed direct modulation of the laser, that cannot be easily performed on all types of SLs. Not all methods are applicable to all types of SL devices. For example, the Hakki–Paoli method cannot be applied to VCSELs for the absence of multiple longitudinal modes.

It is of interest to determine whether the alpha-factor is to be considered a property of the active material or, rather, it can be regarded as an operative parameter of the SL, in particular when it has a complex structure. Examples of lasers with an effective alpha value different from the “material” one are reported in the literature\textsuperscript{5}. A dependence of the $\alpha$–factor on the emitted power was also predicted due to non-linear effects\textsuperscript{6}, but this was never clearly demonstrated experimentally to date.

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As a matter of fact, nowadays there exists no standard method for the measurement of the linewidth enhancement factor, and it can be said that the present scenario is not different from the one that was thoroughly analysed by Osinski and Buus back in 1987. An attempt to compare the results obtained from measurement of the $\alpha$–factor performed on the same laser device using different methods is being carried out in a Round–Robin exercise within the frame of the COST 288 Action.

In the present work we use the new self–mixing method for the measurement of the $\alpha$–factor to characterise several types of SLs. Interesting features of this technique (described in Section 2) are: the easy practical implementation, and the fact that it is capable of measuring the $\alpha$–factor above threshold, i.e. in operating conditions. The technique is based on the analysis of the small perturbations that occur when the SL is subjected to moderate optical feedback, and it is based on the well–known Lang–Kobayashi equations. The goal of the work is to investigate whether the measured $\alpha$–factor is constant for a given laser, or it can vary for varying operating conditions above threshold. Possible physical causes for variations of the $\alpha$–factor are non–linear effects, a change in the longitudinal profile of the carrier density within the laser cavity, or cavity effects. Our results show that relevant $\alpha$–factor variations are found in an External Cavity Laser (ECL), and in some Fabry–Perot lasers. These variations are shown to be correlated with simultaneous variations in the linewidth of the laser.

2. SELF–MIXING METHODS FOR THE MEASUREMENT OF $\alpha$–FACTOR AND LINEWIDTH

The interferometric technique called self–mixing (or optical feedback interferometry) has been recently introduced and used in the field of metrology of mechanical quantities, but it can also be used as diagnostic tool for the measurement of SL parameters. In this Section, the self–mixing methods for the measurement of the $\alpha$–factor and the linewidth are presented.

2.1 Measurement of $\alpha$–factor

In the self–mixing interferometric configuration, a small fraction of the light backreflected or backscattered by a remote target re–enters the laser cavity, thus generating a modulation in the power emitted by the laser through an interferometric waveform $F(\phi)$ which is a periodic function of the back–injected field phase $\phi = 2ks$, where $k = 2\pi/\lambda$ is the wavenumber and $s$ is the distance from SL to target. The power emitted by the SL can be written as:

$$P(\phi) = P_0[1 + mF(\phi)]$$  \hspace{1cm} (2)

where $P_0$ is the power emitted by the unperturbed SL and $m$ is a modulation index which is around $10^{-3}$ for typical operating conditions. The shape of the interferometric function $F(\phi)$ depends on the so–called optical feedback parameter $C$, which depends on the effective power reflectivity of the target. The $C$ parameter is of great importance, because it discriminates between different optical feedback regimes. For $C \ll 1$ (very weak feedback) the function $F(\phi)$ is a cosine, just like the usual interferometric waveform. For $C < 1$ (weak feedback) the function $F(\phi)$ is a distorted cosine, and the asymmetry in the waveform arises due to the fact that $\alpha \neq 0$ in SLs. For $C > 1$ (moderate feedback, which corresponds to a power effective target reflectivity of $10^6–10^8$) the function $F(\phi)$ becomes sawtooth–like and it allows to discriminate

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Figure 1. Experimental self-mixing signal waveforms.

Upper-left trace: loudspeaker drive signal at 657 Hz, 1.2 µm/div; a) $C << 1$; b) $C \approx 1$; c) $C > 1$.  

(a)  

(b)  

(c)
the sign of target displacement, as shown by the examples of experimental interferometric waveforms reported in Fig. 1. One interferometric fringe occurs every time the target is displaced by an amount $\lambda/2$.

2.1.1 Theory
The self–mixing method for the $\alpha$–factor measurement is based on the fact that the SM waveform depends on both the $\alpha$–factor and the optical feedback strength (C-factor), as predicted by the Lang–Kobayashi theory$^{9,10}$. Hence, a measurement of some specific features of the interferometric waveform allows to determine the $\alpha$–factor and the optical feedback strength.

Fig. 2 reports a calculated plot of $F(\phi)$ for $C = 2$ and $\alpha = 3$. Within each period, there are two points with infinite slope (X and Y), and the stability analysis shows that the branch between these two points is unstable. An experimental self–mixing signal corresponding to this case is shown in Fig. 1c. For our purpose, let us call $\phi_1$ and $\phi_4$ the phase values corresponding to a zero–crossing of the function $F(\phi)$, and $\phi_2$ and $\phi_3$, the phase values corresponding to points of $F(\phi)$ with infinite slope. Then, by solving the phase equation for the laser optical field in presence of feedback, we can determine analytical expressions for the length of the segments $\phi_13$ and $\phi_24$, shown in Fig. 2:

\[
\phi_{13} = \sqrt{C^2 - 1} + \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(\frac{1}{C}\right) - \arctan(\alpha) + \frac{\pi}{2} \tag{3a}
\]
\[
\phi_{24} = \sqrt{C^2 - 1} - \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(\frac{1}{C}\right) + \arctan(\alpha) - \frac{\pi}{2} \tag{3b}
\]

From Eqs. (3) the following adimensional quantities are derived: $X_{13} = \phi_{13}/2\pi$, $X_{24} = \phi_{24}/2\pi$, which can be easily measured experimentally. Fig. 3 reports calculated contour lines in the $X_{24}$–$X_{13}$ plane corresponding to constant values for the linewidth enhancement factor $\alpha$ and the feedback coefficient $C$. The knowledge of $X_{13}$ and $X_{24}$ leads univocally to the determination of $\alpha$ and $C$, either by the inverse solution of the set (3), or by graphical analysis carried out with the aid of Fig. 3.

2.1.2 Experimental Set–up
A typical self–mixing experiment for the determination of the $\alpha$–factor is shown in Fig. 4. The SL is biased with a dc current, a microscope objective focuses light on the target (that can be either a mirror, a piece of white paper or a Scotchlite™ retroreflector), and a variable attenuator is used to control the optical feedback strength. The target is mounted on a loudspeaker driven by a sine signal at 40 Hz frequency. The self-mixing signal is obtained from the monitor photodiode connected to a transimpedance amplifier and it is acquired by PC or a digital oscilloscope.
2.1.3 Procedure

The acquired interferometric waveform (that comprises a number of 10-15 fringes in each semi-period) is stored and subsequently analysed by an automated software working under Matlab\textsuperscript{TM}. The software automatically identifies the fringes of the signal, and for each fringe the quantities $X_{13}$ and $X_{24}$ are determined. Average values for $X_{13}$ and $X_{24}$ are then calculated for the whole waveform, and the corresponding values for the $\alpha$-factor and the C factor are estimated. A typical self-mixing waveform used to determine the $\alpha$-factor is shown in Fig. 5.

To reduce the uncertainty on the measured values for $\alpha$, and to check the validity of the method, the acquisition of the self-mixing interferometric waveform is repeated for the same laser current, but with a different optical feedback strength (i.e., different $C$ factor). In this way, it can be checked that the experimental points in the $X_{24}$-$X_{13}$ plane lie very close to a contour line that represents a constant $\alpha$-factor value. Fig. 6 reports an example where several experimental points are measured for different values of the $C$ factor, showing that the estimated $\alpha$-factor value is very good. The accuracy on $\alpha$ is estimated to be 0.2–0.6.

2.1.4 Method using weak optical feedback

The method described above requires operation of the SL with moderate optical feedback. To further extend the

![Figure 4. Experimental self-mixing interferometry set-up for the measurement of $\alpha$-factor and linewidth](image1)

![Figure 5. Example of self-mixing signal used to extract the $\alpha$-factor using the automated software.](image2)

![Figure 6. Experimental points plotted in the $X_{24}$-$X_{13}$. From point to point the value of the $C$-factor was varied. The estimated value is $\alpha = 2.20$.](image3)
applicability of the method, a new modified technique has been devised for a laser that is operated in the weak optical feedback regime (i.e., the interferometric waveform has the shape of a slightly distorted cosine, without bistability). This modified technique is described in reference [13], and it allowed to determine the $\alpha$–factor of a Quantum cascade Laser, thanks to its increased sensitivity for small $\alpha$ values.

2.2  Measurement of linewidth

The method for the evaluation of semiconductor laser linewidth is based on the measurement of phase noise in an interferometer. It is well known that when an unbalanced interferometer is read with laser light, the fluctuations of the laser optical frequency cause fluctuations in the interferometric phase, and a measurement of the phase noise can give information on the laser linewidth[14]. However, in conventional interferometers (i.e., Mach–Zehnder or Michelson) the only way to measure the phase noise is to exploit the conversion of the phase noise into intensity noise, when the interferometer works in the quadrature condition. The phase noise spectrum can then be measured using a RF electrical spectrum analyzer. This method hardly offers practical advantages with respect to the self-heterodyne technique[15].

A notable advantage can be found in a self–mixing interferometer operated in the moderate optical feedback regime (i.e. the interferometric waveform is a sawtooth with hysteresis). When the external mirror is periodically displaced by a sine waveform, the phase noise generates a small randomness in the occurrence times of sawtooth–like interferometric fringe transitions, or, equivalently, in the randomness of the duration of the fringes (see Fig. 7). Hence, the phase noise can be measured by repeated acquisitions of the same interferometric fringe using a digital oscilloscope or an acquisition card connected to a PC. Then the fringe duration statistics is determined, and in particular the variance, which is proportional to the linewidth and the external mirror distance $L$. Hence, by knowledge of the mirror distance $L$, the laser linewidth can be measured with a good accuracy.

2.2.1  Theory

When light from a laser source enters an interferometer, the fluctuation of the laser frequency (represented by the linewidth $\Delta\nu$) generates phase noise. The interferometric phase is given by:

$$\phi = \frac{4\pi}{c} v_0 \langle \Delta L^2 \rangle + L_0^2 \langle \Delta\nu^2 \rangle,$$

where $v_0$ is mean optical frequency, and the target distance has a deterministic value $L_0$ to which a random fluctuation $\Delta L$ is added. If phase noise is measured for different values of target distance $L_0$ so that $L_0^2 \langle \Delta\nu^2 \rangle >> v_0^2 \langle \Delta L^2 \rangle$, a linear dependence $\sqrt{\langle \Delta\phi^2 \rangle} = \frac{4\pi v_0}{c} \sqrt{\langle \Delta\nu^2 \rangle}$ is obtained, and the slope of the curve $\sqrt{\langle \Delta\phi^2 \rangle}$ vs. $L_0$ is proportional to the laser
linewidth. So, the linewidth can be recovered from subsequent measurement of the RMS phase noise performed at different target distances.

The sawtooth-like self–mixing signal obtained in the moderate optical feedback regime allows an easy and accurate measurement of phase noise. The hysteresis in the power–phase characteristic of the self–mixing waveform prevents the occurrence of multiple switching for a single fringe transition.

The main advantages of using the self–mixing method for linewidth measurements are: 1) it does not require RF measurements; 2) it does not require a measurement path with a length comparable to the coherence length (like the fringe visibility method16) or much longer than that (like the self–heterodyne method15); 3) it can be applied to all types of single–mode semiconductor lasers, irrespective of the wavelength.

As a final remark, we observe that the required level of optical feedback is moderate, and hence the linewidth value is not altered with respect to that of the solitary laser.

2.2.2 Procedure

The experimental set–up for the linewidth measurement is the same as that used for the measurement of the α–factor, shown in Figure 4. The self–mixing signal is acquired by a PC using a 250 kS/s acquisition card(DAQ-CARD, National Instrument).

First of all, a calibration of the set–up shall be carried out to determine the entity of the overall environmental mechanical vibration that determine the fluctuation \( \Delta L \). This can be made by measuring the phase noise for a short distance, e.g. \( L_0 = 0.1–0.2 \) m. After the calibration, a set of phase noise measurements are preliminarily carried out for varying external mirror distance \( L_0 \) from 0.2 m up to 1.5 m or even 4 m, depending on the linewidth (and hence the coherence length) of the specific laser under test. Generally, it is sufficient to take measurements up to a distance where the phase noise is 0.1–0.5 rad. Once the expected linear trend of linewidth–related phase noise vs. mirror distance \( L_0 \) is observed, the effective linewidth measurements are then carried out by performing a single phase noise measurement at a chosen fixed mirror distance.

The loudspeaker is driven by a sine waveform of 10 Hz frequency, producing an oscillation amplitude of 10–20 \( \mu \)m peak–to–peak, so that at least 20 interferometric fringes are contained in each oscillation period. The Self-Mixing signal is acquired by the PC and the duration of a specific fringe (located midway from the loudspeaker’s change of directions) is measured in real time by the Labview™ software. This measurement is iterated for a number of times \( N = 1000 \). After that, the statistics of the fringe duration is evaluated, and the RMS phase noise \( \Delta \phi \) is calculated from the variance of the fringe duration statistics. Then the laser linewidth is calculated from equation (4). Figure 8 shows a typical result for the fringe duration statistical distribution, alongside with the fitting Gaussian curve having the same variance. The repeatability of the method is better than 3%, while the accuracy is estimated to be around 5%.

The minimum linewidth value measurable with the self–mixing method is estimated to be around 0.5 MHz. As a final validation of the method, for some lasers the linewidth measurement has been carried out also using the self–heterodyne

![Figure 9. Measurement of α–factor and linewidth vs. emitted power for laser Hitachi HL8325g.](image)
method, finding a good agreement between the two techniques.

3. EXPERIMENTAL RESULTS

Several types of semiconductor lasers have been measured in different operating conditions using the self–mixing methods described above. The goal of the measurement is to investigate on whether the \( \alpha \)–factor is constant, or it varies with the emitted power. The linewidth is measured simultaneously with the \( \alpha \)–factor, because the linewidth of a semiconductor laser is supposed to be proportional to the quantity \( (1 + \alpha^2)^{1/2} \), and it is interesting to correlate possible \( \alpha \)–factor variations with variations in the measured linewidth.

3.1 Fabry–Perot lasers

Two Fabry–Perot lasers with emission in the infrared have been measured. The first laser is an Hitachi HL8325g, that emits 50 mW @ 830 nm, with a narrow linewidth and a very stable single–longitudinal mode (no mode–hops are observed from threshold up to maximum power). The values for \( \alpha \) and linewidth measured as a function of the emitted power are reported in Figure 9. The \( \alpha \)–factor is roughly constant, and it ranges from 4.4 to 4.8. The linewidth has the expected hyperbolic dependence for low power, while at higher powers a linewidth floor of 2 MHz is observed. As the \( \alpha \)–factor is constant, the linewidth floor cannot attributed to an increase of the effective \( \alpha \)–factor.

The second Fabry–Perot laser is a Mitsubishi ML5101, that emits 15 mW @ 850 nm, with a large linewidth and a poorly stable single–longitudinal mode. This laser mostly operates on a single–longitudinal mode, but it exhibits several mode–hops when the current is increased. For this laser the linewidth does not follow the expected hyperbolic trend with the emitted power, because as soon as a mode–hop is about to occur, or it has just occurred, the linewidth value deviates appreciably. Figure 10a and 10b report examples of measured values for \( \alpha \)–factor and linewidth for varying emitted power. The \( \alpha \) values seem to be randomly distributed between 5 and 7.7, but an enlightening hint comes from Figure 10c, where the product Power*Linewidth is plotted as a function of \( \alpha \), showing a good agreement with the \( (1 + \alpha^2)^{1/2} \) dependence. The latter plot is consistent with the prediction of the linewidth formula\(^1\), and it shows that for the considered laser the measured \( \alpha \)–factor varies for different operating conditions. The corresponding variations of the linewidth suggest that the \( \alpha \)–factor value effectively varies in this laser. A satisfactory theoretical explanation for the above effect is still to be found, but a possible interpretation is that additional gain/index dispersion is caused by the pulling effect of the detuned gain profile, hence altering the gain/index dispersion typical of the active material. This mechanism is similar to the detuned loading effect\(^5\).

3.2 VCSELs

The VCSEL model Kodenshi KCx-T46-85DA, that emits 1 mW in a single–transversal mode @ 850 nm, has also been characterised. The measured \( \alpha \)–factor and linewidth are shown in Figure 11. The \( \alpha \)–factor lies in the range 2.4–2.8, and it can be considered as constant. The linewidth also has a normal trend. Hence, it is concluded that for this VCSEL no appreciable variations in the \( \alpha \)–factor are observed, and this is in agreement with the linewidth trend vs. optical power. As the VCSEL is a low–power single–longitudinal mode laser, no additional gain/index dispersion is expected.

![Figure 10. Measurement of \( \alpha \)–factor and linewidth vs. emitted power for laser Mitsubishi ML5101. In c), the power*Linewidth product is plotted vs. of the \( \alpha \)–factor, along with a curve proportional to \( (1 + \alpha^2)^{1/2} \).](image-url)
3.3 External Cavity Laser

An interesting device to be tested is an External cavity Laser (ECL). The ECL is made of a semiconductor laser chip with an anti-reflection coating on one facet, placed in an external cavity that is terminated by a rotating dispersion grating which acts as frequency-selective external mirror. A previous work\(^1\) showed that for such a laser the measured linewidth exhibits large variations as the reflection frequency of the external grating is varied within one free-spectral range of the external cavity. Our goal is to perform simultaneous measurement of the linewidth and the linewidth enhancement factor of the ECL, to investigate possible correlations between these two quantities.

The ECL is a commercial device from Sacher Lasertechnik, in Littman configuration. It emits 40 mW maximum power, and the wavelength is tuneable between 830 and 870 nm.

As the external grating is finely tuned by means of a PZT through a FSR of the external cavity, the measured \( \alpha \)-factor varies between 2.2 and 5. The linewidth of the laser is simultaneously measured in the same operating conditions, and it is also shown to vary. Combined results for linewidth and \( \alpha \)-factor are plotted in Fig. 12a, showing some dispersion. However, these data can be arranged in a more meaningful way by considering the Henry’s formula for the linewidth \( \Delta \nu \), modified to take into account the power-independent term\(^2\):

\[
\Delta \nu = \frac{v_s^2 \cdot B \cdot \frac{g_{\text{at}}}{g_{\text{at}}} \cdot n_m \cdot \alpha_m}{8\pi \cdot P} \cdot \left[1 + \alpha^2\right] + \Delta \nu_0
\]  

(4)

where \( P \) is emitted power, and \( \Delta \nu_0 \) is the power-independent linewidth term. The reason for introducing the above equation is that for this ECL the emitted power varies appreciably as the mirror reflectivity is tuned by the PZT. In figure 12b, the quantity \( P(\Delta \nu - \Delta \nu_0) \), is plotted as a function of \( \alpha \) (using the values: \( \Delta \nu_0 = 1.55 \text{ MHz} \)). This plot is in good agreement with the expected \((1+\alpha^2)\) dependence of the Power*Linewidth term.

A possible interpretation is that linewidth variations can be ascribed to variations of the effective linewidth enhancement factor of the compound cavity laser, which occur in accordance to the principle of detuned loading.

![Figure 11. Measurement of \( \alpha \)-factor and linewidth vs. emitted power for the VCSEL Kodenshi KCx-T46-85DA.](image)

![Figure 12. Measurements for the ECL. a) linewidth vs. \( \alpha \)-factor; b) Plot of \( P(\Delta \nu - \Delta \nu_0) \) vs. \( \alpha \)-factor](image)
3.4 Quantum Cascade Laser

Application of the self–mixing method to a Quantum Cascade Laser (QCL) gave very interesting results, that are extensively described in reference 13. To summarise, the measured $\alpha$–factor of a QCL has very low values just above threshold ($\alpha = 0.25$), while it drastically increases well above threshold (reaching $\alpha = 2.7$ at 1.55 times the threshold). In this case the large increase of $\alpha$ can be attributed to the increase of carriers density above threshold.

4. CONCLUSIONS

The self–mixing technique for the measurement of the linewidth enhancement factor has been described, illustrating the advantages of a simple experimental set–up, and the possibility to perform measurements above threshold in the real operating conditions of the laser. Different types of semiconductor lasers have been tested, and the measured $\alpha$–factor values have been correlated with linewidth measurements. For some lasers, it has been shown that the $\alpha$–factor may vary appreciably, and the linewidth varies accordingly.

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REFERENCES

7. see Cost 288 Homepage: http://www.een.bristol.ac.uk/cost288/home.html