Laser interferometry by induced modulation of cavity field

S. Donati

Istituto di Elettronica, Università di Pavia, Pavia, Italy
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The field backscattered from a remote surface into the laser cavity induces an efficient modulation, both in amplitude and frequency, of the cavity field. The modulating signals are the interferometric components (sin and cos) of object optical path length, which can be recovered by heterodyne detection through a separate reference-frequency mode. Using a dual polarization Zeeman stabilized He-Ne laser, a compact self-aligning interferometer is developed.

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Amplitude modulation of laser cavity field due to backscattering from a remote illuminated object has been observed[1–4] in He-Ne and CO_2 sources, and the possibility of using the laser as a heterodyne preamplifier[2] or a velocimeter[3] has been reported. Actually, the effect of induced modulation leads to amplitude as well as frequency modulation of the cavity field, and the driving terms are the in-quadrature interferometric signals $\cos 2ks$ and $\sin 2ks$ (where s is the object distance and k is the wave number). By the use of a dual-polarization frequency-stabilized laser, which supplies two decoupled orthogonally polarized modes oscillating at slightly different frequencies, one mode can be selected for the propagation to carry the induced modulation, while the other mode confined into the cavity serves as a reference oscillator to perform a heterodyne conversion at the photodetector down to electrical frequencies. After AM and FM demodulations, the interferometric signals are made available for electronic processing. Since the laser mirrors behave as the interferometer mirrors, the structure has inherent self-alignment and a compactness which improves immunity from ambient microphonics. Spatial mode selectivity is provided by the mirror resonator, thus allowing operation of the interferometer on diffusing surfaces without special care. A last distinct feature of the method is that interferometric information is contained in the laser beam[1] and can be picked off at any point of the beam, e.g., at the object site.

Expressions for AM- and FM-induced modulation can be easily derived from the theory of the laser with a transmitting window subjected to an external field, developed by Spencer and Lamb.[5] Their analysis relates the unperturbed cavity field $\mathcal{E}$ to the incident field $\mathcal{E}_I$, assumed as the external field entering the cavity at a mirror of field transmittance $T\exp(i\psi)$; accordingly, in terms of slowly-varying amplitude E and phase φ, and for an external field coming from an illuminated object, these fields can be written

$$\mathcal{E} = E\exp[i(\nu t + \varphi)], \tag{1}$$

$$\mathcal{E}_I = \alpha T E\exp[i(\nu t + \varphi + \psi + 2ks)], \tag{2}$$

where α is the attenuation of the mode propagated on a distance $2s$. Using Eqs. (1) and (2), the following expressions of self-consistency equations are obtained [Eqs. (58) and (59), Ref. 5]:

$$\dot{E} = [g_0(1-\beta E^2) - \Gamma]E + (c/2L)\alpha T^2 E\cos 2(ks + \psi), \tag{3}$$

$$\dot{\varphi} = \xi g_0(1-\beta E^2) - \Delta + (c/2L)\alpha T^2 \sin 2(ks + \psi), \tag{4}$$

where g_0 is the gain parameter, β is the gain saturation, $\Gamma = \nu/2Q$ is the effective cavity bandwidth of the loaded laser, ξ is the detuning of the atomic line, and Δ is the frequency deviation from the cavity resonance.[5] Added to saturation and frequency pulling terms, the last terms in Eqs. (3) and (4) describe the induced modulation. At first order of perturbance, i.e., for $\alpha \ll 1$, Eq. (3) can be solved for E by letting $\dot{E} = 0$, while in Eq. (4) the saturation term can be neglected[5] since $\xi \ll 1$; dropping also the unessential factor ψ, one has

$$E = E_0\left(1 + \frac{(c/2L)\alpha T^2}{2(g_0 - \Gamma)}\cos 2ks\right), \tag{5}$$

$$\dot{\varphi} = -\Delta + (c/2L)\alpha T^2 \sin 2ks, \tag{6}$$

where E_0 is the unperturbed field amplitude. Equations (5) and (6) show that the first-order effect of induced modulation is an amplitude and frequency modulation of the cavity field. A predetection extrinsic amplification $A = (c/2L)T^2[2(g_0 - \Gamma)]^{-1}$ is provided by the laser, as can be seen by a comparison of received and emitted fields $\mathcal{E}_I$ and $T\mathcal{E}$; however, the S/N ratio of interferometric signals at most attains the quantum noise limit associated to the field amplitude $\alpha T^2 E_0$, received in the cavity and responsible for the induced modulation. The mode attenuation α can be calculated as the expansion coefficient of the received field on the cavity mode distribution; its evaluation for a Gaussian beam transmitted without optics on a diffusing surface yields $\alpha = \sqrt{2}\,w_0/s$ in the far field and $\alpha = \lambda/\pi w_0$ in the near field, where w_0 is the beam waist radius and s is measured from the waist.

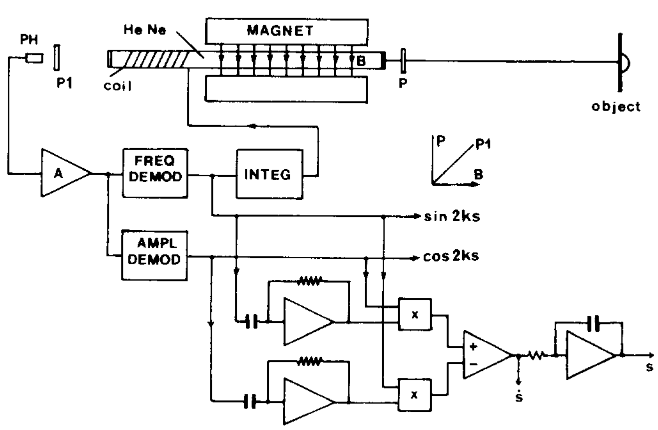

FIG. 1. Scheme of the interferometer.

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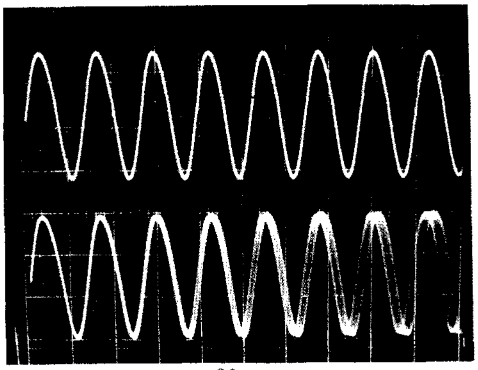

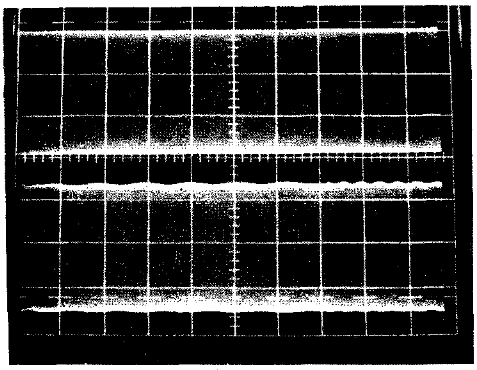

FIG. 2. Signals from the photodiode for an absorbed beam (upper trace) and a propagated beam (lower trace). Time scale in (a) is 20 μs/div and jitter is indicative of FM; in (b) 1 ms/div and AM is shown as a ripple.

To implement the interferometer, a He-Ne Zeeman stabilized laser is used as the source. A moderate transverse magnetic field (300 G) was applied to a 15-cm section of the plasma tube of a 0.5-mW laser[6] by means of permanent magnets. The random polarized mode of the internal mirror cavity is split by birefringence into two decoupled orthogonally polarized modes. Because of the difference in their pulling effects,[7] the parallel and perpendicular modes exhibit a frequency difference, which varies from 0 to about 95 kHz as the cavity line sweeps the atomic line. The beat signal is detected by a photodiode with a front polarizer

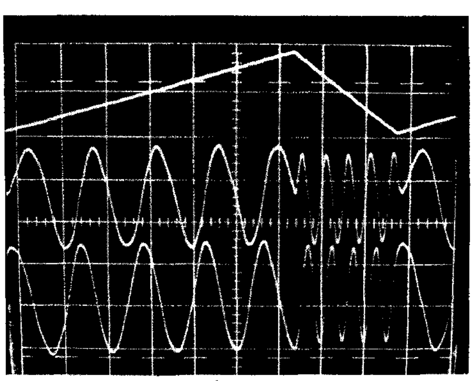

FIG. 3. The waveform driving the loudspeaker displacement (top), and interferometric signals sin $2ks$ and cos$2ks$ demodulated from the photodiode output.

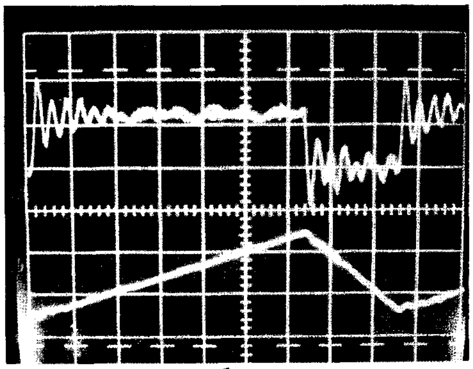

FIG. 4. Velocity and displacement signals obtained by the analog circuit. The ringing is due to the loudspeaker resonance.

oriented at 45° with respect to mode polarization (Fig. 1), and after a frequency-to-voltage conversion is used to feed a coil wound on the plasma tube, so as to control the cavity length by thermal expansion.[7] With a loop gain of about 700, the frequency pulling is stabilized at a selected value of 50 kHz, with drifts of less than 2 Hz/s in the laboratory environment.

A polarizer (P in Fig. 1) inserted in the object path allows one to select one mode for the induced modulation, while the other mode is unaffected and serves as a reference oscillator for heterodyne conversion at the photodiode.[8] The magnetic field applied to the capillary tube shall be carefully controlled to avoid out-of-plane components along the tube section, which would couple the modes and transfer the modulation from one mode to the other. The signal obtained at the photodiode output using a loudspeaker at $s = 50$ cm as the diffusing object is reported in Fig. 2 to illustrate AM and FM due to object motion. After demodulations, the signals cos$2ks$ and sin$2ks$ are available, as displayed in Fig. 3.

To recover the velocity $\dot{s}$ and the displacement s, a simple analog processing was found satisfactory. Through operational amplifiers (Fig. 1), the interferometric signals are time differentiated, their outputs are cross multiplied to the inputs in hybrid multiplier circuits, and subtracted so as to obtain $\dot{s}(\cos^2 2ks + \sin^2 2ks) = \dot{s}$; finally, the velocity $\dot{s}$ is integrated to yield s (Fig. 4).

In regards to the accuracy of the interferometer, the analog multiplier error due to feedthrough and dc offset is measured as 0.01 of full-scale interferometric signals, or 15 Å. Pulling frequency fluctuations have an effect equivalent to a 1-Å rms error for 1-s measure time. On short distances, heterodyne and electrical noise at the output is 0.8 mV/(Hz)$^{1/2}$, which divided by the responsivity (1 mV/Å), yields a noise-equivalent displacement of 0.8 Å/(Hz)$^{1/2}$, a factor of ~2 within the calculated quantum noise limit.[9] The bandwidth of the interferometric signals attains 5 kHz at short distances, and a 0.2-Hz low-frequency cutoff is found on the FM signal, due to the frequency-stabilizing loop.

On diffusing surfaces, two additional limitations to the interferometer operation originate from speckle pattern statistics. To keep constant the random phase contribution added to the returned field, the range of displacement (longitudinal and trasversal) shall be smaller than the speckle grain size[10] projected in the cavity. Within a particular grain, the amplitudes of returned field and interferometric signals are Rayleigh distributed, and an automatic gain control is necessary to ensure scale calibration.

The average values of interferometric signal amplitudes, measured over diffusing surfaces, are in reasonable agreement with those calculated from Eqs. (5) and (6) for $T^2 = 0.01$, $c/2L = 550$ MHz, $g_0 - \Gamma = 2$ MHz (extimated), $w_0 = 0.45$ mm, and for a range of object distance ($s = 0.2 \div 15$ m) covering both near- and far-field signal dependence on $\alpha(s)$.

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