Reconstruction of Displacement Waveforms with a Single-Channel Laser-Diode Feedback Interferometer

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***Abstract*— Using a laser-diode feedback interferometer, we show how to reconstruct without ambiguity the displacement waveform of an external target from a single interferometric signal. We present the underlying theory with numerical simulations and report an example of actual reconstruction from experimental data. Reconstruction accuracy is on the order of tens of nanometers for displacements of a few micrometers.**

***Index Terms*— Displacement measurement, laser interferometry, optical feedback, vibrometry.**

I. Introduction

In CLASSICAL interferometry, the reconstruction of an arbitrary displacement waveform $s(t)$ without directional ambiguity requires two interferometric channels. They are usually obtained either as signals in quadrature, i.e., $\cos 2ks(t)$ and $\sin 2ks(t)$, or in the presence of a carrier at frequency ω_c, i.e., $\cos[\omega_c t + 2ks(t)]$ and $\cos[\omega_c t]$, where $k = 2\pi/\lambda$ is the wavenumber; these signals are then processed in fairly standard ways, analog or digital, to recover $s(t)$. The requirement of two interferometric channels thus increases the complexity of the optical setup. In a previous paper [1], we introduced a new method for the unambiguous measurement of displacement, based on the use of a single interferometric channel which is provided by direct detection of the output power of a semiconductor laser in presence of optical feedback. Briefly, we use a single-mode laser diode, a collimating objective, and an optical attenuator to configure our feedback interferometer. The output signal is detected by the monitor photodiode placed in front of the rear facet of the laser diode (Fig. 1). The objective lens focuses the laser spot onto the distant target while the attenuator serves to adjust the amplitude of the returning signal fed-back into the laser cavity so that the laser is driven in the weak- or moderate-feedback regime. In the weak-feedback regime, the familiar $F = \cos 2ks(t)$ interferometric signal is progressively distorted until it resembles a sawtooth function and, with moderate feedback, it exhibits bistable, abrupt transitions for each $\lambda/2$ displacement Δs. Since the direction of the transitions carries information on whether $s(t)$ is increasing or decreasing, the ambiguity inherent to the cosine function is suppressed, and an interferometer capable of measuring the displacement $s(t)$ in steps of $\lambda/2$ can be readily implemented [1].

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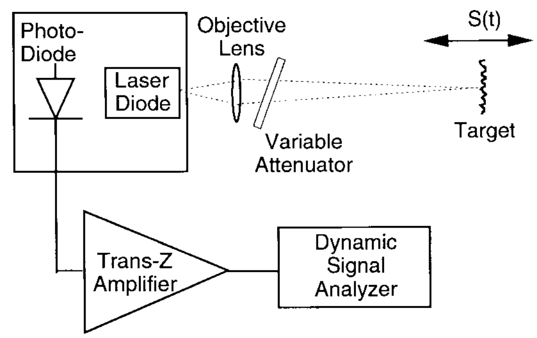

Fig. 1. Optical configuration of the feedback interferometer for the displacement-waveform reconstruction.

In this paper, we demonstrate that the weak- or moderate-feedback interferometric signal $F(t)$ can also be processed so as to recover the analog displacement waveform $s(t)$ even for amplitudes comparable to $\lambda/2$, without directional ambiguity and with enhanced resolution with respect to the counting (or digital) readout mode. Signal processing is performed by means of a personal computer by exploiting the analytical expressions derived in [1], which relate $s(t)$ to the interferometric signal $F(t)$. Numerical simulations and experimental results show that the laser-diode feedback interferometer could be succesfully applied in the implementation of vibrometers or enhanced resolution interferometers.

II. Theory

The starting point for waveform reconstruction is the analysis of the weak and moderate optical-feedback regimes in a laser diode [1]–[3]. From this analysis, based on the Lang and Kobayashi equation [4], it has been found that the laser power is amplitude-modulated by the interferometric signal $F(t) = \cos(\omega_f \tau)$, where $\tau = \tau(t) = 2[s_o + s(t)]/c$ is the external cavity round-trip time, s_o is the quiescent distance, and $\omega_f = \omega_f(\tau)$ is the actual oscillation frequency, which differs from the unperturbed value ω_o and satisfies the equation [1]

$$\omega_o \tau = \omega_f \tau + \frac{C}{\sqrt{1+\alpha^2}}[\alpha \cos(\omega_f \tau) + \sin(\omega_f \tau)] \quad (1)$$

where C is the feedback parameter [5]

$$C = \kappa \frac{\tau\sqrt{1+\alpha^2}}{\tau_L} \quad (2)$$

and α is the linewidth enhancement factor (which assumes a value between 3 and 7, typically $\alpha = 6$), κ is the fraction of the emitted electric field which is reinjected inside the cavity, and τ_L is the diode cavity round-trip time (see also

(a)
(b)
(c)
(d)

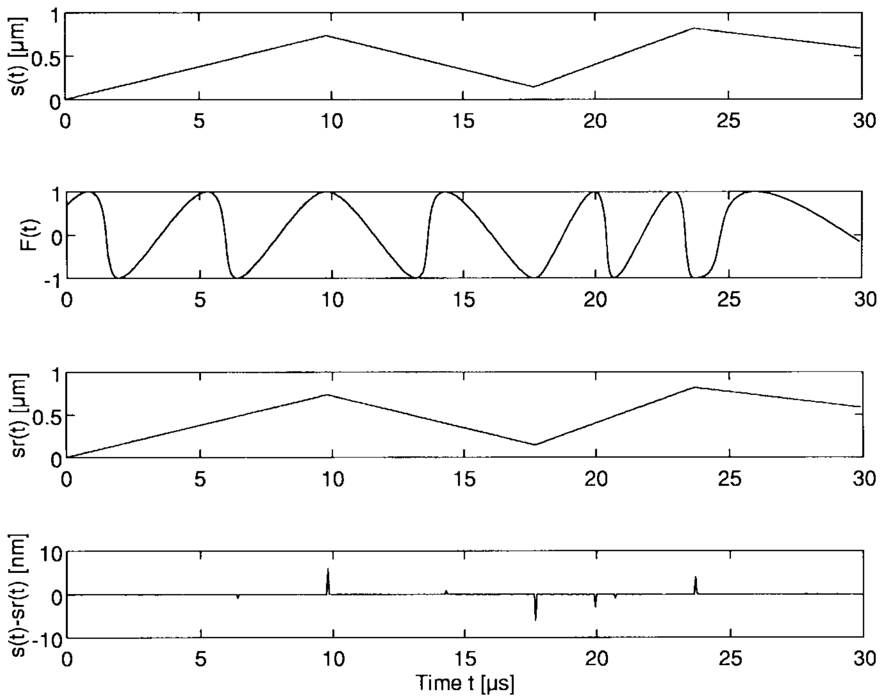

Fig. 2. Numerical simulation of a waveform reconstruction. (a) The displacement signal $s(t)$ [μm] employed in the calculation of $F(t)$. (b) The interferometric signal $F(t)$. (c) The reconstructed signal $sr(t)$ [μm]. (d) The reconstruction error $s(t) - sr(t)$ [nm]. The sampling period is 0.1 μs.

[1]). Since C depends on τ, it depends on the instantaneous distance of the target and, thus, on the time t; however, since $\Delta C(t)/C = s(t)/s_o$, if the interesting displacement $s(t)$ is superposed on a quiescent distance such that $s_o \gg s(t)$, the variations of C with t can be neglected during a displacement measurement.

The function $F(t)$ has a distorted cosinusoid shape; however, it preserves the normal periodicity of $2ks$ for interferometric signals. As found in [1], $F(t)$ can be applied to compute the displacement signal $s(t)$ superposed on the quiescent distance s_o by means of the following expressions:

$$s(t) = \left(\frac{1}{2k}\right)\left\{ -\phi_o + \arccos(F(t)) + \frac{C}{\sqrt{1+\alpha^2}} \cdot [\alpha F(t) + \sqrt{1 - F^2(t)}] + m2\pi \right\} \tag{3a}$$

for

$$\left(\frac{dF}{dt}\right) \cdot \left(\frac{ds}{dt}\right) < 0$$

and

$$s(t) = \left(\frac{1}{2k}\right)\left\{ -\phi_o - \arccos(F(t)) + \frac{C}{\sqrt{1+\alpha^2}} \cdot [\alpha F(t) - \sqrt{1 - F^2(t)}] + (m+1)2\pi \right\} \tag{3b}$$

for

$$\left(\frac{dF}{dt}\right) \cdot \left(\frac{ds}{dt}\right) > 0.$$

In (3a) and (3b), m has to be increased (or decreased) by one for $ds/dt > 0$ (or $ds/dt < 0$) at each 2π crossing of the phase term in braces; in addition, ϕ_o is the initial phase $2ks_o$ [modulus 2π], and we may take without loss of generality $s(t) = 0$ at $t = 0$.

The sign of ds/dt can be determined by inspection of the $F(t)$ waveform in contrast to what happens with a normal interferometric signal. Indeed, $s(t)$ is increasing when $F(t)$ exhibits downward transitions ($F(t)$ decreases from $+1$ to -1) characterized by a sharper slope with respect to what happens when $F(t)$ increases from -1 to $+1$; *vice versa* $s(t)$ is decreasing when a sharper slope is associated with upward transitions of $F(t)$ (see, e.g., [1, Figs. 4 and 6] and Figs. 2 and 3 below). Also, we can discern where the signal $s(t)$ is stationary by looking at the points in $F(t)$ where two consecutive transitions (from -1 to $+1$ and then *vice versa*) exhibit similar slopes, either both sharp or both gradual. It turns out, then, that the reconstruction routine treats $F(t)$ piecewise in intervals from -1 to $+1$.

The above rules can be implemented to reconstruct $s(t)$ from $F(t)$ with the aid of (3) on a PC. To test the method, we have first numerically simulated $F(t) = \cos(\omega_f \tau)$ by solving (1) for different displacement waveforms $s(t)$. The calculated values of $F(t)$ have then been used as the input of the reconstruction routine.

Two results, representative of several simulations, are shown in Figs. 2 and 3. In Fig. 2, $s(t)$ is a triangular waveform with different slopes in consecutive periods and direction changes in correspondence of the usually ambiguous conditions $F = \pm 1$. In Fig. 3, $s(t)$ is the sum of three sinusoids with different frequency. As can be seen, both waveforms are reconstructed correctly with a small residual error $s(t) - sr(t)$ (the lowest trace in both figures), thus demonstrating that the proposed method of reconstruction is well conditioned and little sensitive to numerical errors.

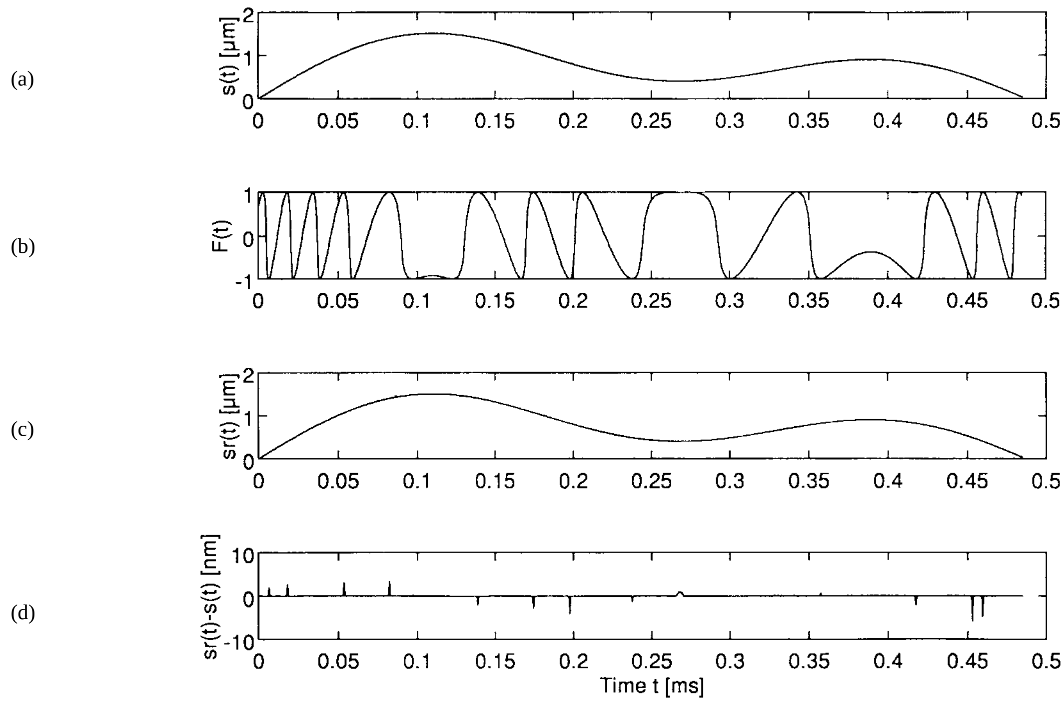

Fig. 3. The same as in Fig. 2 but for a different displacement waveform $s(t)$. The sampling period is 0.5 μs.

III. Experiment

Experiments were carried out with a 673-nm commercial InGaAlP Fabry–Perot laser (SDL-7311-G1), yielding a single longitudinal mode when biased well above threshold (at I =43 mA, i.e., $I/I_{\rm th}$ =1.44).

A point worth noting is that good feedback-interferometer waveforms were also obtained by other commercially available Fabry–Perot laser diodes as long as the residual side-modes were down with respect to the main mode by 20/30 dB; however, one should remember that the theoretical model of Lang and Kobayashi was derived under the assumption of a single-mode laser.

As a target, we used white paper glued on a loudspeaker, placed about 50 cm away from the laser and driven by a function generator. The attenuated laser spot was focused on the target and the interferometric signal was detected by the monitor photodiode incorporated into the laser package via an ac-coupled 100-kΩ transimpedance amplifier. The output voltage $V_{\rm ph}$ was sampled at an appropriately fast rate (e.g., some hundreds of samples per period) by a Dynamic Signal Analyzer (HP35665A) set in the time measurement mode. Data acquisition was in a Standard Data format and the files were then converted to Matlab format for processing on a PC computer.

For the computing routine, we need to determine the parameters C and α of (3). The feedback parameter C is related [1] to the asymmetry of the $F(t)$ waveform, and, for a ramp-like (linearly decreasing) $s(t)$, can be determined from

$$\frac{t_r}{t_f} = \frac{\sqrt{1+\alpha^2}\pi - 2C\alpha}{\sqrt{1+\alpha^2}\pi + 2C\alpha} \cong \frac{\pi - 2C}{\pi + 2C} \qquad (4)$$

where t_r and t_f are the time duration of increasing and decreasing semiperiods of $F(t)$. One should note that (4) provides a sort of calibration algorithm to determine the feedback parameter C of a practical setup, just before the actual measurement. At this purpose, one could exploit the following convenient feature. In presence of optical feedback from a (reflecting) standing target, amplitude and frequency modulation are induced by modulating the laser emission wavelength, i.e., by simply modulating the laser pumping current. By applying a well-known positive-slope current ramp to the laser, the interferometric, sawtooth-like signal $F(t)$ is generated even with a standing target, and the value of C can then be obtained from the measurement of t_r and t_f.

We performed some numerical simulations to demonstrate that α does not critically affect the reconstruction. Starting with the same displacement waveform $s(t)$ considered in Fig. 2, we calculated the residual error $s(t) - sr(t)$, when in the reconstruction of $sr(t)$ we used values of the linewidth enhancement factor which were different from the value $\alpha = 6$ used in the simulation of $F(t)$. This residual error is reported in Fig. 4: (a) and (b), respectively, are related to the reconstruction with $\alpha = 5.5$ and $\alpha = 6.5$ and display systematic errors (with opposite sign) within ± 2 nm for a maximum displacement of the order of 700 nm. These traces should be compared with the lowest trace of Fig. 2, which shows the residual error $s(t) - sr(t)$ only due to numerical (quantization) errors, when the same value of α for simulation and reconstruction ($\alpha = 6$) is applied. With the above observations in mind, we verified with numerical simulations that we could actually evaluate the linewidth enhancement factor α by a preliminary, separate measurement on a well-known ramp-like displacement signal $s(t)$ or current modulation. By

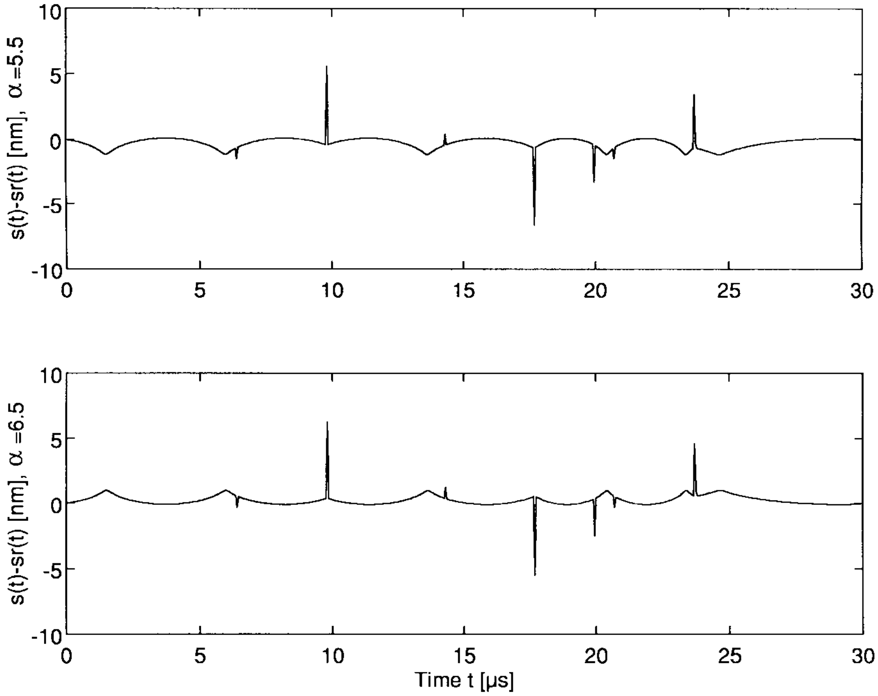

Fig. 4. Reconstruction error $s(t) - sr(t)$ for the displacement signal $s(t)$ of Fig. 2 when the simulation of $F(t)$ and the reconstruction of $sr(t)$ are performed with different values of the linewidth enhancement factor α. (a) Reconstruction with $\alpha = 5.5$. (b) Reconstruction with $\alpha = 6.5$. In the numerical simulation of $F(t)$, we had $\alpha = 6$. The vertical scale is in nanometers.

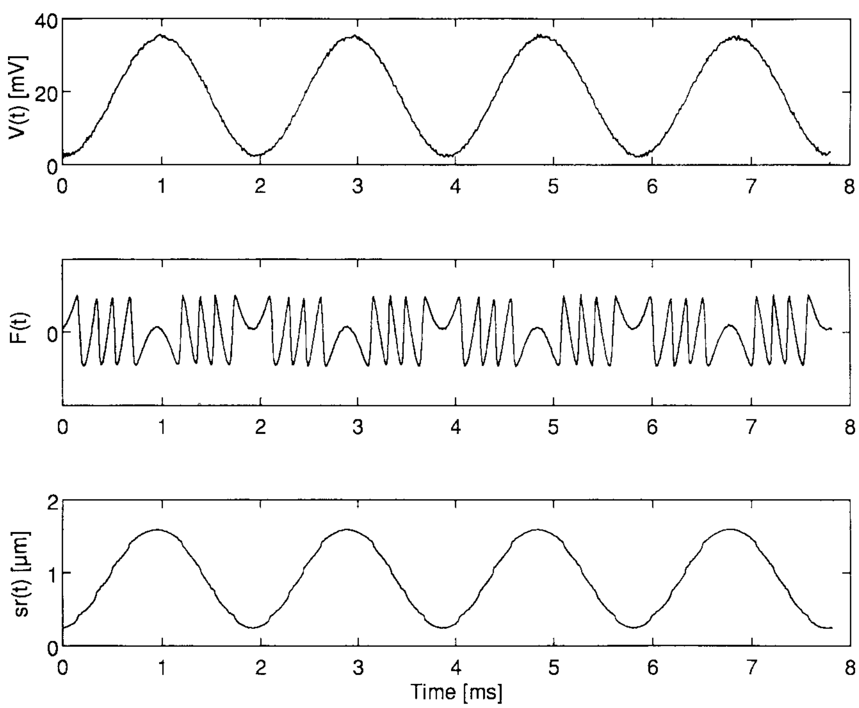

Fig. 5. Reconstruction of $s(t)$ from the experimental data on $F(t)$. (a) The loudspeaker driving signal $V(t)$ [mV]. (b) The interferometric signal $F(t)$. (c) The reconstructed signal $sr(t)$ [μm]. Sampling frequency ≈130 kHz.

applying the reconstruction routine with several trial values for α, the best estimate of the linewidth enhancement factor α was clearly identified as the one yielding the reconstructed waveform closest to an ideal ramp.

Fig. 5 reports the experimental result relative to the reconstruction of a sinusoidal signal $s(t)$ of peak-to-peak amplitude ≈1.2 μm, at $f = 49$ Hz, performed with $C = 0.81$ and $\alpha = 6$. The parameters are in agreement with the theoretical calculation that yields $\kappa \approx 10^{-3}$ for $C = 0.81$. The small ripple-like error on the reconstructed displacement $sr(t)$ is on the order of 50 nm peak to peak and can be attributed to residual vibrations producing misalignments in the setup, which might alter the collection efficiency of the backreflected light, and to electronic noise on the photodetected signal. For reconstruction of other $s(t)$ waveforms (not reported here for brevity), the residual errors were even smaller (e.g., <10 nm).

With regard to the suitable targets, using an almost ideal diffuser like white paper, we actually have to attenuate the backreflected signal, as shown in Fig. 1, in order to obtain $C < 1$. Still, it would be possible to work with a target not as good as white paper if we remove the attenuator. In the case of really poor reflectors, the illuminated area could be covered with a high-diffusing surface, usually obtained by deposition of glass particles, which could be attached onto the target without any particular alignment problem.

In conclusion, we have shown that the waveform reconstruction can be achieved in the weak-feedback regime of a laser-diode interferometer and that the analysis of the feedback regime allows one to determine all of the parameters needed in the calculations.

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