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AN EXACT CALCULATION OF TIME RESOLUTION WITH THE SCINTILLATION DETECTOR

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Starting from the results of a statistical analysis, we have computed exactly, in particular case, the time resolution of a model of the scintillation detector. The accuracy of well-known approximated expression has been checked.

In preceeding papers we have analyzed, by means of statistical methods, the amplitude and time resolution attainable with scintillation detectors[1-4]). A fairly general model has been assumed, which consists of cascaded branching processes and time of flights, both described by their random features. As a result, we have obtained the generating function $\Phi_0(s,t)$ associated with the probability $P(n,t)$ of collecting at the output the nth electron in the time interval $0-t$ [2-4]):

$$\Phi_0(s,t) = \textstyle\sum_{n=0}^{\infty} P(n,t)\, s^n$$
$$= M_0\left[f_0 \cdot M_1\{f_1 \cdots M_{\nu-1}[\Phi_{\nu-1}(s,t)]\}\right], \quad (1)$$

where

ν is the number of stages;

$f_i(t)$ is the probability density function of a time of flight t for an electron between electrodes i and $i+1$;

$M_i(s) = \Sigma_{n=0}^{\infty}\, p_i(n)\, s^n$ is the generating function associated with the probability $p_i(n)$ of obtaining n secondary electrons for one primary at the ith electrode;

$\Phi_{\nu-1}(s,t) = 1+(s-1)\int_0^t f_{\nu-1}(t)\,\mathrm{d}t$ is the generating function associated with the probability of collecting at the anode n electrons for one leaving the last dynode at time $t=0$.

The generating function Φ_0 given by eq. (1), or by slightly more generalized forms quoted elsewhere[3-5]), yields all the interesting informations on the random output pulse. For example, one can calculate the mean and the amplitude variance of the output current pulse, even in the case of single channel selection on the total pulse charge; experimental results have shown to be in good agreement with theory[5]). For what concerns the time resolution, an approximate expression has been widely used[1,2,6]) namely:

$$\varepsilon_t^2 = \varepsilon_n^2 \Big/ \left[\frac{\mathrm{d}}{\mathrm{d}t}\bar{n}(t)\right]^2. \quad (2)$$

This expression comes from the simple rule of deviding the amplitude variance ε_n^2 for the mean slope squared to obtain the time variance ε_t^2. This is a good approximation if the single random output pulses are not very different from the average one; moreover, the slope should be constant at least in a time interval of the order of ε_t. This condition is asymptotically satisfied when the number R of photoelectrons is large. However, the exact time variance can be computed, even if in a not very straight way, from the generating function $\Phi_0(s,t)$. In fact, the probability density function $W(n,t)$ of collecting the nth electron *just* at time t, is related to the $P(n,t)$ by the equation[2,7]):

$$W(n,t) = \textstyle\sum_{k=n}^{\infty} \dfrac{\partial}{\partial t} P(k,t),$$

and therefore a formal generating function $\Phi_W(s,t)$ associated with $W(n,t)$ can be calculated, in terms of $\Phi_0(s,t)$, by the following equation[7]):

$$\Phi_W(s,t) = \textstyle\sum_{n=0}^{\infty} W(n,t)\, s^n = \sum_{n=0}^{\infty} s^n \left[\sum_{k=n}^{\infty} \dfrac{\partial}{\partial t} P(k,t)\right]$$
$$= \frac{\partial}{\partial t}\, \frac{1-s\,\Phi_0(s,t)}{1-s}. \quad (3)$$

By putting $s = e^{j\sigma}$, we can attribute to $\Phi_W(\sigma,t)$ the meaning of discrete Fourier transform of the probability density function $W(n,t)$. In this way, numerical values of $W(n,t)$ can be obtained by means

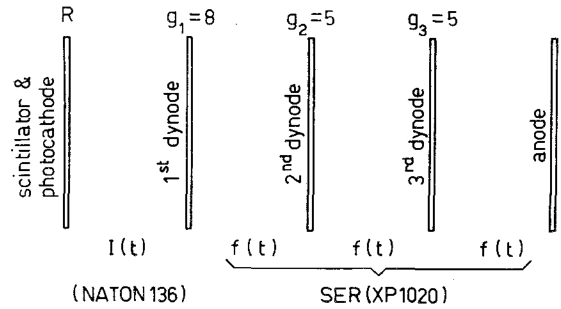

Fig. 1. The model assumed in the calculation of the time standard deviation.

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of fast Fourier inverse transform calculations on an electronic computer[8]).

The results obviously refer to the integral response case, since $W(n, t)$ is the probability density function of the time at which the nth electron is collected at the output. However, other response cases might be analyzed since it is possible to take account, through the generating function, of filters cascaded at the output*.

For a simple check of the accuracy of expression (2), we have considered the scintillation detector as sketched in fig. 1.

In the calculations the gain at the three dynodes has been assumed to follow a Polya distribution [see ref.[9])] with mean values $g_1 = 8$, $g_2 = g_3 = 5$ and a shape parameter $b = 0.05$, while for the scintillator-photocathode set the statistic is taken Poissonian with a mean value R.

We have taken into account experimental equivalent illumination $I(t)$ and SER measured[5]) for the Naton 136–XP 1020 assembly. That is, referring to expression(1), we have assumed $f_0(t) = I(t)$ and equal time of flight distributions $f_i(t) = f(t)$ so that:

$$f(t)\cdot f(t)\cdot f(t) = \mathrm{SER}.$$

The Fourier transform $\Phi_W(\sigma, t)$ has been calculated in 300 steps of 0.1 ns for a fixed σ, then repeating for σ from 0 to $1/(R\, g_3\, g_2\, g_1)$ in 1024 steps.

By multiplication for t and t^2 of the obtained $W(n, t)$, one gets the mean value and the variance of the crossing time at a threshold n. (Actually, for computer memory space saving, the moments were calculated on the $\Phi_W(s, t)$ and afterwards the inverse Fourier transform was performed.) Results are shown in fig. 2 (dotted lines) in which the standard deviation ε_t of threshold crossing time is plotted against the threshold C/R ($C/R = 1$ means a threshold at a level equal to the average output charge).

These "exact" results are to be compared to the approximate ones given by eq. (2), which are also plotted in fig. 2 for two distinct cases. Broken lines refer to the results of eq. (2) in which the exact expression of the amplitude variance has been inserted; full lines to the case of the rigid SER approximation[2-4]) to compute the amplitude variance, i.e. for

$$\varepsilon_n^2 = (1+\varepsilon_A^2)\, R I(t)\cdot F^2(t) \qquad (4)$$

* In fact, if $F(t)$ is the step response of such a filter, the generating function is still given by eq. (1), in which the following expression holds for $\Phi_{\nu-1}(s, t)$:

$$\Phi_{\nu-1}(s,t) = \int_0^\infty s^{F(t-\tau)} f_{\nu-1}(\tau)\,\mathrm{d}\tau\,.$$

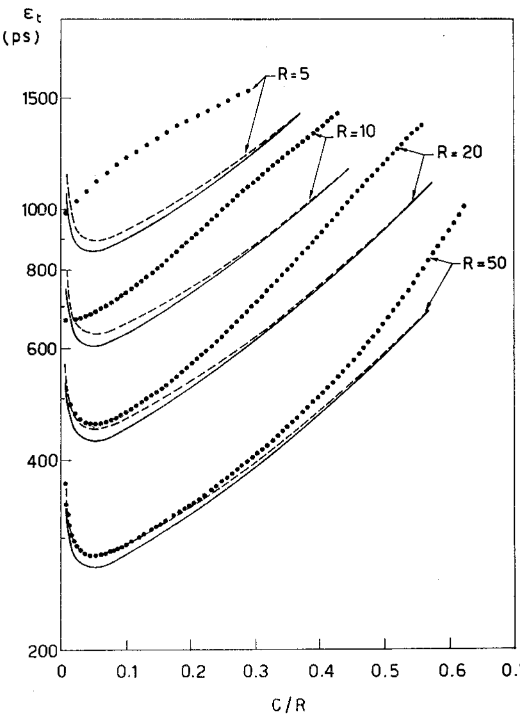

Fig. 2. The standard deviation ε_t vs fractional charge threshold C/R. Points represent the exact results calculated from the generating function by means of numerical methods; full lines represent the results obtained by eq. (2) in which the exact expression of amplitude variance was inserted; broken lines refer to the same case but using the approximate expression (4) for computing the amplitude variance.

(ε_A^2 being the relative variance of electron-multiplier gain, and $F(t)$ being the SER integral). A comparison of the results shows that for small values of the mean number of photoelectrons R, exact results may be as much as 30% larger than the approximate ones, whilst a good fit is obtained above $R = 20$ provided the threshold is not too high.

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