Responsivity and Noise of Self-Mixing Photodetection Schemes

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***Abstract*— Responsivity and noise properties of the self-mixing (SM) detection process are analyzed. Starting with the photodiode and SM laser scheme as a detector of weak optical echoes from remote targets, we find equivalence to a coherent homodyne detection of the returning signal. In particular, in the VIS-NIR wavelength range, echoes can be detected down to about −90 dB of relative amplitude, typically. Then we consider the laser-diode voltage self-mixing (LV-SM) detection and find it is also a coherent homodyne scheme, though noise performance is limited by shot-noise of the bias current and by the relatively small signal supplied as an output. Theoretical results are finally compared to recent experimental data, finding a substantial agreement and confirming that the LV-SM is an attractive alternative to conventional photo-detection, especially for THz waves and other spectral ranges where good low-noise detectors may be difficult to employ.**

***Index Terms*— Coherent photodetection, optical injection, photodetection schemes, self-mixing detection.**

I. Introduction

RECENTLY, the concept of *self-mixing* photodetection (also known as *injection* photodetection) has been demonstrated with T-waves (i.e., at THz frequency), in an imaging experiment carried out with a quantum cascade laser (QCL) [1].

Self-mixing effect (SME) is a well-known phenomenon belonging to the class of coupling (or injection) interactions. When a weak-amplitude field from a remote target is injected back into the laser, the interaction with the cavity field produces amplitude and frequency modulations, which carry information about both amplitude and phase of the injected signal [2].

Previously, the self-mixing scheme has been mainly used in instrumentation to sense the *phase* of the optical signal propagated on a path external to the laser, that is, to build a special configuration of interferometer [2], [3].

Yet, not just phase but also *amplitude* can be sensed [2], and a few examples have been already reported on echo sensing for the measurement of optical isolation-factor and return-loss [2]–[4] in the 2^{nd}–3^{rd} windows of fibers.

In all these measurements, the SME (self-mixing effect) is special in that it dispenses with adding any optical component in the measurement path [4]. Yet, measurement is still carried out by a normal photodiode sensing the field emitted by the laser. This is not the only possibility, however, because the SMI signal also appears across the laser diode junction.

In Fig. 1 the schematic of two most commonly used options for an SME setup are sketched: we can either have (i) the PD-SM configuration, where a photodiode PD, conveniently placed on the rear mirror of the laser diode senses a fraction of the output power, and (ii) the LV-SM that uses the voltage drop across the laser diode junction as the output signal readout of emitted power.

The PD-SM configuration, the most common in applications, can be regarded also as the special case of self-injection of a more general injection-detection scheme, in which a weak signal is injected into the laser cavity field instead of being superposed to the laser field as in a conventional coherent detection scheme [2]–[5].

The LV-SM configuration is very attractive because the laser is used also as a kind of detector of the returning signal, and no photodiode is necessary. Despite the relatively small dynamic resistance found in parallel to the junction will ultimately limit the achievable SNR, the configuration is viable for applications in which a photodiode is missing (e.g., VCSEL) or is available only with poor noise performance (e.g., T-waves).

In this paper we present for the first time, to the best of our knowledge, an analysis of the performances of both the PD-SM and LV-SM detector schemes, from the standpoint of quantity of response (or responsivity) and noise (or SNR). We find that both SM schemes belong to the class of conventional *homodyne* detectors, with the cavity field playing the role of local oscillator. The PD-SM has a larger signal and can approach the quantum limit in the measurement of small signals or high attenuation, whereas the LV-SM provides a smaller output signal and is limited by thermal noise in practice (at least for results obtained so far).

In the following Section, the analysis of SMI photodetection is carried out in the approximation of small perturbations and linear regime of operation for the quasi-static amplitude and phase of the cavity field. This approximation is justified because, on detecting minute signals, we stay in the *weak coupling* regime of interaction [2], and the constitutive equations (Lamb's or Lang and Kobayashi equations) are in the linear range, well below the *strong coupling* regime [2], characterized by hysteresis, bifurcation and chaos. As already reported in literature [2] and observed experimentally [4], the typical range of weak coupling regime typically extends from a minimum detectable signal about −90 dB to a maximum of about −30 dB, the incipient saturation due to the onset of external cavity modes [2].

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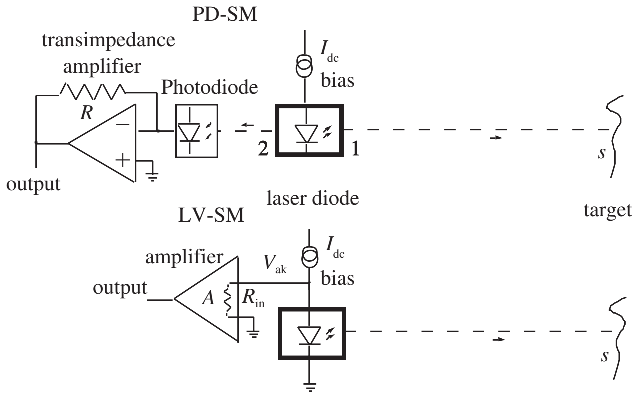

Fig. 1. Self-mixing configurations top, PD-SM using a photodiode (PD) to detect a fraction of the laser power, usually by means of the monitor photodiode mounted in the laser diode package and bottom, the LV-SM based on the readout of voltage across the pn junction of the laser diode.

II. Analysis of the PD-SM Scheme

The field ΔE returning from the remote target and added in the cavity after re-entering at the output mirror (point 1 in Fig. 1) is written in terms of the cavity field E_0 by taking into account the transmissions in the path to the remote target and back, and the round trip propagation in the cavity. We get:

$$\begin{aligned}\Delta E &= E_0 i t_1 \exp(2iks) A^{1/2} i t_1 \exp(\gamma L + 2ikL) r_2 \\ &= E_0(-t_1^2)\exp(2iks) A^{1/2}/r_1 \\ &= E_{in}\exp(\gamma L) i t_1/r_1 \end{aligned} \tag{1}$$

where: E_0 is the cavity field (taken at point 1), $t_1 t_2$ (and $r_1 r_2$) are the field transmissions (and reflections) at front/back mirrors; $T_1 = t_1^2$, $R_1 = r_1^2$, etc., in terms of powers; i is the 90° phase shift at each mirror transmission (respect to 0° of the reflection), $\exp(2iks)$ is the propagation term to target and back; $A^{1/2}$ is the field attenuation suffered in propagation, including the diffusion at the target surface; $\exp(\gamma L)$ is the active medium *field* gain (on the round trip $2L$), equal to $1/r_1 r_2$ because in the permanent regime of oscillation it shall be $|r_1 r_2 \exp(\gamma L + 2ikL)| = 1$; γ is the power gain per unit length of the active medium; $\exp(2ikL)$ is the phase of the round trip loop in the laser cavity [it is $|\exp(2ikL)| = 1$ in the permanent regime of oscillation], and in Eq.1 we have let $E_{in}\exp(\gamma L) = E_0\, \mathrm{it}_1 \exp(2iks) A^{1/2}$ to allow for the case of injected signal E_{in} physically different from a returned fraction of E_0.

The output field E_2 at the rear mirror can be expressed in terms of E_0 as:

$$E_2 = E_0\, r_1 \exp(\gamma L/2 + ikL) i t_2 = E_0 (r_1/r_2)^{1/2} i t_2. \tag{2}$$

Now, the extra field ΔE is the perturbation term we have to add into the Lamb [3] (or Lang-Kobayashi [4]) equation, written for simplicity in the constant-gain form as:

$$(d/dt)E = [\alpha - \beta E^2 - \Gamma]E + (c/2L) Re\{\Delta E\} \tag{3}$$

where: α is the linear (field) gain per unit time [$\alpha = \gamma c/2$ in term of power gain per unit length], β is the gain saturation coefficient; $\alpha - \beta E^2$ is the effective gain [approximation of $\alpha/(1+\beta E^2)$]; Γ is the field loss per unit time, $\Gamma = -(\ln r_1 r_2)$ (c/2L) when mirrors dominate; L is the laser cavity (optical) length ($n_{eff}L$ when the effective index is considered); and $\mathrm{Re}\{\Delta E\} = -A\ [E_0 t_1^2/r_1] \cos 2ks$ is the self-mixing or induced-modulation term.

Eq.3 can be solved in the weak perturbation regime (small ΔE) on letting $E = E_0 + \Delta E_{sm}$, for the perturbed field solution, where ΔE_{sm} is the self-mixing signal, and the result reads:

$$\Delta E_{sm} = \Delta E[(c/2L)/2(\alpha - \Gamma)] \cos 2ks. \tag{4}$$

Recalling that $\alpha = \gamma c/2$ and $\Gamma = -(\ln r_1 r_2)$ (c/2L) and rearranging terms, we can rewrite Eq.4 as

$$\Delta E_{sm} = \Delta E(2\gamma L + ln R_1 R_2)^{-1} \cos 2ks \tag{4'}$$

where ΔE is the field returning from the target (Eq.1), and positions $R_1 = r_1^2$, $R_2 = r_2^2$ have been used. The unperturbed field at the back mirror output is, from Eq.2:

$$E_{02} = (r_1/r_2)^{1/2}\ i\ t_2 E_0 \tag{5}$$

whereas the self-mixing contribution ΔE_{sm2} is obtained, using Eq.1 and 4', as:

$$\begin{aligned}\Delta E_{sm2} = -(r_1/r_2)^{1/2} i t_2\ E_0(t_1^2/r_1) \\ A^{1/2}(2\gamma L + ln R_1 R_2)^{-1} \cos 2ks.\end{aligned} \tag{5'}$$

About the strength of the optical echo from the remote target, let us introduce the ratio of powers $\rho = \Delta P_{sm2}/P_{02}$ associated with signals, $P_{02} = E_{02}^2$ and $\Delta P_{sm2} = \Delta E_{sm2}^2$. In the average limit $<\cos^2 2ks> = 1/2$, the ratio ρ is given by:

$$\rho = \Delta P_{sm2}/P_{02} = (t_1^2/r_1)^2 A\ 2^{-1}(2\gamma L + ln R_1 R_2)^{-2} \tag{6}$$

or, it is proportional to A.

Now, let us write the photo-detected current as $I_{ph} = \sigma <E^2>$, (σ being the spectral sensitivity [5]), the field at the rear mirror being $E = E_{02} + \Delta E_{sm2}$. Developing the square and assuming $\Delta E_{sm2} << E_{02}$ (for small signals) we get $<E^2> = <E_{02}^2> + 2<\Delta E_{sm2} E_{02}>$. The first term is simply the dc component $I_{ph0} = \sigma\ <E^2>$, the second is the useful SM signal. Expressing the total current as the sum of a dc term and a SM term i_{ph}, letting $P_0 = <E_{02}^2>$, and using Eqs.5 and 5', we get:

$$I_{ph0} = \sigma\ <E_{02}^2> = \sigma\sqrt{P_{02}} = \sigma(r_1/r_2) T_2 P_0, \tag{7}$$

$$\begin{aligned} i_{ph} &= 2\sigma\ <\Delta E_{sm2} E_{02}> = \sigma(\Delta P_{sm2} P_{02})^{1/2} = \\ &= I_{ph0}\left[2(t_1^2/r_1) A^{1/2} \cos 2ks (2\gamma L + ln R_1 R_2)^{-1}\right]. \end{aligned} \tag{7'}$$

Worth a remark, the dependence of i_{ph} from field attenuation from $A^{1/2}$ (in place of A of the power) indicates that the PD-SM process is a *coherent* detection. Indeed, if the large E_{02} and the small ΔE_{sm2} are mutually coherent, then they behave at the photodetector as the local oscillator and the small received signal, and the scheme is just the one we define as homodyne coherent detection, known as readily attaining the quantum noise limit [5].

Specifically, if all the extra noise (preamplifier input resistance noise $4kTBr_{in}$, background noise, i.e. $2eI_{bg}B$, excess noise, etc.) are small respect to the Johnson noise [5] of the feedback resistance R (see Fig. 1), that is $4kTB/R$, as

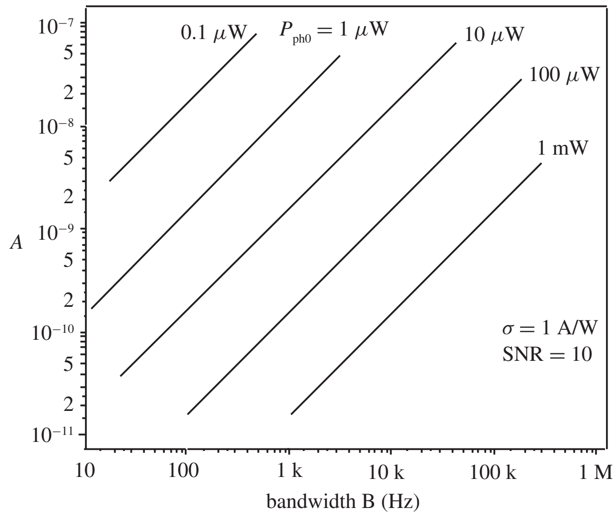

Fig. 2. Theoretical maximum attenuation A of optical power which is measurable by a PD-SM detection scheme, plotted as a function of bandwidth and with the used optical power P_{ph0} assumed as a parameter. It is assumed $\kappa^2_{PD} = 0.1$, $\sigma = 1$ A/W, and SNR = 10.

usually ensured by low-noise design of the preamplifier, then the quantum noise limit is attained, provided that $2eI_{ph0}B$ is dominant respect to $4kTB/R$, or for:

$$I_{ph0} > (2kT/e)/R. \tag{8}$$

This condition is not critical, as for 2kT/e = 25 mV and with a reasonable value of feedback resistance, e.g., $R = 10$ kΩ, we get a minimum local oscillator current $I_{ph0} > 2.5\mu$A. In the quantum noise regime, the SNR is given by [5]:

$$SNR = i^2_{ph}/<\Delta i^2_{ph}> = i^2_{ph}/[2eI_{ph0}B] \tag{9}$$

where e is the electron charge and B the bandwidth of observation. Using Eqs.7 and 7', and $<\cos 2ks^2> = 1/2$ gives:

$$SNR = 2\ I_{ph0}(t_1^2/r_1)^2(A/eB)(2\gamma L + ln R_1 R_2)^{-2}$$
$$= \kappa^2_{\rm PD} I_{ph0} A/eB \tag{9'}$$

where we let $\kappa^2_{\rm PD} = 2(t_1^2/r_1)^2/(2\gamma\,{\rm L}+{\rm lnR_1R_2})^2$.

With reasonable values of the parameters for a semiconductor diode laser, i.e., $t_1^2 = 0.3$, $r_1 = 0.55$, $2\gamma L = 2.7$ to 4, ln $R_1R_2 = -0.6$, so that $2\gamma L+$ In $R_1R_2 = 2.1-3.4$, we get for $\kappa^2_{\rm PD}$ in Eq.9 a typical value 0.05 to 0.14.

Now, we can rewrite Eq.9' in term of the maximum (power) attenuation A we can measure in the path to the target and back as:

$$A = SNR\ eB/(I_{ph0}\ \kappa^2_{PD}). \tag{10}$$

With a typical SNR = 10, taking $\kappa^2_{\rm PD} = 0.1$ and a bandwidth of 1 kHz, Eq.10 supplies for $I_{\rm ph0} = 10\ \mu A$ an attenuation theoretically measurable of $A = 1.6 \times 10^{-9}$ or −88 dB.

Actually, experimental values approaching such sensitivity have been reported [4] in the measurement of return echoes from a laser diode feeding fibers and optical isolators, in the 3rd window. Saturation level for the detected signal were found in the range −18 to −35 dB depending on the diode laser type and on target distance [4]. Thus, assuming a sensitivity of −88 dB, the dynamic range of SMI attenuation measurement may go up to 70 dB.

In Fig. 2 we plot the diagram of measurable attenuation versus measurement bandwidth B, as per Eq.10.

We may compare the above results on SNR with that can be obtained by *direct* detection of the weak signal $i_{\rm ph0}$. After being attenuated along propagation to the target and back, $i_{\rm ph0}$ may be down to nA level or less. Using direct detection with the same feedback resistance R we will obtain a $\rm SNR_{direct}$:

$$SNR_{direct} = i^2_{ph}/<\Delta i^2_{ph}> = i^2_{ph}/(2eI_{ph0}B + 4kTB/R)$$
$$\cong i^2_{ph0}/(4kTB/R) \quad (\text{for } I_{ph0} < 2kT/eR). \tag{9A}$$

Eq.9 A tells us that, on direct detection of the weak echo, we likely work in the well-known *thermal regime* [5] of detection.

Finally, along with the detected current (or quantity of response) $i_{\rm out}$, we may consider the *responsivity* $R = i_{\rm out}/P_{\rm in}$ of the PD-SM scheme, and evaluate it in the most general case of a small signal $\Delta E_{\rm in}$ arriving at the front mirror, and being detected by a current $i_{\rm out}$ at the PD placed on the rear mirror.

Provided the signal is the same frequency and coherent with the in-cavity field, taking account of Eqs.1, 5 and 7, and that the input power is $P_{\rm in} = <\Delta E^2_{\rm in}> = E_0^2\ T_1^2 A^2/R_1$, we find:

$$i_{out} = \sigma 2(P_{in}P_0)^{1/2}(2\gamma L + ln R_1 R_2)^{-1}\cos 2ks. \tag{11}$$

Also Eq.11 tells us that the PD-SM scheme is a *coherent* detector, because it supplies an output proportional to *square root* of input power, and is also a *homodyne* scheme because it is phase-sensitive through the term cos 2ks [5]. The in-cavity power P_0 plays the role of the local oscillator of the detection process.

The only remaining term is $(2\gamma L + \ln\ R_1R_2)^{-1}$, with a typical value of 0.3−0.5 summarizing the self-mixing process efficiency. Additionally, from Eq.11, the responsivity $R = i_{\rm out}/P_{\rm in}$ is written as:

$$R = \sigma 2(P_0/P_{in})^{1/2}(2\gamma L + ln R_1 R_2)^{-1}\cos 2ks \tag{11'}$$

and we can see that the signal is amplified by a *coherent gain* factor $G_{\rm coh} = (P_0/P_{\rm in})^{1/2}$, exactly like in a normal coherent photodetection process [5].

III. Analysis of the LV-SM Scheme

In the LV-SM scheme, the laser diode junction behaves as a sort of a detector of the returning signal, and thus is very attractive for those applications where adding a photodetector is undesirable.

Let us analyse LV-SM scheme, using the Lang and Kobayashi (L-K) equations of field amplitude and carrier concentration [6]:

$$(d/dt)E = 1/2[G(N - N_t) - 1/\tau_p]E + Re[\Delta E] \tag{12}$$
$$(d/dt)N = J/ed - N/\tau_r - G(N - N_t)E^2 \tag{12'}$$

where symbols are used with the usual meaning, see e.g. [2], [3]: in particular, N_t is the carrier concentration at transparency, $\tau_{\rm r}$ and τ_p are the carrier lifetime and the photon lifetime, J is the current density, and d the junction thickness.

Letting E and N vary of small quantities around the quiescent value in Eq.12, yields the following relation between ΔE and ΔN:

$$\Delta N(1/\tau_r + GE^2) = -G(N - N_t)2Re\{E\Delta E\}. \quad (13)$$

Additionally, at equilibrium and in unperturbed conditions ($\Delta E = 0$), we get from Eqs.12 and 12':

$$G(N - N_t) = 1/\tau_p, \quad J_t/ed = N/\tau_r,$$
$$E^2/\tau_p = (J - J_t)/ed \quad (14)$$

where J_t is the current density at transparency and in the last equation we have neglected $(N-N_t)/\tau_r$ because much smaller than (J-J_t)/ed at currents densities starting from J_t.

Last, we use Boltzmann's law of carrier concentration produced by a bias voltage V_{ak}:

$$N = N_0 \exp eV_{ak}/2kT \quad (15)$$

where the factor 2 multiplying kT is because at high injection rates the generation-recombination contribution dominates [7], and N_0 is the carrier concentration at equilibrium ($V_{ak} = 0$). Upon differentiating and rearranging, we get

$$\Delta V_{ak} = (2kT/e)\Delta N/N. \quad (16)$$

Inserting Eq.13 in Eq.16, and using Eqs.14 we get:

$$\Delta V_{ak} = -(2kT/e)$$
$$2Re\{E\Delta E\}/[N_t G_t(\tau_p^2/ed)(J - J_t)]. \quad (17)$$

In Eq.17 we have taken the safe approximations $N \sim N_t$, and $G \sim G_t$, valid above threshold where N and G are pinned close to their threshold values [7].

Using Eq.4' to take account of the term actually interacting with the cavity field in the SM process, we get:

$$\Delta V_{ak} = -(2kT/e)2Re\{E_0\Delta E_{sm}\}$$
$$\times\left[N_t G_t(\tau_p^2/ed)(J - J_t)\right]^{-1}$$
$$\times\left[(2\gamma L + lnR_1R_2)\right]^{-1}. \quad (17')$$

Expressing the field internal to the laser as the square root of power, $E = \sqrt{P_0}$, and $\Delta E = \sqrt{P_{sm}}$ for the signal power, and recalling that the real part Re{..} gives a phase term cosϕ = cos 2ks, we can rewrite Eq.17 as:

$$\Delta V_{ak} \approx -(2kT/e)2(P_0P_{in})^{1/2}\ \chi(J - J_t)^{-1}\cos 2ks \quad (18)$$

where χ $[G_tN_t(\tau_p^2/ed)\ (2\gamma L+\ln R_1R_2)]^{-1}$.

About responsivity R, dividing by $<\Delta E^2> = P_{in}$ we have:

$$R = -(2kT/e)(P_0/P_{in})^{1/2}\ \chi(J - J_t)^{-1}\cos 2ks. \quad (18')$$

From Eqs.18 and 18' we can recognize that also the LV-SM scheme has the coherent homodyne dependence from field amplitude $\Delta E = \sqrt{\Delta P}$ and from the cosine of the phase term 2ks, and the coherent gain $G_{coh} = (P_0/P_{in})^{1/2}$.

About the quadratic effect in the LV-SM scheme, responsible for the beating of E_0 and E_{in} and counterpart of the photodiode quadratic response in the PD-SM, this is the stimulated emission term E^2 (Eq.12'), directly affecting the junction voltage through the carrier concentration N and Boltzmann's law (Eq.15).

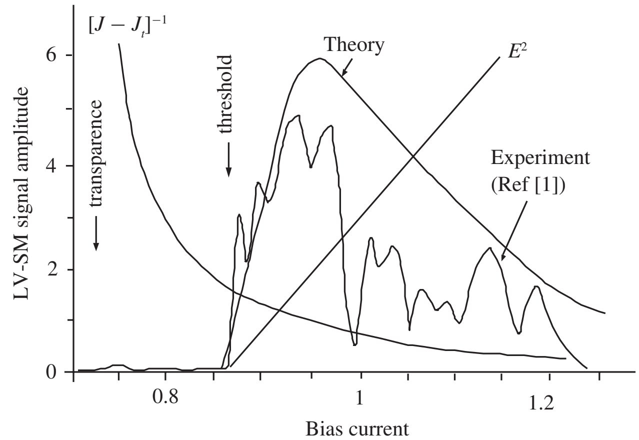

Fig. 3. Response of a LV-SM scheme of detection, theory, calculated from (19), zigzag curve, result of measurement from [1].

The scale factor of voltage in Eq.18 depends on the inverse of current density excess J$-J_t$ respect to transparency. Using the typical data and parameter values commonly used for a laser diode (as in Ref.[2]), we obtain the diagram of ΔV_{ak} versus bias current I_{bias} plotted in Fig. 3. There is an optimum bias J to obtain the largest response, and in practice the maximum is located close to threshold of laser oscillations, at J = 1.1 to 1.5 J_{thr} typically.

We also plot in Fig. 3 the experimental results measured in Ref.[1] for a quantum cascade THz laser. Despite the rather large fluctuations of the data, the trend of signals is clearly in good qualitative agreement.

Now, let's turn to the attenuation which is measurable by the LV-SM scheme. By developing P_{in} in Eq.18, we get for the amplitude of the voltage signal:

$$\Delta V_{ak} = -(2kT/e)2^{-1/2}PA^{1/2}(J - J_t)^{-1}\chi. \quad (19)$$

Recalling that $\chi = [G_tN_t(\tau_p^2/ed)\ (2\gamma L+\ln R_1R_2)]^{-1}$, and from Eq.14 that J-$J_t$ = ed E^2/τ_p = ed P/τ_p, Eq.19 becomes:

$$\Delta V_{ak} = -(2kT/e)2^{-1/2}A^{1/2}(G_tN_t\tau_p)^{-1}$$
$$\times(2\gamma L + lnR_1R_2)^{-1}. \quad (19')$$

The last term (see Eq.9') is evaluated in the range $(2\gamma L+\ln R_1R_2)^{-1}$ = 0.3 to 0.5, whereas the new multiplicative term $G_tN_t\tau_p$ is found to be $\sim$ 2.94 with the currently assumed values of parameters [2]. So we have $\Delta V_{ak} \approx -(2kT/e)\ A^{1/2}$ within a multiplicative factor of the order of unity.

As 2kT/e = 50 mV at ambient temperature (T = 300 K) it is clear that the attenuation measurement will be limited by the small voltage available. For example, to achieve -60 dB or 10^{-6} attenuation we shall be able to measure a 50-nV voltage difference across the junction, a value at the boarder line of low-noise preamplifier performance [5]. Even more, at the temperature of operation of a quantum cascade THz laser [1] of 186 K, the scale factor 2kT/e is only 31 mV and the low-noise requisite is crucial to LV-SM operation.

To evaluate noise of the LV-SM scheme, we can still use the Lang-Kobayashi equations, in which a Langevin term is introduced in the field equation to account for fluctuations [4]. The calculation is lengthy, however, and basically yields the same result as obtained by just considering the thermal and shot noises of the diode junction, the approach we prefer to develop below.

Noise of a normal diode junction is made up of two contributions [8]: (i) Johnson (or thermal) noise associated with the finite conductance dI/dV_{ak} found across the junction, and (ii) shot noise of the reverse saturation current I_{rev}. Additionally, in a laser diode subjected to self-mixing, an extra direct current $+I_{SM}$ is generated, to which a shot noise fluctuation shall be attributed, similar to a detected current of a photodiode, even though the current is now flowing the opposite direction.

Writing Shockley's equation for the self-mixing laser diode junction as:

$$I = I_{rev}[\exp(eV_{ak}/2kT) - 1] + I_{SM} \tag{20}$$

where I is the total current, I_{rev} is the reverse saturation current, I_{SM} is the self-mixing current, and we have taken the ideality factor $\eta = 2$ for direct bias (when the generation/recombination mechanism is dominant [9]).

At the working point $I = I_{bias}$, the differential resistance $r_{ak} = dV_{ak}/dI_{bias}$ across the junction is found from Eq.20 (neglecting I_{rev}, $i_{SM} << I_{bias}$) as:

$$r_{ak} = (2kT/e)/I_{bias} \tag{21}$$

Associated to r_{ak} we find a Johnson-noise voltage fluctuation, in series with the useful signal voltage, whose quadratic mean value is [5]:

$$v_n^2 = 4kTBr_{ak} = 2eB(2kT/e)^2/I_{bias}. \tag{22}$$

The shot noises associated with I_{rev} and I_{SM} are given by

$$i_{n,rev}^2 = 2eI_{rev}B, \text{ and } i_{n,SM}^2 = 2eI_{SM}B, \tag{22'}$$

and applying Thevenin's theorem, the total voltage noise is accordingly:

$$\begin{aligned} v_n^2 &= \left[2eB(2kT/e)^2/I_{bias}\right] + 2eBr_{ak}^2(I_{rev} + I_{SM}) \\ &= 2eB(2kT/e)^2\left[(1/I_{bias}) + (I_{rev} + I_{SM})/I_{bias}^2\right]. \end{aligned} \tag{23}$$

At reasonably high bias, the term in square brackets becomes $\approx 1/I_{bias}$ and we can write the junction noise as

$$v_n^2 \approx 2eB(2kT/e)^2/I_{bias}. \tag{23'}$$

To this contribution, intrinsic to the laser device, we shall add the extrinsic noise introduced by the preamplifier that handles the junction voltage. As we will read the junction voltage with an amplifier whose input resistance is much larger than the source resistance, $R_{in} >> r_{ak}$, the dominant noise we shall consider is the input voltage noise v_{AMP}^2, while the effect of the input current noise i_{AMP}^2 is negligible, in view of the relatively low value of R_{in} [5]. Assuming $R_{in} >> r_{ak}$, it is easy to find that the total noise is:

$$\begin{aligned} v_n^2 &= 2eB(2kT/e)^2/I_{bias} + v_{AMP}^2 \\ &= F2eB(2kT/e)^2/I_{bias} \end{aligned} \tag{23''}$$

where, in the last equation, we have introduced the excess noise factor $F = 1 + v_{AMP}^2/(4kTBr_{ak})$ due to the amplifier non-ideality.

Now let's evaluate the SNR of the power attenuation measurement. Using Eqs.18, 14 and with $<\cos^2 2ks> = 1/2$ we get:

$$SNR = 2(P_0 P_{in})(J - J_t)^{-2}\chi^2/[2eBF/I_{bias}]$$

and using Eq.14 for $J\text{-}J_t = ed\, P_0/\tau_p$

$$\begin{aligned} SNR = 2(P_{in}/P_0)I_{bias}/\big(2eB \\ F[G_t N_t \tau_p(2\gamma L + \ln R_1 R_2)]^2\big). \end{aligned} \tag{24}$$

About attenuation, noting that $P_{in}/P_0 = A$ and letting $\kappa_{LV}^2 = [G_t N_t \tau_p(2\gamma L + \ln R_1 R_2)]^2$e can write

$$SNR = A\kappa_{LV}^2(I_{bias}/2eB)F. \tag{25}$$

This expression is formally coincident to Eq.9' of the PD-SM. The differences are that: (i) the current I_{ph0} is now replaced by the larger bias current I_{bias} (typically, hundreds mA instead of μA's) and (ii) the multiplicative factor κ_{LV}^2 is now much smaller than the corresponding κ_{PD}^2. Using the currently accepted values for the parameters [2], we have now a typical value of $\kappa_{LV}^2 = 0.005 \ldots 0.014$. This is considerably smaller than the value of κ_{PD}^2.

Solving Eq.25 for A gives:

$$A = SNReBF/[I_{bias}\kappa_{LV}^2]. \tag{26}$$

The diagram of Fig. 3 can again be used, after appropriate scaling of the parameter (now $\kappa_{LV}^2 I_{bias}$ in place of $\kappa_{PD}^2 I_{ph0}$).

From the experimental data of Ref.[1], we get for the LV-SM detection of THz waves carried out with: $P_0 = 0.32$ mW, $\kappa_{LV}^2 = 5\times10^{-3}$, and bandwidth is estimated B = 300 Hz. From this data, and F = 1, Eq.26 would supply (at SNR = 10) a measurable attenuation of −90 dB. In contrast, only about 4nW or −48 dB of attenuation is obtained experimentally [1].

The discrepancy is reconciled taking into account the amplifier noise. Indeed, a bias current of 900 mA corresponds to a junction dynamical resistance $r_{ak} = 30$ mV/900 mA = 33 mΩ, and the associated voltage Johnson noise is a bare $v_n = 0.018$ nV/$\sqrt{}$Hz. As commercial amplifiers may have an input voltage noise of 2 nV/$\sqrt{}$Hz typically, an excess factor $F = (2/0.018)^2 = 1.2 \times 10^4$ or about 41 dB is obtained, and therefore the observed result is in reasonable agreement with theory.

IV. Conclusion

We have shown that self-mixing schemes perform homodyne coherent detection of the weak incoming signal, provided the signal is coherent respect to the in-cavity field. The external photodiode PD-SM scheme has a very good sensitivity and can tolerate typically −90 dB of attenuation still performing a measurement with a reasonable SNR. The LV-SM scheme also behaves as a homodyne detector, but because of the smaller amplitude of response and the small scale factor of junction voltage, it provides less performance (up to ∼−50 dB has been observed).

Yet the LV-SM is interesting because no photodiode is needed, and it is promising for covering new wavelength spectral ranges in several applications. The upper limit of detectable signal amplitude, as measured in the 3rd window [4] is up to −20…30 dB and indicates that a wide dynamic range (up to 70 dB) is potentially obtained in SMI-based measurement.

REFERENCES

[1] P. Dean, Y. L. Lim, A. Valavanis, R. Kleise, M. Nikolic, S. P. Khanna, M. Lachab, D. Indjin, Z. Ikonic, P. Harrison, A. D. Rakic, E. H. Lindfield, and G. Davies, "Terahertz imaging through self-mixing in a quantum cascade laser," *Opt. Lett.*, vol. 36, no. 13, pp. 2587–2589, 2011.

[2] S. Donati, "Developing self-mixing interferometry for instrumentation and measurements," *Laser Photon. Rev.*, vol. 5, 2011, DOI: 10.1002/lpor.201100002.

[3] S. Donati, *Electrooptical Instrumentation*. Upper Saddle River, NJ: Prentice-Hall, 2004.

[4] S. Donati and M. Sorel, "A phase-modulated feedback method for testing optical isolators assembled into the laser diode package," *IEEE Photon. Technol. Lett.*, vol. 8, no. 3, pp. 405–407, Mar. 1996.

[5] S. Donati, *Photodetectors*. Upper Saddle River, NJ: Prentice-Hall, 1999.

[6] E. M. Randone and S. Donati, "Self-mixing interferometer: Analysis of the output signals," *Opt. Exp.*, vol. 14, no. 20, pp. 9188–9196, Oct. 2006.

[7] Y. Suematsu and A. R. Adams, *Handbook of Semiconductor Lasers and Photonics Integrated Circuits*. London, U.K.: Chapman & Hall, 1994, ch. 5.

[8] M. J. Buckingham, *Noise in Electronic Devices and Systems*. Chichester, U.K.: Ellis Horwood, 1983.

[9] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. New York: Wiley, 1981.

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