

Fig.1 Schematic of a self-mixing interferometer (SMI) using a laser diode to sense the phase shift $2ks$ of field returning from an external reflector (or diffuser) at a distance s .

measure, concurrent to the displacement signal ϕ , the angle α between \underline{k} and \underline{n} , and to resolve it into two orthogonal components, namely tilt θ and yaw ψ .

The paper is organized as follows: in Section II, we develop the theory of both ϕ -dependent modulations and A -dependent cavity losses, relative to the power oscillating in the cavity and measured by the rear photodiode (Fig.1).

In Section III we offer some experimental evidence confirming the theory. Then, we turn to consider a reflective target and its dependence of A on alignment angles α_{tilt} and α_{yaw} , and in Section IV we propose the simultaneous measurement of s , α_{tilt} and α_{yaw} , by properly allocating frequency bands to the three signals, in the single output channel of the monitor photodiode output.

II. THEORY

For sake of clarity, let's write explicitly the well-known Lang-Kobayashi (L-K) equations [4,20] for an SMI as:

$$dE/dt = \frac{1}{2} [G_N(N-N_0) - 1/\tau_p] E + (K/\tau_{\text{in}}) E(t-\tau_{\text{ext}}) \times \cos [\omega_0\tau_{\text{ext}} + \phi(t) - \phi(t-\tau_{\text{ext}})] \quad (2a)$$

$$d\phi/dt = \frac{1}{2} \alpha \{G_N(N-N_{\text{th}}) - 1/\tau_p\} + (K/\tau_{\text{in}}) E(t-\tau_{\text{ext}})/E(t) \times \sin [\omega_0\tau_{\text{ext}} + \phi(t) - \phi(t-\tau_{\text{ext}})] \quad (2b)$$

$$dN/dt = J\eta/ed - N/\tau_r - G_N(N-N_0) E^2(t) \quad (2c)$$

where (with values currently used for a FP laser diode):

G_N = modal gain = $8.1 \cdot 10^{-13} \text{ m}^3 \text{ s}^{-1}$,

K = fraction of field coupled into the oscillating mode

N = carrier concentration (m^{-3}),

N_0 at inversion = $1.2 \cdot 10^{24} \text{ m}^{-3}$, □

N_{th} at threshold = $2.5 \cdot 10^{24} \text{ m}^{-3}$, □

τ_{ext} = $2s/c$ = round trip time of external cavity

s = distance to external cavity reflector, $\phi = 2ks$ external optical phaseshift

τ_{in} = $2n_{\text{in}}L_{\text{in}}/c$ = round trip time of laser cavity = 5 ps,

τ_p = photon lifetime in laser cavity = 2 ps,

τ_r = carrier lifetime = 5 ns,

α = linewidth enhancement factor, (typically 3..6)

$\omega_0 = k_0/c = 2\pi\lambda_0/c$ = unperturbed frequency;

$J\eta$ = pumping current density, with

η = internal pumping efficiency

d = active layer thickness = typ. 0.25 μm

V = active volume = typ. 80 (μm)³

For slowly varying distance such that $2k[ds(t)/dt]\tau_{\text{ext}} \ll 1$, the phase difference $\phi(t) - \phi(t-\tau_{\text{ext}})$ in Eqs.2a-2b is negligible, and then τ_{ext} can be dropped in $E(t-\tau_{\text{ext}})$: we take these assumptions for granted in the following.

Factor K in Eqs.2a-2b can be expressed in terms of cavity mirrors reflectivity as:

$$K = \eta_s (1-r_2^2)(r_3/r_2) \quad (3)$$

where η_s is the mode superposition factor, and $r_2 = \sqrt{R_2}$ and $r_3 = \sqrt{R_3}$ are the field reflectivities of output mirror and target ($r_2^2 \approx 0.35$ for a typical semiconductor cleaved facet).

Now, let's start considering the SMI ϕ -dependent injection and calculate the amplitude of the resulting SMI signal $P(\phi)$. Solving Eq.2b for small K , and after some rearrangements we arrive to the constitutive self-mixing frequency equation, counterpart of Adler's locking equation [4], written in the form [20-22]:

$$\omega t = \omega_0 t - C \sin(\omega t + \text{atan } \alpha) \quad (4)$$

In Eq.4 we have introduced the feedback factor C :

$$C = (1 + \alpha^2)^{1/2} K \tau_{\text{ext}} / \tau_{\text{in}} \quad (4a)$$

The C factor determines [4,19-23] the regime of feedback, from *weak feedback* ($C \ll 1$) when the waveform $F(\cdot)$ is almost sinusoidal and the amplitude of modulation m is proportional to C , to *moderate feedback* ($C \approx 1$) when mode-hopping starts to show up and $F(\cdot)$ exhibits switchings while the amplitude doesn't increase any more, to *strong feedback* ($C \gg 1$) when $F(\cdot)$ becomes chaotic with multiple switching per period and the amplitude (peak-to-peak swing) is saturated.

In the case of weak feedback, we can solve the L-K equations (Eqs.2a-2b), on letting $K \ll 1$ and calculating field amplitude and phase in the small signal regime as $E = E_0 + \Delta E$, and $\omega = d\phi/dt = \omega_0 + \Delta\omega$, where E_0 and ω_0 are the quiescent unperturbed values obtained for $dE/dt=0$, $d\phi/dt=0$ and $K=0$. On doing so, we find for small C :

$$\Delta E = [E_0 \kappa C \cos \omega_0 \tau_{\text{ext}}] / [1 + \kappa C \cos \omega_0 \tau_{\text{ext}}], \quad (5a)$$

where $\kappa = (1 + \alpha^2)^{-1/2} \tau_p / \tau_{\text{ext}}$ and

$$\Delta \nu = -(C/\tau_{\text{ext}}) \sin(\omega_0 \tau_{\text{ext}} + \text{atan } \alpha) / (1 + C \cos \omega_0 \tau_{\text{ext}}) \quad (7a)$$

Also, for $C \ll 1$ we can approximate ΔE and $\Delta \nu$ to:

$$\Delta E = E_0 \kappa C \cos \omega_0 \tau_{\text{ext}}, \quad (5b)$$

$$\Delta \nu = -(C/\tau_{\text{ext}}) \sin(\omega_0 \tau_{\text{ext}} + \text{atan } \alpha) \quad (7b)$$

Thus, at weak feedback ($C \ll 1$) both AM and FM modulations are sinusoidal and phase-shifted by the difference $\arg(\Delta E) - \arg(\Delta \nu) = \zeta$ of the sine and cosine function, or by $\zeta = \pi/2 - \text{atan } \alpha$, from Eqs.5b and 7b.

About the output signal found at the photodiode, the photogenerated current $I_{\text{ph}} = \sigma P$ is proportional to power P through the spectral sensitivity σ of the photodiode, and so we can limit ourselves to consider power and in particular the signal-dependent part of it, $\Delta P = m P_0$ (as in Eq.1).

Writing $P_0 + \Delta P = \langle [E_0 + \Delta E]^2 \rangle$ and using Eq.5b we get, for weak feedback ($C \ll 1$):

$$\Delta P = 2 E_0^2 \kappa C \frac{1}{2} = P_0 K \tau_p / \tau_{\text{in}} \quad (8a)$$

whereas for strong feedback (when $C \gg 1$) Eqs.5 and 7 are no more valid and we shall solve numerically the L-K equations (Eq.2a-2b). Doing so, we find [14,15] that the signal saturates at the value corresponding to $C \approx (1 + \alpha^2)^{1/2}$ in Eq.8a, so that:

$$\Delta P \approx E_0^2 \kappa(1+\alpha^2)^{1/2} = P_0 \tau_p/\tau_{\text{ext}} \quad (8b)$$

In conclusion, the SMI ϕ -dependent signal is governed by the C parameter both in shape (the F function) and amplitude (the modulation index m): at weak feedback F is about a sine wave and $m \approx \kappa C$, whereas at moderate and strong feedback function F is a distorted sine (and then chaotic) waveform and the modulation index is $m \approx \tau_p/\tau_{\text{ext}} \approx \text{const}$.

For a semiconductor laser, with the target placed at a distance $s=60\text{mm}$, we get from Eq.8b a saturation at $m \approx 0.01$, independent from external mirror reflectivity, in good agreement with experimental observation.

Second, let's now consider the A -dependent cavity losses induced by the external reflector.

The reflectivity r_3 of the external target of course affects also the factor K of the coherent injected term (Eq.2a) already taken into account, but also the cavity losses summarized by the photon lifetime in the laser cavity τ_p . With a simple loop gain argument, by noting that the rate of decay of power inside the cavity is given by the inverse of the loss per transit time, it easy to see that for an unperturbed cavity formed by mirrors with (power) reflectivity $R_1=r_1^2$ and $R_2=r_2^2$, and with $R_3=0$, the unperturbed photon lifetime τ_{p0} is found as:

$$\tau_{p0} = \tau_{\text{in}} [-\ln(R_1 R_2)]^{-1} \quad (9a)$$

whereas, on adding the third mirror with (power) reflectivity R_3 , we have also the external cavity formed by R_2 and R_3 and the (perturbed) photon lifetime is increased to:

$$\tau_p \approx \tau_{\text{in}} \{-\ln R_1 [R_2 + (1-R_2)^2 R_3 / (1-R_3 R_2)]\}^{-1} \quad (9b)$$

or, the effective reflectivity R_2^* of the output mirror is changed to:

$$R_2^* = R_2 + (1-R_2)^2 R_3 / (1-R_3 R_2), \quad (9c)$$

where term $(1-R_2)^2 R_3$ is the contribution returning from target R_3 after a double transmission through R_2 , and $(1-R_3 R_2)$ accounts for multiple reflections between R_3 and R_2 .

Photon lifetime affects two quantities readily measured from the static P-I characteristics of the laser: (i) the threshold current J_{th} and (ii) the slope efficiency $S=P/(J-J_{\text{th}})$.

To find their dependence, we solve Eq.2c for E^2 in the stationary conditions $dN/dt=0$ and above threshold, where the term $(N-N_{\text{th}})/\tau_r$ can be assumed much smaller than $(J-J_{\text{th}})/ed$ (a condition called pinning of N). Noting also, from Eq.2a, that $G(N-N_0)=1/\tau_p$, we get:

$$E^2/\tau_p = (J-J_{\text{th}}) \eta/ed - (N-N_{\text{th}})/\tau_r \approx (J-J_{\text{th}}) \eta/ed \quad (10a)$$

and introducing $P=E^2$ in Eq.10a, the slope efficiency S is found to be directly proportional to photon lifetime τ_p :

$$S = P/(J-J_{\text{th}}) = \tau_p \eta/ed \quad (10b)$$

If we start from $R_3=0$ ($\tau_p=\tau_{p0}$) at which $S_0=\tau_{p0}\eta/ed$ and then let R_3 increase so that $\tau_p>\tau_{p0}$, the slope S will vary as $\Delta S=(\tau_p-\tau_{p0})\eta/ed=\Delta\tau_p\eta/ed$, and then the relative slope $\Delta S/S_0$ is:

$$\Delta S/S_0 = \Delta\tau_p/\tau_{p0} \quad (10c)$$

About threshold J_{th} , from Eqs.2c we have $J_0/ed=N_0/\tau_r$ and therefore also $J_{\text{th}}/ed=N_{\text{th}}/\tau_r$. Moreover, from Eq.1c we get $N_{\text{th}}=N_0+1/G\tau_p$ and $J_{\text{th}}/J_0=N_{\text{th}}/N_0=1+1/GN_0\tau_p$.

Now, starting from a value $J_{\text{th}0}$ of unperturbed threshold (i.e., for $R_3=0$) at which the photon lifetime is τ_{p0} , and going to a perturbed condition ($R_3 \neq 0$) with a new value τ_p , we can compute the threshold variation $\Delta T=J_{\text{th}}-J_{\text{th}0}$, and then the relative threshold variation $\Delta T/T_0=(J_{\text{th}}-J_{\text{th}0})/J_{\text{th}0}$ as a function of $\Delta\tau_p=\tau_p-\tau_{p0}$. The result reads:

$$\begin{aligned} \Delta T/T_0 &= (J_{\text{th}}-J_{\text{th}0})/J_{\text{th}0} = [1/GN_0\tau_p - 1/GN_0\tau_{p0}] / (1+1/GN_0\tau_{p0}) \\ &= (\tau_{p0}-\tau_p) / \{\tau_p[1+GN_0\tau_{p0}]\} = -(\Delta\tau_p/\tau_p) / [1+GN_0\tau_{p0}] \quad (11) \end{aligned}$$

$$\approx -(\Delta\tau_p/\tau_{p0}) / [1+GN_0\tau_{p0}] \quad \text{for } \tau_p \approx \tau_{p0} \quad (11a)$$

As a conclusion of the analysis, slope and threshold are both affected by changes in mirror reflectivity R_3 because of the dependence on photon lifetime τ_p given by Eq.9b.

When the external mirror reflectivity R_3 is changed, lifetime τ_p varies according to Eq.9b, and from Eq.10b we see that slope variations are equal to lifetime variations $\Delta\tau_p/\tau_{p0}$; moreover, from Eq.11, we see that threshold variations are just the lifetime variations $\Delta\tau_p/\tau_{p0}$ scaled by the factor $-1/[1+GN_0\tau_{p0}]$, when $\tau_p \approx \tau_{p0}$ (or, R_3 is small).

III. EXPERIMENTAL VALIDATION

We have tested the validity of the model using a VCSEL diode, model PH85-F1P1S2-KC of Optowell Co., emitting 8 mW at 850 nm at a nominal drive current of 20 mA.

Reason for choosing a VCSEL is that the spot is reasonably circular and no asymmetry is introduced in the tilt and yaw measurement; however we also tested FP-laser and found they are described equally well by the model.

We used the simple feedback scheme of Fig.1, with a reflecting target placed at a distance of 60-mm and properly

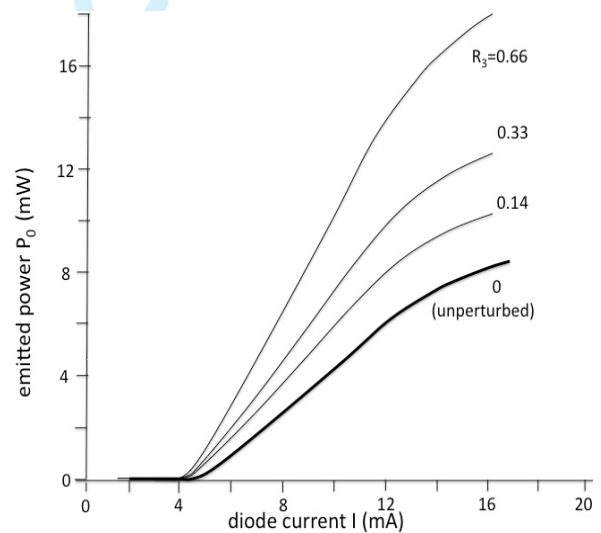


Fig.2 Experimental power-current curves for our VCSEL emitting at 850 nm. Thick line is the unperturbed case ($R_3=0$), thin lines are the power emitted at three values of target reflectivity R_3 .

aligned so as to maximize the return into the laser cavity. The reflecting targets were flats of: sapphire ($R_3=0.14$), Silicon ($R_3=0.33$) and Nickel ($R_3=0.66$), all with the back surface blackened to avoid spurious contributions.

The power-current curve dramatically depends on the target reflectivity, more than doubling for $R_3=0.66$ at $I=16$ mA respect to the unperturbed condition $R_3=0$, see Fig.2.

The slope of the power-current curve also increases with R_3 , and, in the moderate current range where slope $S=P/I$ is almost constant, it supplies the experimental value of $\Delta S/S_0 = \Delta\tau_p/\tau_{p0}$. Also, by drawing the tangent to the initial part of the P-I curve and extrapolating it linearly to zero power we could easily measure experimentally the relative variation of threshold, $-\Delta T/T_0$.

Experimental data we measured for the VCSEL PH85-F1P1S2-KC with external reflector are collected in Table I.

Table I

reflectivity R_3	0	0.14	0.33	0.66
threshold variation $\Delta S/S_0$ (%)	0	2.2	3.5	9.2
slope variation $-\Delta T/T_0$ (%)	0	9	15	48
slope-to-threshold ratio	-	4.1	4.3	5.1

By combining Eqs.10c and 11, we can calculate the ratio of relative slope-to-threshold variations with reflectivity R_3 . For small perturbation ($\tau_p \approx \tau_{p0}$ or small R_3) we get:

$$-(\Delta T/T_0)/(\Delta S/S_0) = 1 + GN_0\tau_{p0} \quad (12)$$

For a VCSEL, the commonly assumed values [24] of gain g_N and of carrier concentration at transparency N_t are: $g_N = 2.5 \cdot 10^{-16} \text{ cm}^2$ and $N_{th} = 1.3 \cdot 10^{18} \text{ cm}^{-3}$. By converting these quantities to our parameters G and N_0 of Eqs.2, we get: $G = c g_N = 2.5 \cdot 10^{16} \cdot 3 \cdot 10^{10} \text{ cm}^3/\text{s} = 7.5 \cdot 10^{12} \text{ m}^3/\text{s}$ and $N_0 = N_t = 1.3 \cdot 10^{24} \text{ cm}^{-3}$.

Consequently, factor $[1 + GN_0\tau_{p0}]$ in Eq.12 is evaluated as $1 + 9.75 \tau_{p0}$, where τ_{p0} is in ps.

Then, we can start by matching the experimental result of slope-to-threshold ratio at small R_3 , that is $-(\Delta T/T_0)/(\Delta S/S_0) = 4.1$ in Table I, and to do so we need $\tau_{p0} = (4.1 - 1)/9.75 = 0.318$ ps. This value corresponds, in a $L=10\text{-}\mu\text{m}$ VCSEL cavity with $\tau_{in}=0.2\text{-ps}$, to a mirror reflectivity product (following Eq.9a)

$$R_1 R_2 = \exp(-\tau_{in}/\tau_{p0}) = 0.533 \quad (13)$$

As the manufacturer didn't provide data about mirror reflectivity of our laser diode, we looked for best match of experimental data to theory. On taking $R_1=0.90$ and $R_2=0.59$ (i.e., a back mirror with high reflectance) we get a reasonable good fit with experimental data, as shown in Fig.3 (full line), where $-\Delta T/T$ and $\Delta S/S$ from Eqs.10c and 11 are plotted.

On the other side, should we assume the cavity loss is equally shared by the two mirrors, i.e., $R_1=R_2=0.73$, the match becomes appreciably worsened (Fig.3, dotted lines).

One reason for the limited accuracy of the match is that the extra contribution in Eq.9c makes the decay rate governed by

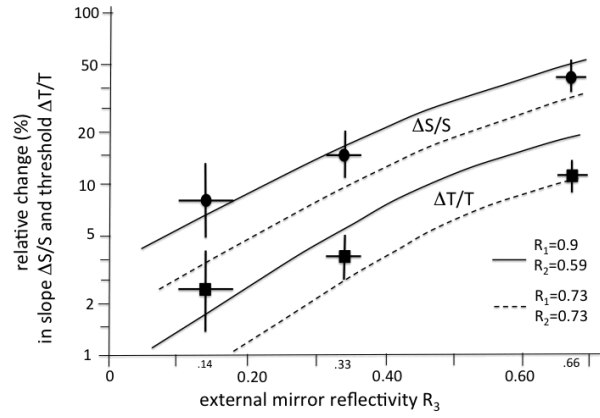


Fig.3 Match of theoretical and experimental values of slope $\Delta S/S$ (dots) and threshold $\Delta T/T$ (squares) as a function of external mirror reflectivity. Full lines represent the best matching, found for a high reflectance back mirror ($R_1=0.9$) and $R_2=0.59$ as derived from the τ_{p0} value, ruling out that the two laser mirrors are same reflectivity $R_1=R_2=0.73$ (dotted lines).

a double-time constant regime, different from the simple exponential assumed to derive Eqs.9b and 13.

Another reason is the eventual existence, in some lasers, of a leak contribution due to backreflected light passing around the laser chip and reaching directly to the rear monitor photodiode. This stray contribution makes the apparent laser power increase, and should be subtracted from the true photodetected current I_{ph} due to the backface-emitted power reaching the photodiode. The existence of a stray photocurrent is revealed by comparing the variations of photodiode current and of voltage V_{LD} across the laser junction, this last being obviously unaffected by the stray contribution. For our VCSEL chip having an area of $500 \times 500 \mu\text{m}^2$, the stray contribution was expected to be

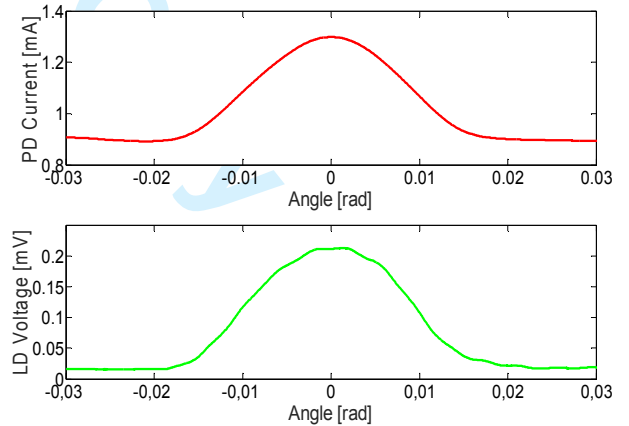


Fig.4 The $P(a)$ dependence on angle a , as measured by the photodetected current I_{ph} at the monitor photodiode (top) and by the diode voltage V_{LD} (bottom) are quite the same and confirm that no appreciable leakage contribution is collected. Setup data: $2H=120$ mm, $w_0=1.5$ mm.

negligible in our working condition of well-collimated output beam, because the returning beam retraces the output rays and isn't spread enough to escape the chip.

This conjecture is confirmed by the nearly coincident trend of detected photocurrent I_{ph} and diode voltage V_{LD} reported in Fig.4. Incidentally, it is worth noting that, if the VCSEL diode hasn't a rear monitor photodiode, the measurement can be anyway carried out by using the diode voltage as the sensing signal (albeit with a reduced SNR respect to the I_{ph} , as indicated by the small ripple in Fig.4, bottom).

IV. INSTRUMENTAL DEVELOPMENT

The angle dependence of power $P(a)$ can be exploited to measure angle a between wavevector \underline{k} of output beam and the normal \underline{n} to the target surface. In particular, we can measure tilt ψ and yaw θ angles of the reflective target surface (see Fig.5). This measurement can be carried out simultaneously to the SMI measurement of displacement s , as detailed below.

Of course, different from the usual operation of an SMI, where for measuring displacements we can also employ a *diffusing* target, we now shall restrict operation to *reflective* target to read the angle signal.

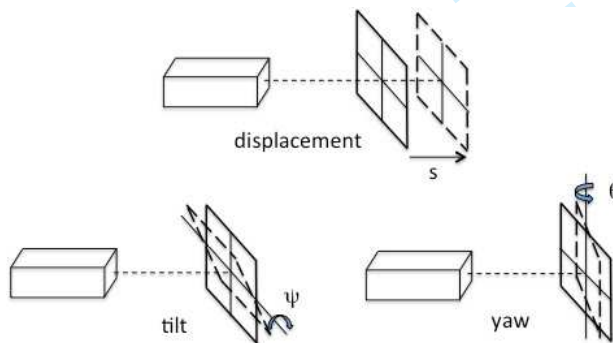


Fig.5 Three quantities describing target movement can be measured simultaneously from the $P(\phi, A)$ signal (see Eq.1a): displacement s as in a normal self-mixing interferometer, and the angles ψ and θ of tilt and yaw of the target surface, here drawn separately for clarity.

When the reflective target is moved of an angle a from alignment of its normal \underline{n} to wavevector \underline{k} , the superposition of retuning beam and unperturbed mode distribution inside the cavity results in a dependence of emitted power $P(a)$ found as [25]:

$$P_0(a) = P_0 + P_{00} \exp -(2aH)^2/w_0^2 \quad (14)$$

where a is either tilt ψ or yaw θ in Fig.5, H is the laser-to-target distance, and w_0 is the spot size of the beam projected by the laser onto the target, assumed Gaussian and nearly constant because of the collimation provided by the objective lens placed at the laser output.

Eq.14 applies for a circular spot size like the one emitted by our VCSEL; if a FP-laser diode is used, Eq.14 should be duplicated for the two spot sizes w_{0X} and w_{0Y} along axes X and Y , the axes that conveniently will be made coincident with the reference system of tilt and yaw (Fig.5) measurement.

The Gaussian shape of response of power $P(a)$ as a function of angle is fairly well matched (within, say a few percent) [25] provided the laser emits a clean spatial fundamental mode and care is exercised that the beam doesn't suffer spurious reflections inside the optical elements.

To transform the Gaussian dependence to an almost-linear one, we use the following well-known technique. A small modulation Δa (say $\Delta a = \Delta a_0 \cos \omega_m t$) is superposed to the angle a to be measured, so that the small increment Δa is multiplied by the *derivative* of the dependence (14), or:

$$P_0(a+\Delta a) = P_0(a) + [dP_0(a)/da] \Delta a \quad (15a)$$

$$= P_0(a) + P_0(a) 2a\Delta a (2H)^2/w_0^2 \quad (15b)$$

Thereafter, we detect the corresponding signal by a lock-in (or phase sensitive) amplifier centered at ω_m . The lock-in output is then the second term at the right hand side of Eq.15b, that is, a linear dependence multiplied by a Gaussian, as already shown in Refs.[25,26].

The useful, almost-linear range of measurement is about $a \approx \pm(1.0-1.6)w_0/H$, before the signal reaches a saturation and bends back (see Fig.7 below and Fig.3 of Ref.25). In our setup, with $w_0=1.5$ -mm and $H=80$ -mm, the useful linear range was about ± 45 mrad (or ± 2.5 deg).

We can do better however, if we compute the *relative derivative* of the power versus angle. We do so by dividing the synchronously detected signal (second term in Eq.15b) by signal (dc) component (first term in Eq.15b), thus obtaining:

$$[dP_0(a)/da] / [P_0(a) - P_0] = a [8H^2/w_0^2] \quad (15c)$$

This a truly linear dependence versus angle, which is extended up to the range where signal $P(a)$ becomes very small (and SNR is impaired), e.g. $a \approx \pm 3-4w_0/H$ typically.

The angle modulation is readily obtained, as already reported in previous papers [25-27], by mounting the objective collimating lens in a holder with two small piezoceramic chips on either side of a diameter, along the X and Y axes, as shown in Fig.6. Note that, different from another possible control developed in Ref.[28], focus is not altered in this arrangement.

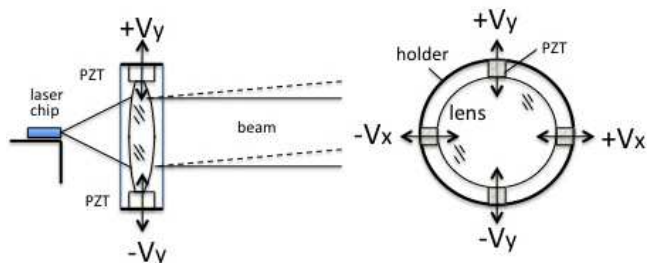


Fig.6 Mounting the objective lens on PZT piezo drivers, arranged along the X and Y axis and driven in push-pull, to modulate tilt θ and yaw ψ angles of the beam.

The piezo are driven in push-pull with $+V_d$ and $-V_d$ to ensure a transversal movement without excessive stress on the lens. With typical piezo of 1-mm by side, the transversal

movement was $\Delta_{\text{tm}}=2\mu\text{m}$, corresponding to a modulation of angle $\Delta a_{\text{mod}}=\Delta_{\text{tm}}/F=2 \cdot 10^{-6}/5 \cdot 10^{-3}=2.2 \text{ mr}$, more than enough in amplitude respect to the 50-mr full range, to preserve a good SNR of the measurement.

To ensure the X and Y channel are independent, the two driving signals are chosen, as in [25], an orthogonal pair, that is:

$$V_{\text{dX}} = V_0 \cos 2\pi f_{\text{mt}}, \quad V_{\text{dY}} = V_0 \sin 2\pi f_{\text{mt}} \quad (16)$$

so that tilt $a_{\text{mod}}=\psi$ and yaw $a_{\text{mod}}=\theta$ angles are:

$$\Delta\psi = \Delta\psi_0 \cos 2\pi f_{\text{mt}}, \quad \Delta\theta = \Delta\theta_0 \sin 2\pi f_{\text{mt}} \quad (16a)$$

and they compound along X and Y axis to give the total angle Δa as:

$$\Delta a = \hat{i} \Delta\psi + \hat{j} \Delta\theta \quad (16b)$$

where \hat{i} and \hat{j} are the unit vectors associated to X and Y axes. Upon synchronous detection (e.g., by a lock-in) of the photocurrent signal $I_{\text{ph}}=\sigma P(a+\Delta a)$ (Eq.16) and thanks to orthogonality, each channel ψ and θ is separated from the other. Indeed, using Eq.16b, the phase V_1 and quadrature V_2 outputs of the lock-in are written as:

$$V_1 = \langle V_{\text{dX}} P_0(a+\Delta a) \rangle = P_0(a) 2\psi\Delta\psi_0(2H)^2/w_0^2$$

$$V_2 = \langle V_{\text{dY}} P_0(a+\Delta a) \rangle = P_0(a) 2\theta\Delta\theta_0(2H)^2/w_0^2 \quad (16)$$

The frequency of driving was chosen in the low-audio range ($\omega_{\text{m}}/2\pi=20\text{-Hz}$) to minimize cross-talk with the displacement signal. Amplitude of the piezo driver was a few Volts. The same waveforms are used as the references of the lock-in for detection of ψ and θ .

After the photodetected current at the output of the rear photodiode is detected, a notch filter is used to remove the modulation carrier ω_{m} to leave a clean, interferometric self-mixing signal (the one of the normal SMI) carrying the 2ks information. The complementary signal, that is the bandpass filtered SMI, is the angle-dependent signal sent to the lock-in. Our lock-in had both in-phase and in-quadrature outputs, so yields simultaneously angles ψ and θ .

To test the linearity of the angle measurement response, a measurement was carried out by mounting the target (a flat with $R=0.33$ reflectivity) on a rotatable platform with arc-minute resolution, and then comparing the lock-in output to platform readout.

No appreciable difference was found between tilt and yaw measurements, and a good linear trend with error less than 2% was found up to about $\pm 6 \text{ mrad}$, and less than 10% up to about 15 mrad (full lines in Fig.7) for a single measurement lasting 50-ms.

These results are obtained with the relative derivative algorithm (Eq.15c), whereas the plain derivative measurement (dotted lines in Fig.7) deviates much earlier, at about 3.5 mrad for a 5% linearity error.

The error bars in Fig.7 correspond to one standard deviation, estimated over 70 repeated measurements. The angle error is limited to 0.2-mrad for the range $\pm 6 \text{ mrad}$, and is mainly due to electrical disturbances. By averaging on several ($N=100$)

samples, the error drops to 12- μr , comparable to the value found in Ref.[25].

For illustrative purposes, we measured signals with an *ad-hoc* jig, the target arrangement shown in Fig.8 chosen because it readily provides simultaneous time-varying displacement and angles waveforms with a single drive.

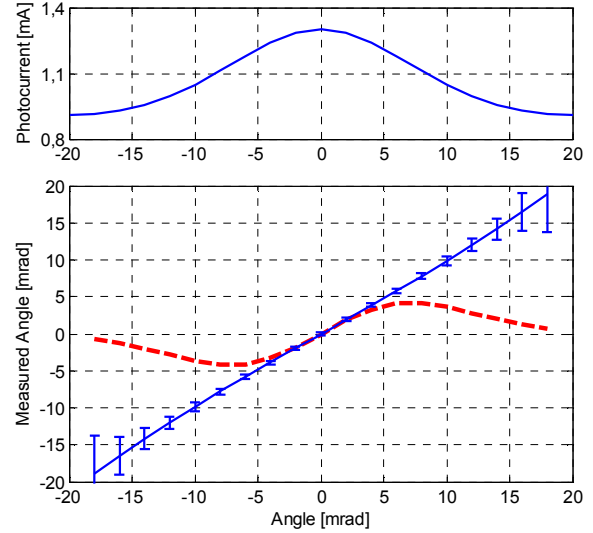


Fig.7 Top diagram: the $P(a)$ Gaussian response, for a setup with $2H=190 \text{ mm}$, $w_0=1.5 \text{ mm}$. Bottom diagram: experimental measurement of the angle (tilt or yaw) response of the self-mix $P(a)$ signal obtained by: (i) piezo modulation of the beam aim (Eq.15b) - dashed line -, and (ii) using the relative derivative linearization of (Eq.15c) - full line. Note the sizeable improvement of the linear dynamic range obtained by the relative derivative algorithm, only limited by the worsening of the SNR (i.e., the increasing size of the bars).

An optical $R=0.33$ flat was pivoted at one fixed end-point along the edge, and was driven by a loudspeaker at the opposite end of the edge, while the piezo scan of objective lens was switched off. In this way, we impart a periodic tilt θ toward the observer, as well as a periodic displacement s .

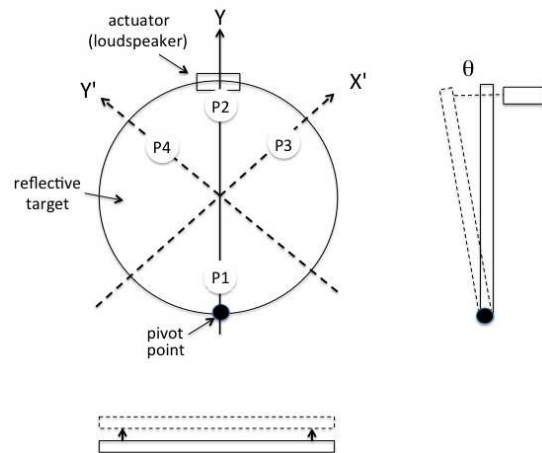


Fig.8 Experimental jig to develop illustrative time-varying SMI and angle signals.

The displacement is a maximum at point P2 and zero at P1, whereas the tilt angle θ is constant all over the target, when we analyze it with the Y axis parallel to the vertical as indicated in Fig.8, while the yaw angle (measured along an axis perpendicular to Y) is zero.

In Fig.9 we report the corresponding experimental SMI signal, detected at P1 and P2. At P1 (at the pivot point) the signal is pure angle, whereas at P2 the signal has a large displacement component superposed to the angle signal (visible as a ripple).

Second, upon moving the measurement points along X' and Y' axes in Fig.8, for example looking at P3 and P4, both tilt θ and yaw ψ angles are developed.

The corresponding experimental waveforms (Fig.7) are reported in Fig.10.

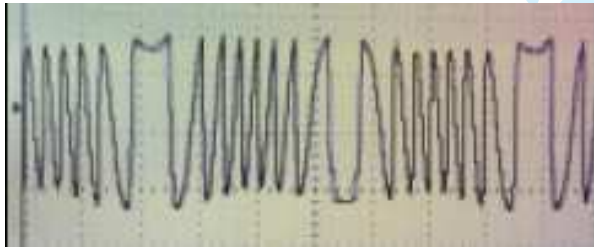
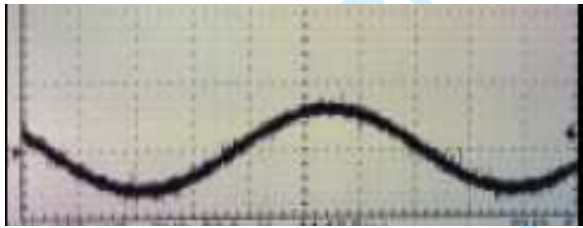


Fig.9 Experimental waveforms of self-mix signals. Top: an almost pure angle a signal is found at point P1 of Fig.8, whereas at P2 the main contribution is the displacement signal $2ks$ and the angle component is revealed by the underlying ripple. Time scale: top, 3 ms per division, bottom 6 ms per division.

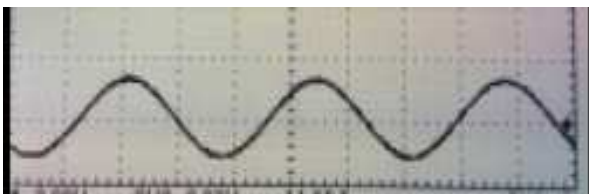
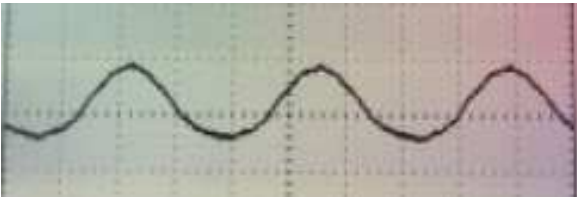


Fig.10 Experimental waveform of tilt (top) and yaw (bottom) angles, detected at points P3 and P4 of Fig.8, after removing the displacement signal by bandpass filtering.

V. CONCLUSIONS

By using the incoherent effect of power dependence upon the alignment angle of a reflective target, together with the usual coherent self-mixing dependence on external pathlength, we have demonstrated that the SMI can perform the simultaneous measurement of displacement and angles, tilt and yaw taking advantage of two orthogonal modulation of the beam aim.

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