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# MEASUREMENTS OF LIGHT INTENSITY CORRELATIONS IN THE SUBNANOSECOND REGION BY PHOTOMULTIPLIERS ‡

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A theory of intensity correlation measurements on a light beam whose coherence time is of the same order of magnitude as the response time of the photodetector is presented and supported by experimental results. Single and two-phototube measurements are compared. Extension of the results to higher order correlations is outlined.

The statistical properties of electromagnetic fields at optical frequencies have been extensively explored in recent years by measuring the associated fluctuations of counts number at the output of a photodetector [1].

It is well known that a statistical distribution of photocounts allows evaluation of intensity correlation functions of any order. We recall that high order correlation functions are useful both to assign the degree of coherence [2] of the electromagnetic field and to estimate the statistical errors made in the evaluation of the lower order correlations [3,4].

The finite response time $\tau_r$ of the photodetector puts however a fundamental limit to the use of photocount statistics for short times. Depending on the value of the coherence time $\tau_c$ of the light field, we may consider three time ranges.

(i) $\tau_c \gg \tau_r$. Since $\tau_r$ is usually of the order of 1 nsec, $\tau_c$ must be longer than 100 nsec [1]. Such long coherence times are peculiar of a single mode laser field not too far from threshold, or of light scattered by the density fluctuations in a pure fluid near its critical point. In this time range, correlation measurements performed on the output current of the detector yield directly the correlation functions of the light intensity. Correlation functions of increasing order can be evaluated either by analog or digital techniques [1] up to a limit set by the accuracy of the measurements. If only the lowest order intensity correlation function has to be measured, the result can be also obtained by a spectral analysis of the photodetector current (self-beating spectroscopy [5]).

(ii) $\tau_c \ll \tau_r$. This is the case, for instance, of the picosecond pulses generated by mode-locked solid state lasers, or of light scattered by material excitations with fast decay times (collision induced scattering) or relatively high transfer frequencies (Raman scattering)‡‡‡. Techniques based on nonlinear optical phenomena such as second harmonic generation [6] or two-photon fluorescence [7], have been devised to yield the intensity correlation function. Very high peak intensities are however required, and the relative errors are remarkably large as compared to the photocount techniques.

(iii) $\tau_c \approx \tau_r$. This boundary region is peculiar of many physical situations, such as a single mode laser far away from threshold, many-modes lasers, Brillouin scattering by a fluid, and light from spectral lamps. Photodetectors can still be used in this region, but the correlations in their outputs are no longer directly related to the correlations in the impinging field.

We show here how light intensity correlations of any order can be measured by taking into account the statistical behaviour of the photodetector and give experimental results which support our theory.

Our treatment can be applied in any of the above mentioned time ranges. It becomes, however,

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‡‡‡ If the spectral width of the field correlation function is wanted, standard optical techniques are used. In this paper, however, we are interested in intensity correlations and shall not deal with field correlations.

particularly relevant for the experimental cases in which $\tau_c \approx \tau_r$. The treatment holds for any photodetector, but here we refer specifically to photomultipliers, since they are the most used detectors in range (iii). Indeed, their output current can be high enough to be processed by fast electronics even for very low light intensities.

The output $i(t)$ of a photocathode illuminated by a light signal may be represented by a train of photoelectrons localized at random times $t_k$, that is,

$$i(t) = \sum_{k=1}^{\infty} \delta(t - t_k) . \tag{1}$$

The first correlation function $R_i(\tau) = \langle i(t)\, i(t+\tau)\rangle$ of the random process $i(t)$ which we assume stationary, is given by

$$\begin{aligned} R_i(\tau) &= \langle \sum_k \sum_j \delta(t - t_k)\, \delta(t+\tau - t_j)\rangle \\ &= \langle \sum_k \delta(t - t_k)\, \delta(t+\tau - t_k)\rangle + \langle \sum_{k\neq j} \sum \delta(t - t_k)\, \delta(t+\tau - t_j)\rangle \\ &= \langle i\rangle \delta(\tau) + \langle i\rangle^2 (1 + \gamma(\tau)) , \end{aligned} \tag{2}$$

where $\langle i\rangle$ is the average rate of photoelectrons, which is proportional to the average light intensity, and $\gamma(t)$ is the reduced intensity correlation function. Note that the first two terms $\langle i\rangle\delta(\tau)$ and $\langle i\rangle^2$ are the usual shot-noise contributions [8], in the limit of $\delta$-like pulses [8, section 7.4]. The last term $\langle i\rangle^2\gamma(\tau)$ expresses the correlations among the arrival times $t_k$ and $t_j$.

Both $\langle i\rangle$ and $\gamma(\tau)$ can be expressed in terms of the first and second order field correlation functions [2,9] as follows:

$$\langle i\rangle = sG^{(1)}(t;t) \tag{3}$$

$$\gamma(\tau) = \frac{G^{(2)}(t,t+\tau;\ t+\tau,t)}{[G^{(1)}(t;t)]^2} - 1 . \tag{4}$$

Here $G^{(1)}(t;t)$ is the average light intensity, and $s$ is the photodetection efficiency.

The photomultiplier, considered as a linear stochastic filter [10], is described by a random response function $h(t)$ which is the output for a single photoelectron emitted at time $t = 0$. Consequently the random output current $u(t)$ will be the time convolution of the two random functions $h(t)$ and $i(t)$, that is,

$$u(t) = \int_{-\infty}^{\infty} dt'\, i(t - t')h(t') \equiv i(t) * h(t) . \tag{5}$$

In performing this convolution one should remember that the different $\delta(t - t_k)$ of the input (1) are associated with independent realizations $h_{(k)}(t)$ of the random process $h(t)$. Therefore the above equation is in more detail

$$u(t) = \sum_k \delta(t - t_k) * h_{(k)}(t) = \sum_k h_{(k)}(t - t_k) . \tag{6}$$

The correlation function $R_u(\tau) = \langle u(t)u(t+\tau)\rangle$ of the output can then be written as

$$\begin{aligned} R_u(\tau) &= \langle \Big[ \sum_k \delta(t - t_k) * h_{(k)}(t) \Big] \Big[ \sum_j \delta(t+\tau - t_j) * h_{(j)}(t+\tau) \Big] \rangle \\ &= \langle \sum_{k\neq j} \sum \delta(t - t_k)\, \delta(t+\tau - t_j) * h_{(k)}(t) h_{(j)}(t+\tau)\rangle + \langle \sum_k \delta(t - t_k)\, \delta(t+\tau - t_k) * h_{(k)}(t) h_{(k)}(t+\tau)\rangle . \end{aligned} \tag{7}$$

Taking into account that the statistical properties of the photomultiplier operating in the linear range are independent of the statistics of $i(t)$, interchanging ensemble averaging with convolution operations, and recalling that $h_{(k)}$ and $h_{(j)}$ for $k \neq j$ are statistically independent samples, we finally obtain:

$$R_u(\tau) = \langle i \rangle^2 (1 + \gamma(\tau)) * G(\tau) + \langle i \rangle C(\tau) , \tag{8}$$

where

$$G(\tau) = \int_{-\infty}^{\infty} \langle h(t) \rangle \langle h(t+\tau) \rangle \, dt \tag{9}$$

and

$$C(\tau) = \int_{-\infty}^{\infty} \langle h(t) h(t+\tau) \rangle \, dt . \tag{10}$$

A full description of the effect of the photomultiplier on the intensity correlation function $R_u(\tau)$ requires therefore two different functions $C(\tau)$ and $G(\tau)$. $C(\tau)$ modifies the $\delta$-like term of eq. (2), which comes from the quantized character of the detection process, whereas $G(\tau)$ modifies the term describing the joint probability of two different photon events. The difference can be explained as follows. We decompose the response $h(t)$ in its average plus a fluctuating part

$$h(t) = \langle h(t) \rangle + \Delta h(t) .$$

Eq. (10) can then be written as

$$C(\tau) = G(\tau) + \int_{-\infty}^{\infty} \langle \Delta h(t) \Delta h(t+\tau) \rangle \, dt ,$$

that is, $C(\tau)$ is the sum of the correlation of the average response $\langle h(t) \rangle$, plus the correlation of the fluctuation $\Delta h$, which always relaxes more rapidly than the first term. As a consequence, $C(\tau)$ is narrower than $G(\tau)$.

The fluctuations of $h(t)$ are generated by essentially three phenomena [11]: gain statistics at the dynodes, spread in transit times of secondary electrons from dynode to dynode, spread in the transit time of photoelectrons from the photocathode to the first dynode. If only the first effect is present, $C(\tau)$ will be proportional to $G(\tau)$, whereas spreads in transit times affect differently the two functions. In particular, the spread associated with the transit time of photoelectrons affects only the location of each $h(t)$ and hence does not affect $C(\tau)$.

We have made an experimental determination of $C(\tau)$ and $G(\tau)$ and the results are given in fig. 1. The function $C(\tau)$ has been obtained by performing a correlation measurement at the output of a photomultiplier Philips XP 1210, illuminated by a broadband thermal source. The photomultiplier signal is sampled by a 5 T 3 unit (90 psec sampling time) of a Tektronix 661 oscilloscope whose output is connected with a two-dimensional multichannel analyzer. The correlation function is then computed from the information stored in the analyzer. A detailed description of this experiment is given in ref. [12]. $G(\tau)$ has been computed from the results of a measurement of $\langle h(t) \rangle$. This latter function has been obtained by an oscilloscope (Textronix 519) picture of the output of the same photomultiplier illuminated by a mode-locked ruby laser. Laser pulses have a duration of 40 psec and induce the emission of many simultaneous photoelectrons, so that the output current pulse due to a single laser pulse can be considered, within a good approximation, coinciding with $\langle h(t) \rangle$. The two functions in fig. 1 have been normalized to the same initial value. As expected from the discussion reported above, $G(\tau)$ is broader than $C(\tau)$.

To obtain information on the light intensity correlation $\gamma(\tau)$ from the measured correlation $R_u(\tau)$, the term $\langle i \rangle C(\tau)$ must be subtracted. In this operation experimental errors strongly affect the results, especially for low photon fluxes. However the term $\langle i \rangle C(\tau)$ does not appear at all in the expression for $R_u(\tau)$ when the correlation is performed with two phototubes, as in the Hanbury-Brown and Twiss experiment.

In this case, repeating the calculations, eq. (8) is modified as follows

$$R_u(\tau) = \langle u_1(t) u_2(t+\tau) \rangle = \langle i_1 \rangle \langle i_2 \rangle (1 + \gamma(\tau)) * G_{12}(\tau) , \tag{11}$$

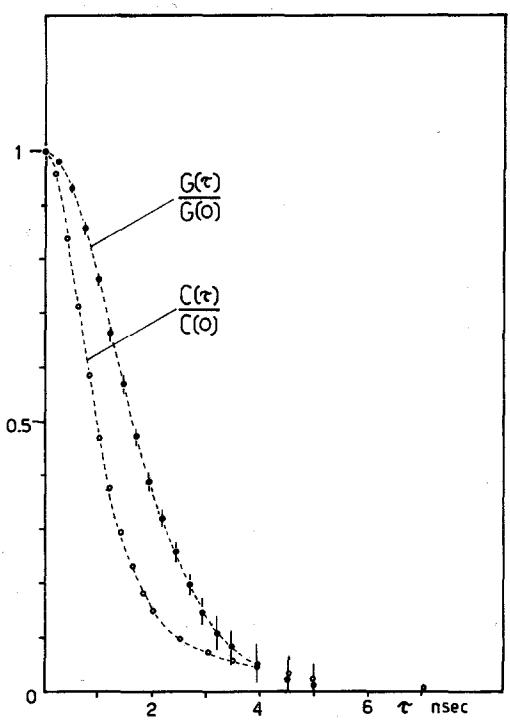


Fig. 1. Plot of the functions $C(\tau)$ and $G(\tau)$ for the Philips XP 1210 photomultiplier, with the socket including bypass capacitors supplied by the manufacturer. The operating voltage is 3800 V. The dashed lines interpolate the experimental points.

where the indexes 1 and 2 refer to the two phototubes and

$$G_{12}(\tau) = \int_{-\infty}^{\infty} \langle h_1(t)\rangle \langle h_2(t+\tau)\rangle \, dt \, . \tag{12}$$

In fig. 2 we show a comparison of a single phototube experiment with a two-phototube experiment. The light source is a broadband thermal source spatially incoherent. For such a field $\gamma(\tau)$ can be assumed to be negligible compared with 1 and therefore the bump appearing in $R_u(\tau)$ for short delays is due only to $C(\tau)$. In the two-phototube experiment the bump disappears, in agreement with eq. (11).

As a conclusion we present some critical remarks and comparisons with previous work. We have shown that a measurement of intensity correlations in the range $\tau_c \approx \tau_r$ implies knowledge of the statistical properties of the phototube. The phototube contributes with two functions $C(\tau)$ and $G(\tau)$, whose measurement requires different operational procedures. Precisely $C(\tau)$ is measured by an experiment with a white-noise light source [$\gamma(\tau) = 0$], whereas $G(\tau)$ can be easily computed from the average response $\langle h(t)\rangle$.

Previous attempts to perform measurements in this range with a single photomultiplier [13] have not distinguished between $C(\tau)$ and $G(\tau)$. Knowledge of $C(\tau)$ is no longer necessary when the correlation function is measured by two phototubes, since in this case there is no way of correlating a single realization $h_{(k)}(t)$ with itself. Therefore some recent measurements [14,15] of intensity correlations on the 4358 Å line from a mercury lamp, are correctly described by eq. (11) and require only previous knowledge of the function $G_{12}(\tau)$ given in eq. (12).

Our results can easily be generalized to the measurement of higher order correlation functions. As an example we report the expression of the third order correlation function as measured by a single

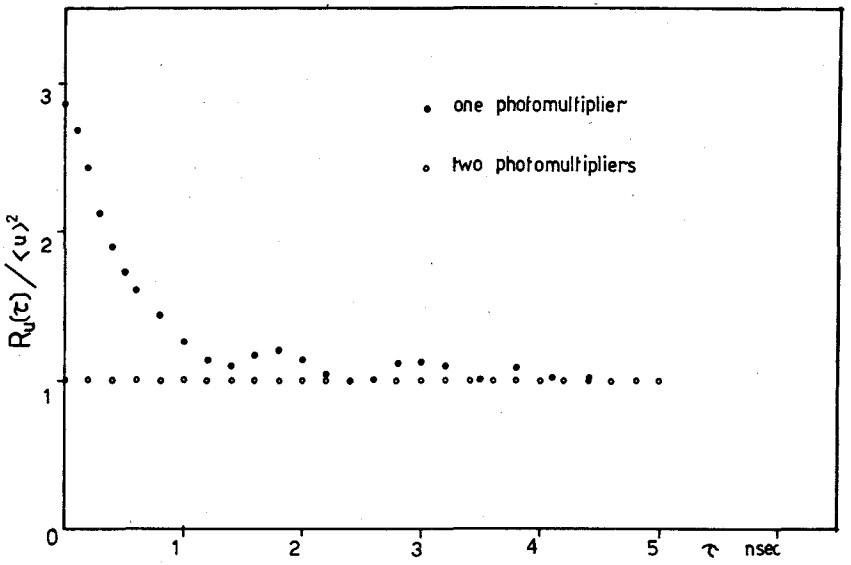


Fig. 2. Plot of the normalized correlation functions for one phototube and two-phototube experiments. The light beam comes from a broadband thermal source. Two identical RCA C70045 C photomultipliers have been used. The operating voltage is 5000 V. Experimental errors are within the dot size.

phototube. In the most general case of non-stationary processes this can be written as

$$\langle u(t_1)u(t_2)u(t_3)\rangle = \langle i\rangle^3 \langle h(t_1)\rangle \langle h(t_2)\rangle \langle h(t_3)\rangle \overset{*}{*}\overset{*}{*} g^{(3)}(t_1,t_2,t_3)$$
$$+ \langle i\rangle^2 [\langle h(t_1)h(t_2)\rangle \langle h(t_3)\rangle \overset{*}{*} g^{(2)}(t_1,t_2) + \{2,3;\ 1\} + \{3,1;\ 2\}] + \langle i\rangle \langle h(t_1)h(t_2)h(t_3)\rangle * g^{(1)}(t_1)\ . \tag{13}$$

Here the two curly brackets denote permutation of the indexes $\{1,2;\ 3\}$ of the first term in the square brackets, and $g^{(i)}$ are the normalized Glauber $i$th order correlation functions [2]. If the optical field is stationary, when we express eq. (13) in terms of the new times $\tau = t_2 - t_1$, $\tau' = t_3 - t_1$, one of the convolutions in each term disappears. For a three-phototube measurement the third order correlation function is given by only one term, as follows

$$\langle u_1(t_1)u_2(t_2)u_3(t_3)\rangle = \langle i_1\rangle \langle i_2\rangle \langle i_3\rangle \langle h_1(t_1)\rangle \langle h_2(t_2)\rangle \langle h_3(t_3)\rangle \overset{*}{*}\overset{*}{*} g^{(3)}(t_1,t_2,t_3)\ . \tag{14}$$

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## REFERENCES

[1] F. T. Arecchi, in: Quantum optics, ed. R. J. Glauber (Academic Press, New York, 1969).
[2] R. J. Glauber, in: Quantum optics and electronics, eds. C. de Witt et al. (Gordon and Breach, New York, 1965).
[3] F. T. Arecchi, A. Berné, A. Sona and P. Burlamacchi, IEEE J. Quantum Electron. 2 (1966) 341.
[4] V. Degiorgio and J. B. Lastovka, to be published.
[5] G. B. Benedek, in: Statistical physics, Vol. 2, eds. M. Chrétien et al. (Gordon and Breach, New York, 1968).
[6] J. A. Armstrong, Appl. Phys. Letters 10 (1967) 16.
[7] J. A. Giordmaine, P. M. Rentzepis, S. L. Shapiro and K. W. Wecht, Appl. Phys. Letters 11 (1967) 216.
[8] W. B. Davenport and W. L. Root, Random signals and noise (McGraw-Hill, New York, 1958).
[9] L. Mandel and E. Wolf, Rev. Mod. Phys. 37 (1965) 231.
[10] G. Adomian, Rev. Mod. Phys. 35 (1963) 185.
[11] S. Donati, E. Gatti and V. Svelto, Advan. Electron. Electron Phys. 26 (1969) 251.
[12] M. Corti and A. Vendramini, to be published.
[13] V. V. Artemyev, Radio Eng. Electron. Phys. 13 (1968) 1616.
[14] D. B. Scarl, Phys. Rev. Letters 17 (1966) 663.
[15] D. T. Phillips, H. Kleiman and S. P. Davis, Phys. Rev. 153 (1967) 113.