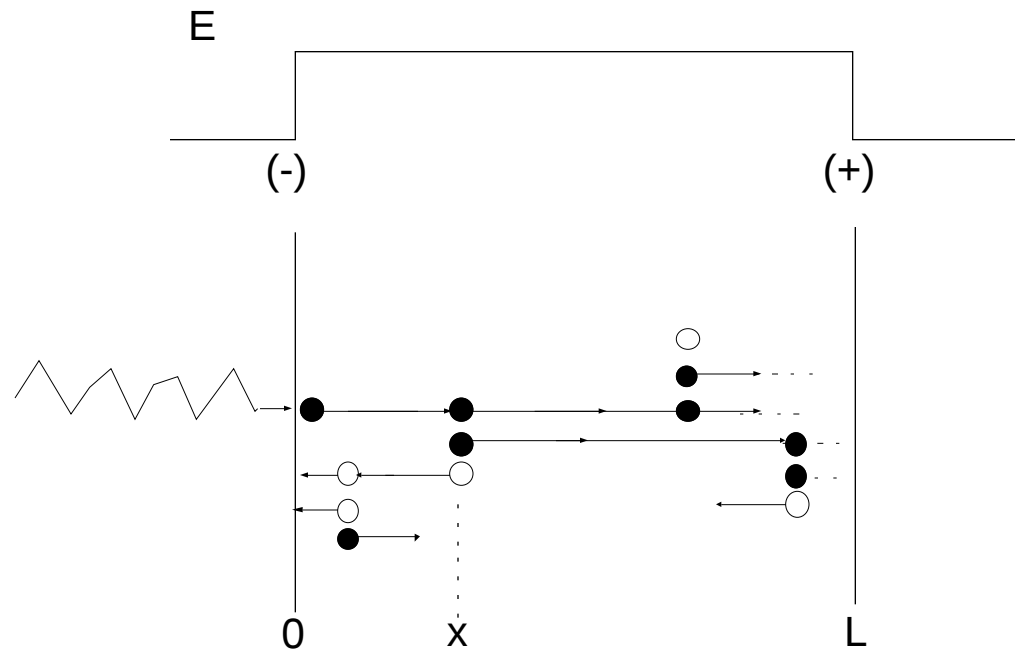
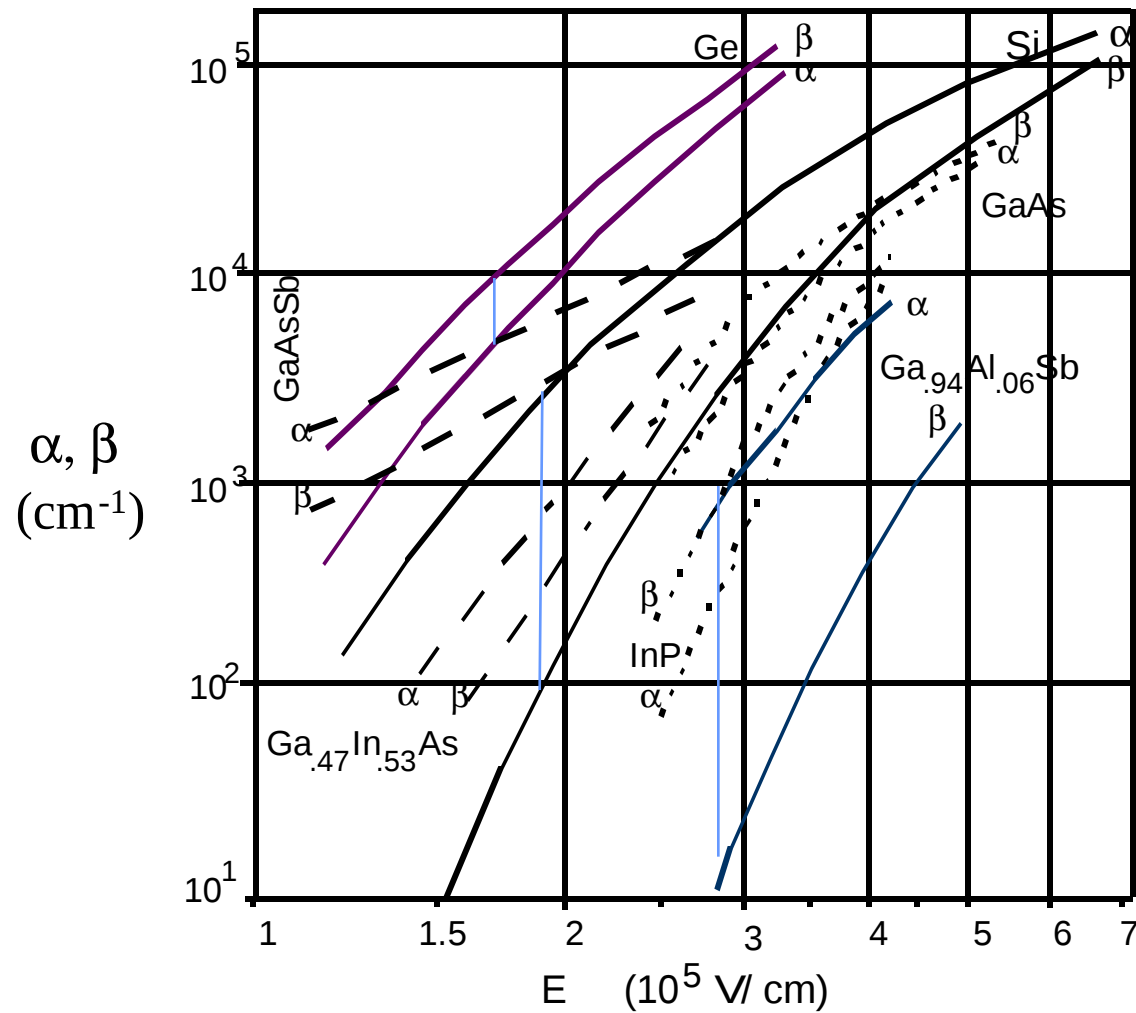


Avalanche multiplication



Ionization coefficients α, β



from: "Photodetectors", by S. Donati, Prentice Hall 2000

APD Gain

$$di_n/dx = \alpha i_n + \beta i_p, \quad di_p/dx = -(\alpha i_n + \beta i_p)$$

$$\text{with: } i_n(0) = I_{ph}, \quad i_n(L) = I, \quad i_p(L) = 0$$

then, $d^2i_n/dx^2 = (\alpha - \beta)di_n/dx$,

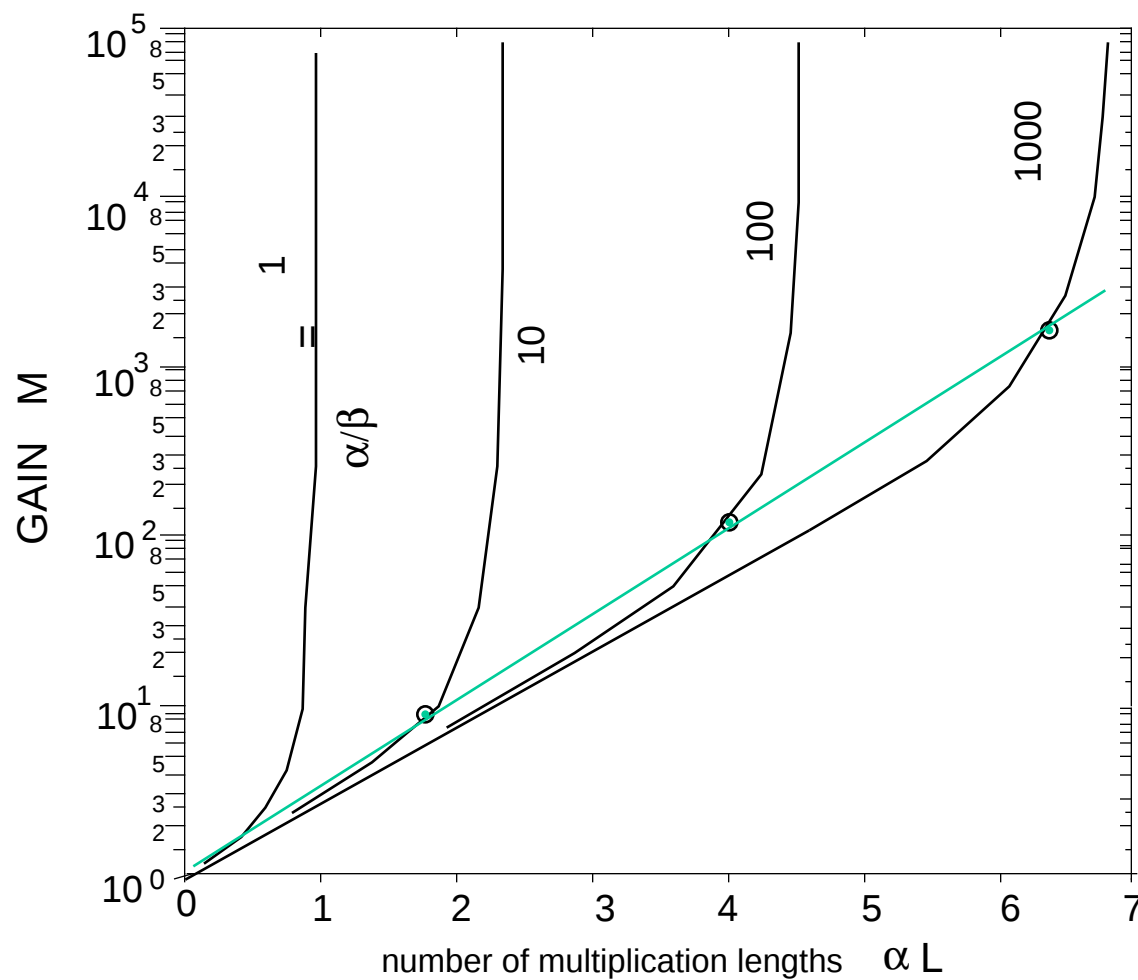
$$i_n = [C_1/(\alpha - \beta)]\exp(\alpha - \beta)x + C_2, \quad \beta i_p = -[\beta C_1/(\alpha - \beta)]\exp(\alpha - \beta)x - \alpha C_2.$$

$$M = \frac{(\alpha - \beta) \exp(\alpha - \beta)L}{\alpha - \beta \exp(\alpha - \beta)L}$$

special cases: for $\beta \ll \alpha$: $M = \exp \alpha L$,

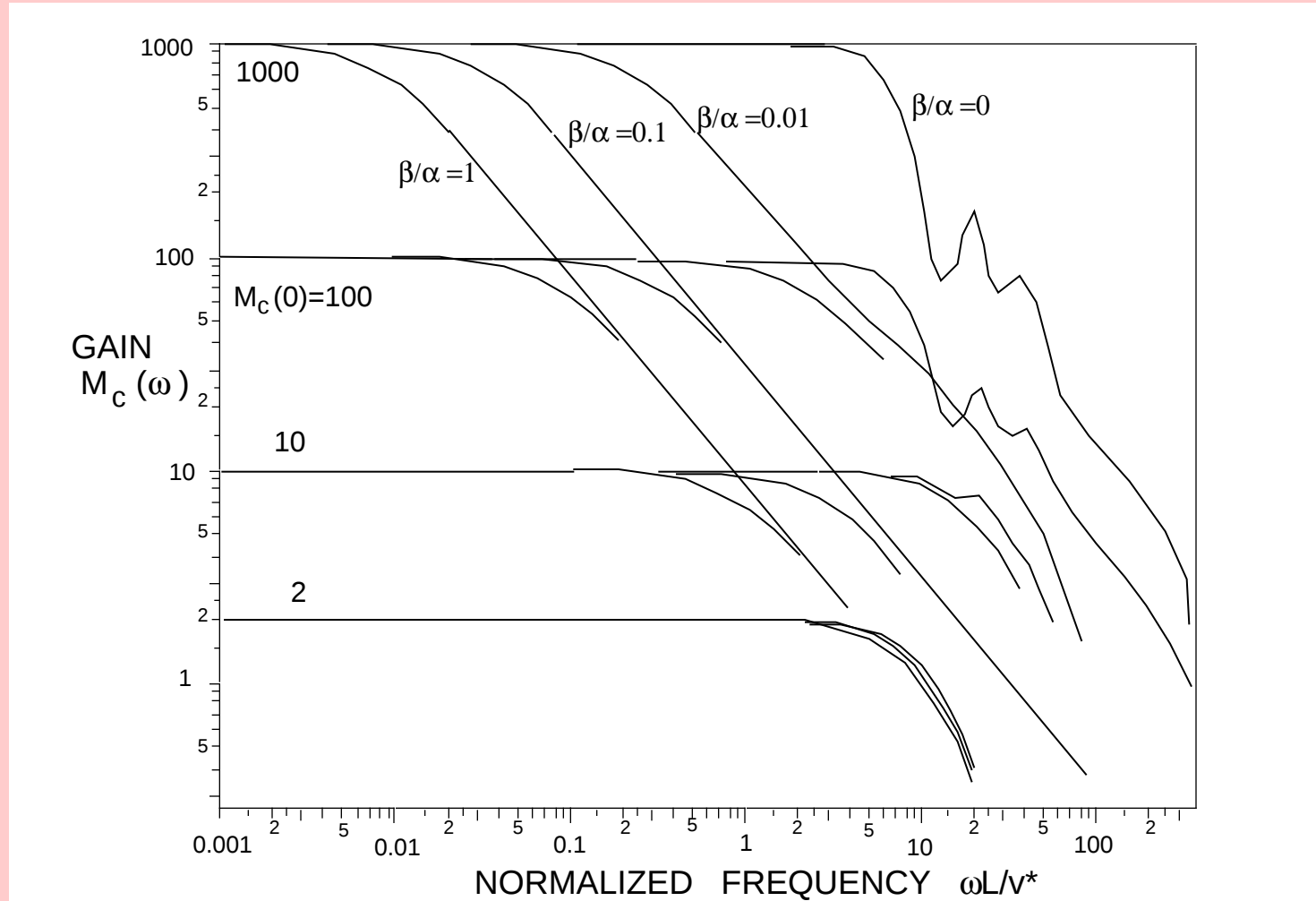
for $\beta = \alpha$: $M = 1/(1 - \alpha L)$

APD gain vs αL



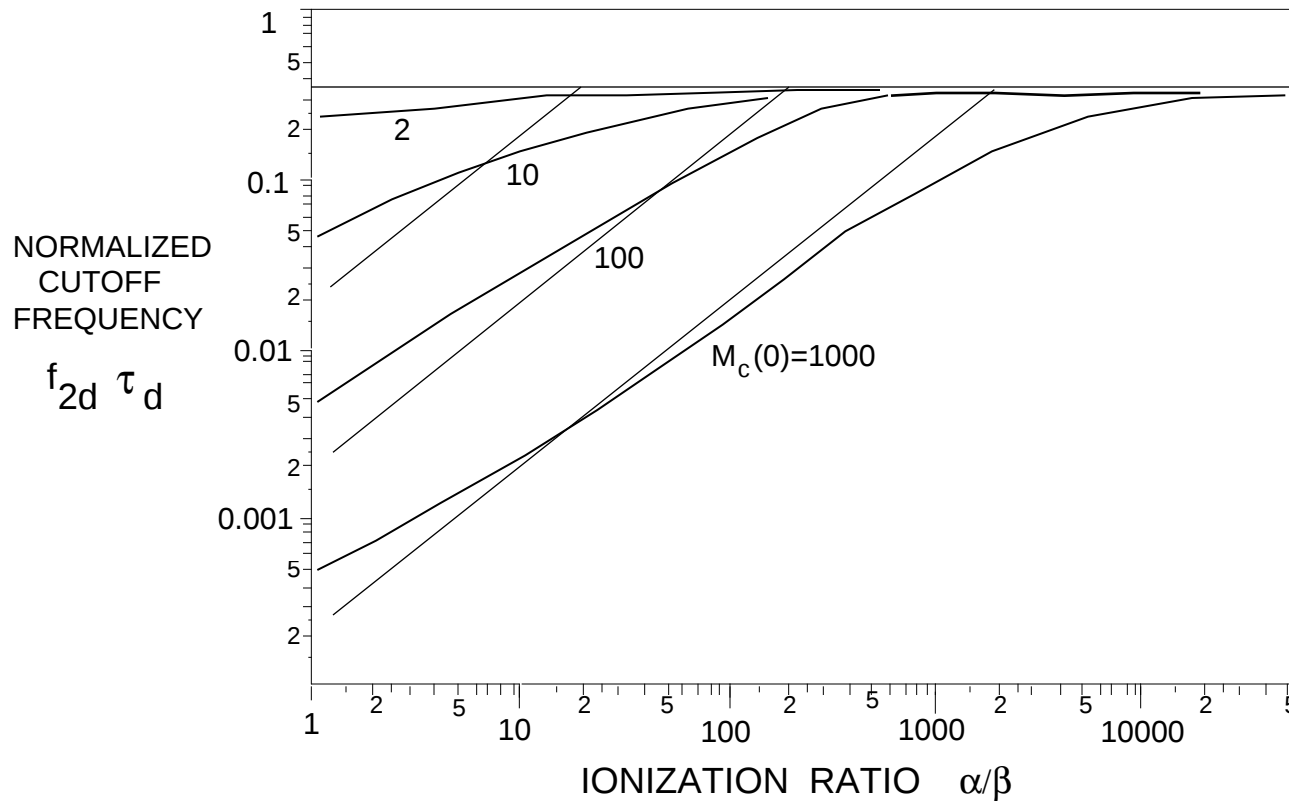
$M_o = \alpha/\beta$ is
the optimal
gain

Frequency response



from: "Photodetectors", by S. Donati, Prentice Hall 2000

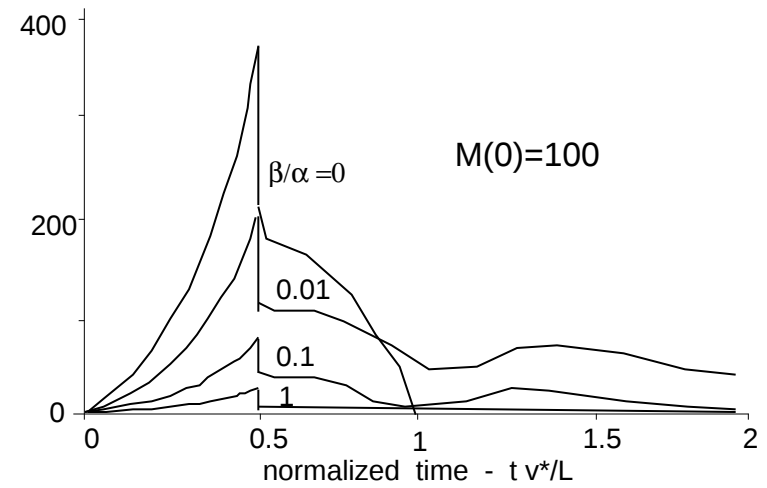
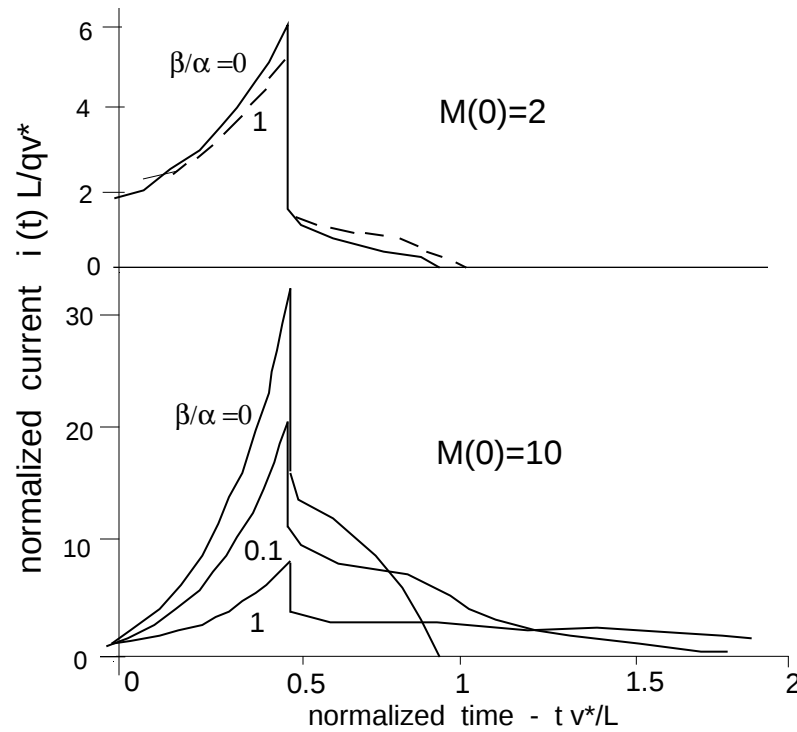
3-dB cutoff frequency



$M_o = \alpha/\beta$ is
the optimal
gain

$$f_{2d} \cong 0.36 / \tau_d \quad (M \ll M_o), \quad f_{2d} M \cong 0.36(\alpha/\beta) / \tau_d \quad (M > M_o)$$

Delta response



noise

relative variance of output charge for 1 pair at $x=0$:

$$\begin{aligned}\sigma_M^2(0)/M_c^2(0) &= 1 + \beta M_c(0)/(\alpha - \beta) - \alpha M_c^2(L)/M_c(0)(\alpha - \beta) \\ &= 1 + M_c(0) \{ \beta/(\alpha - \beta) - [\alpha/(\alpha - \beta)] \exp(-2(\alpha - \beta)L) \}\end{aligned}$$

for moderate gains M_c :

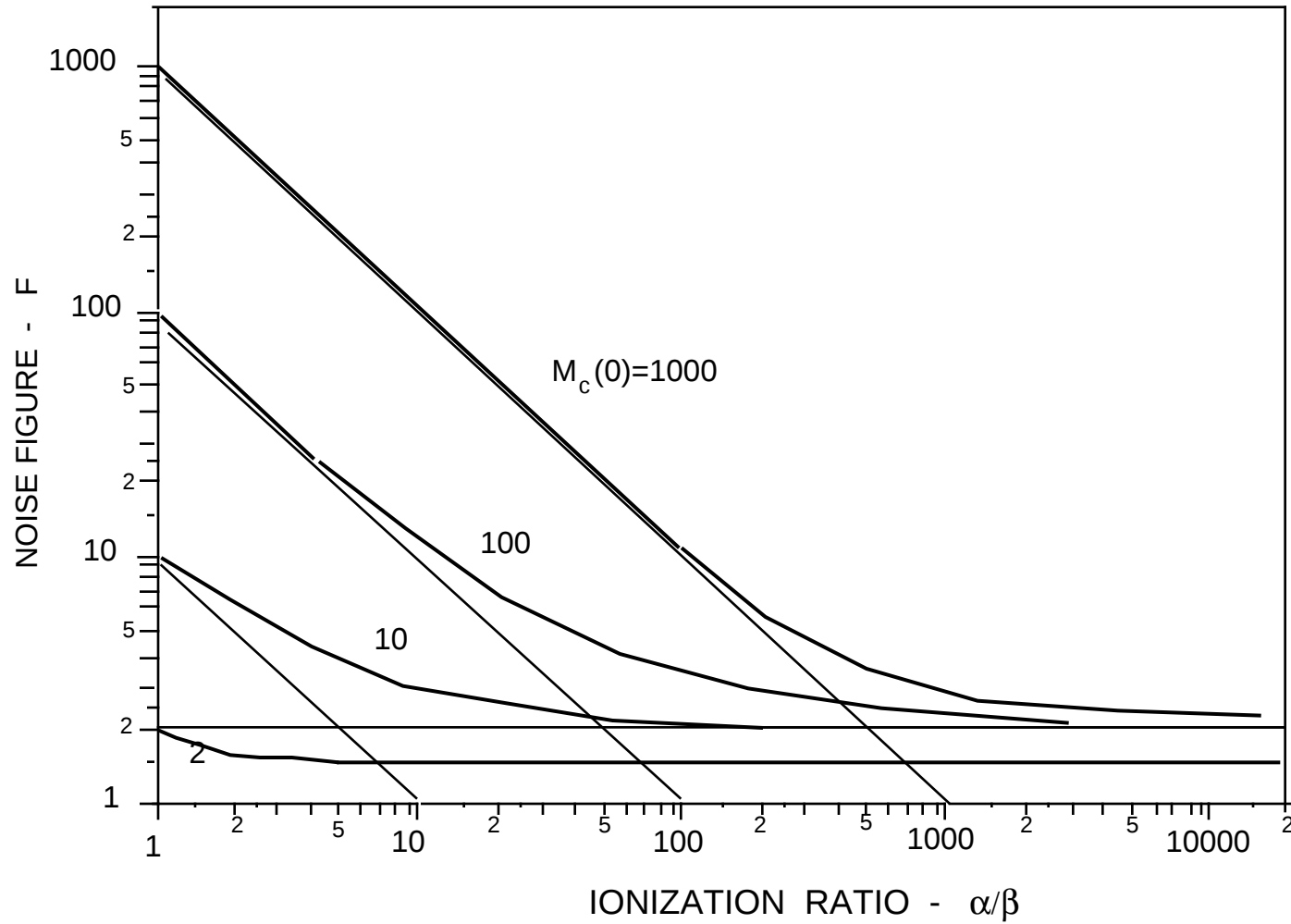
$$\begin{aligned}\sigma_M^2(0)/M_c^2(0) &\approx 1 && \text{for } M_c(0) < \alpha/\beta \\ &\approx M_c(0)\beta/\alpha && \text{for } M_c(0) > \alpha/\beta\end{aligned}$$

once again, $M_o = \alpha/\beta$ is the optimal gain. Noise current:

$$\sigma_I^2 = 2eB I_{ph} (M^2 + \sigma_M^2) = 2eB I_{ph} M^2 F$$

$$F = 1 + \sigma_M^2/M^2$$

Noise figure



from: "Photodetectors", by S. Donati, Prentice Hall 2000

Optimum gain and beyond

$$\sigma_I^2 = 2eB (I_{ph} + I_o) M^2 F + 4kTB/R$$

$$\sigma_{in}^2 = 2eB (I_{ph} + I_o) F + 4kTB/RM^2$$

$$\text{but, } S = I_{ph}, \quad N^2 = \sigma_{in}^2, \quad \text{and } F = M\beta/\alpha$$

$$(S/N)^2 = I_{ph}^2 / [2eB(I_{ph} + I_o) M\beta/\alpha + 4kTB/RM^2]$$

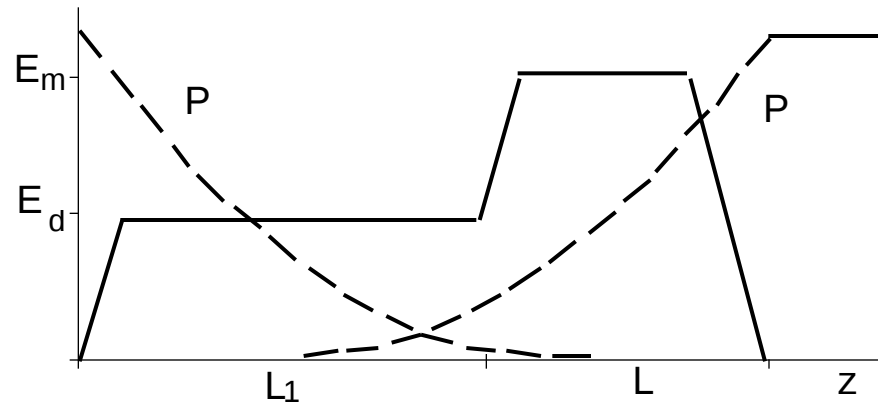
S/N is maximum at a M_{max} found as:

$$M_{max} = [(\alpha/\beta)(4kT/e) / R(I_{ph} + I_o)]^{1/3}$$

for instance, for $R=50\Omega$, $I_{ph} + I_o = 1\text{nA}$ and $\alpha/\beta=1$: $M_{max}=125$

practical limit for M: bandwidth decreasing by $M\beta/\alpha$ at $M > \alpha/\beta$

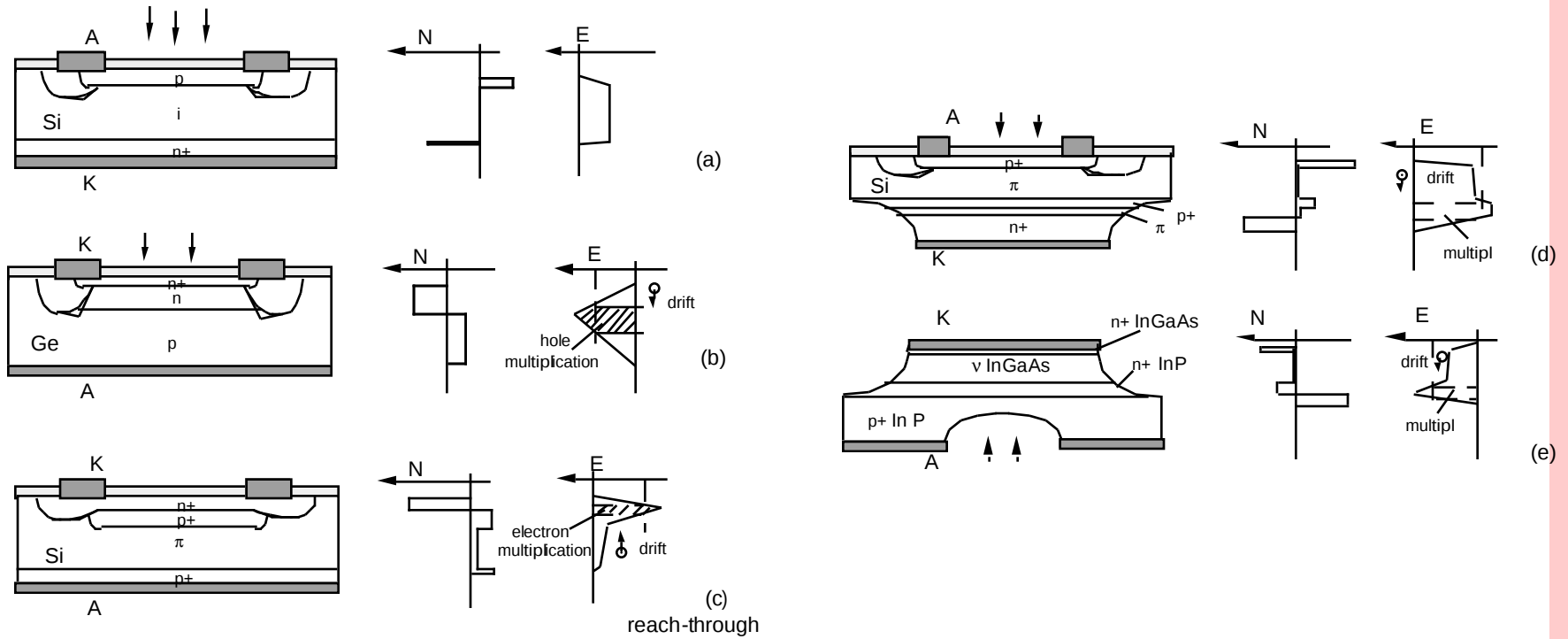
APD desirable structure



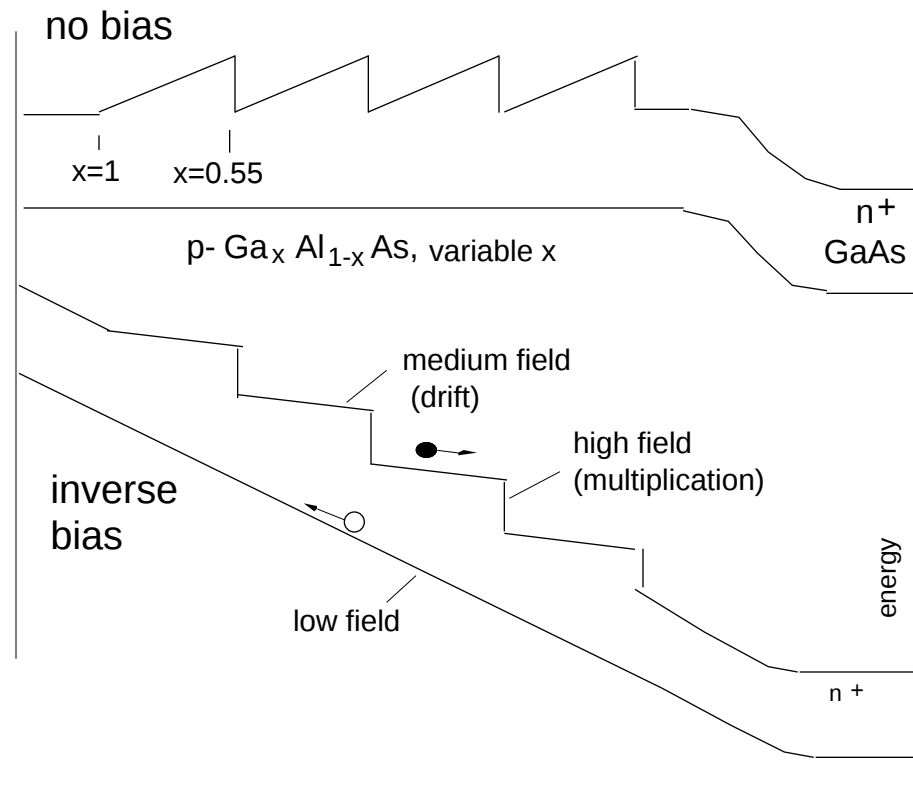
two regions desirable, to separate :

- photon dissipation in $L_1 (\approx 1/\alpha)$ and the carrier drift (medium E_d)
- multiplication in L of most efficient ionizing carrier (high E_m)

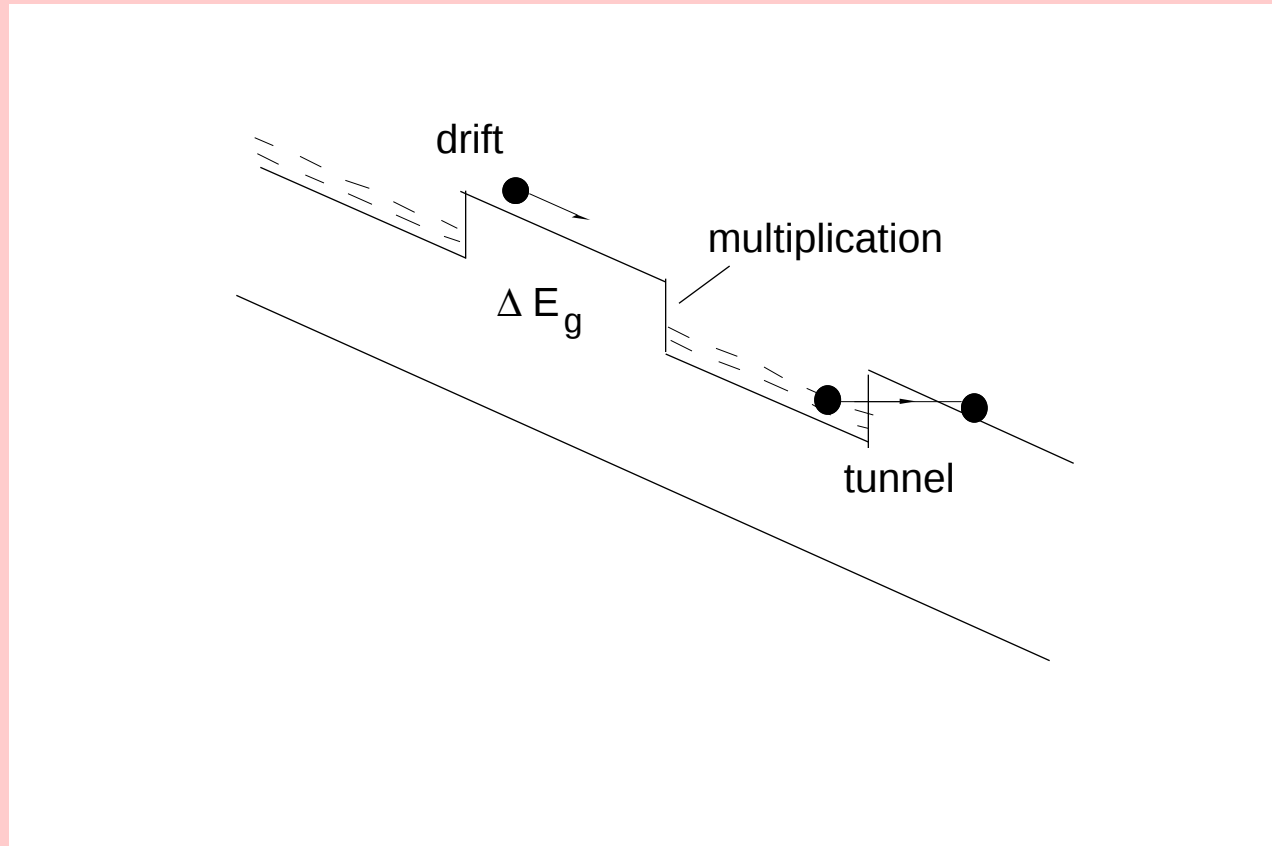
APD structures



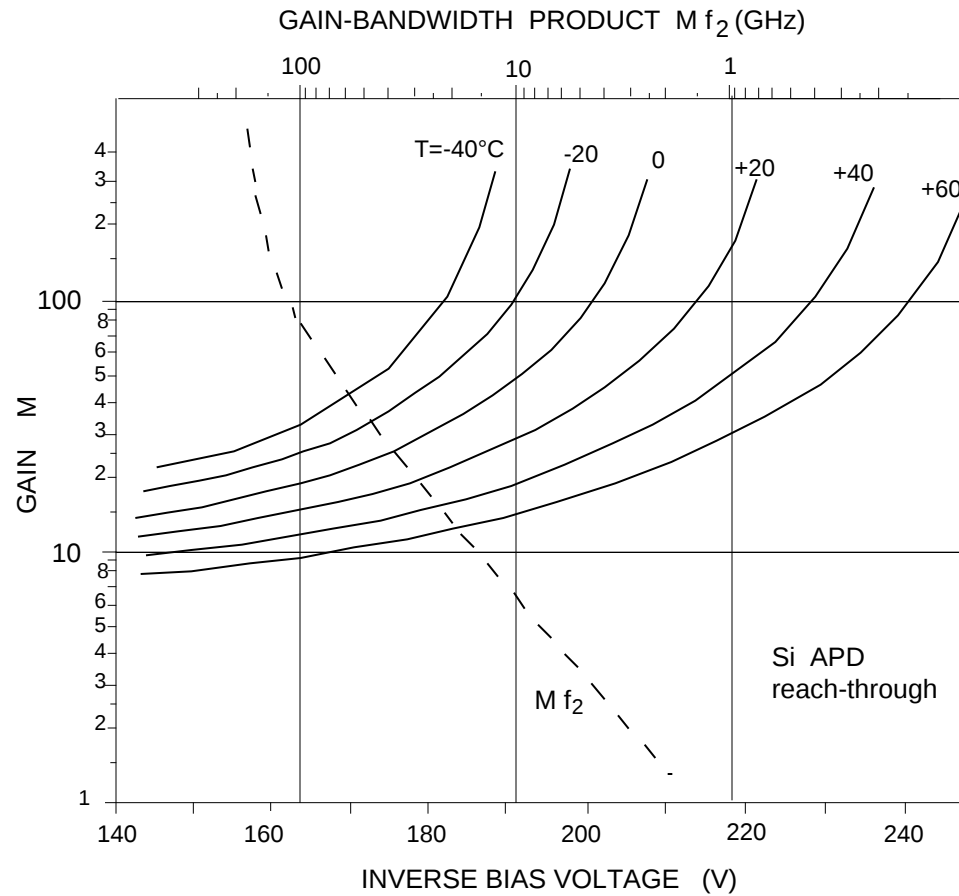
Staircase APD



MQW APD

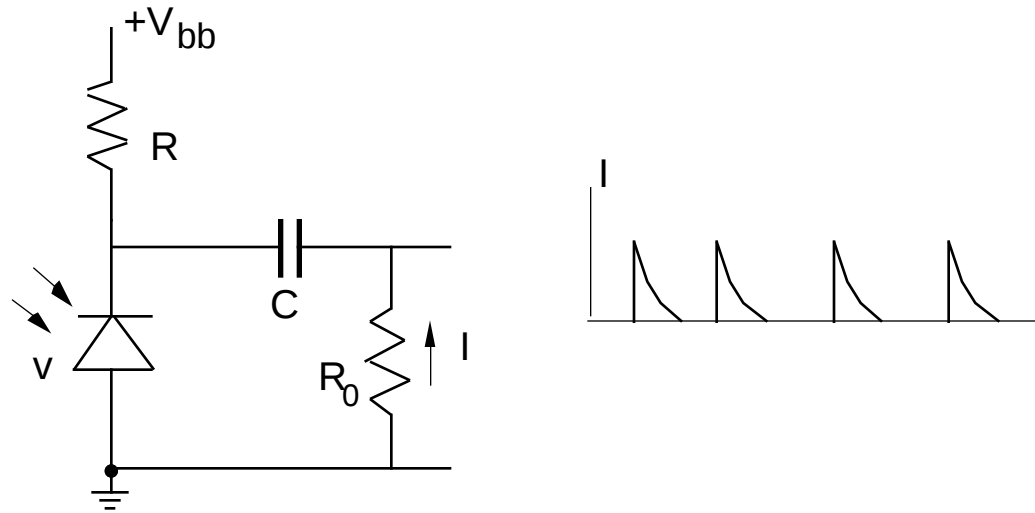


APD Biasing and Use Requisites



from: "Photodetectors", by S. Donati, Prentice Hall 2000

SPAD



in quiescent conditions, $V_{bb} > V_{\infty}$, $M = \infty$

a single charge-pair makes I increase fast up to $V_{ak} = V_{bb} - RI < V_{\infty}$;
then M returns to a finite value

Max. photon rate: $F_M < \tau_{rec} = R_0 C$; dark rate: $F_b = J_0 A / q$

Dynamic range: $F_M / F_b = 10^3$ typ.