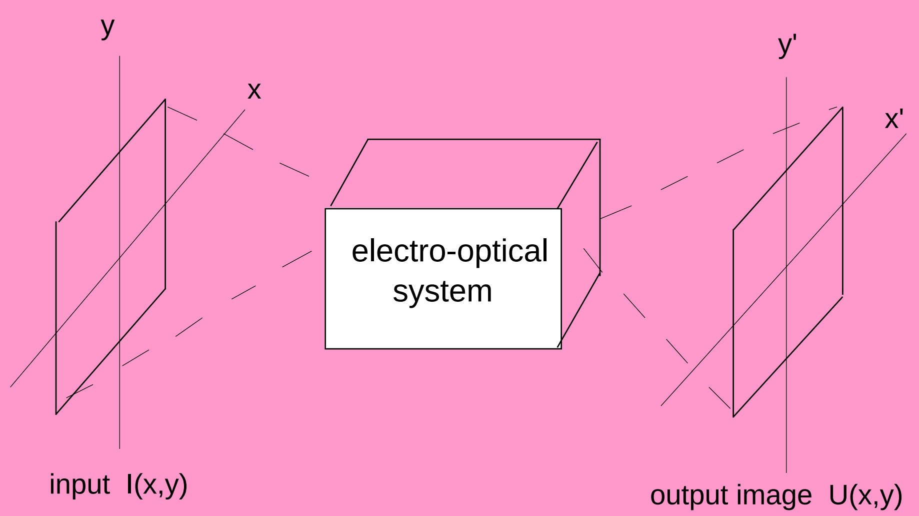
# Optical Transfer Function




The OTF describes resolution of an image-system. It parallels the concept of electrical transfer function, about transformation of an object I(x,y) to its image U(x,y). Signals are now the spatial distributions of the radiant power density in the coordinates x,y (instead of time t).

## Fourier domain representation

Fourier transforms **I** and **U** give frequency content of I and U :

$$\mathbf{I}(k_x ,k_y) = \int_{-\infty}^{+\infty}\int \exp 2\pi i(k_x x+k_y y)\, I(x,y)\, dx\, dy$$

$$\mathbf{U}(k_x ,k_y) = \int_{-\infty}^{+\infty}\int \exp 2\pi i(k_x x+k_y y)\, U(x,y)\, dx\, dy$$

Antitransforms are:

$$I(x,y) = \int_{-\infty}^{+\infty}\int \exp -2\pi i(k_x x+k_y y)\, \mathbf{I}(k_x ,k_y)\, dk_x\, dk_y$$

$$U(x,y) = \int_{-\infty}^{+\infty}\int \exp -2\pi i(k_x x+k_y y)\, \mathbf{U}(k_x ,k_y)\, dk_x\, dk_y$$

In a linear system, ratio **U**/**I** is independent of the input and is called the OTF (*optical transfer function*) of the system.

# OTF and MTF

Taking **I**=1, we see that OTF is the system response to the spatial Dirac-delta $\delta(x,y)$ input image, or:

$$OTF(k_x,k_y) = \mathbf{U}(k_x,k_y)/\ \mathbf{I}(k_x,k_y) = \mathbf{U}_{\delta(}k_x,k_y)$$

The antitransform $U_\delta(x,y)$ of $\mathbf{U}_\delta$ is the *confusion distribution* of the image system (its width is the *confusion circle* of classical optics).

In a system invariant to rotations/translations of the input image:

$$OTF(k_x,k_y) = OTF(k_x)\ OTF(k_y)$$

and we can limit ourselves to consider one component OTF(k). We can express in general the OTF(k) as the product of a modulus and a phase factor:

$$OTF(k) = MTF(k)\ \exp i\ PTF(k)$$

# MTF

Modulus MTF is called the *modulation transfer function*, and PTF is called the *phase transfer function*.

In linear systems, PTF vanishes because $U_\delta(x,y)$ is symmetrical to the origin x,y=0 and its transform – the sine of the imaginary part – has a zero integral. Thus:

$$OTF(k) = MTF(k)$$

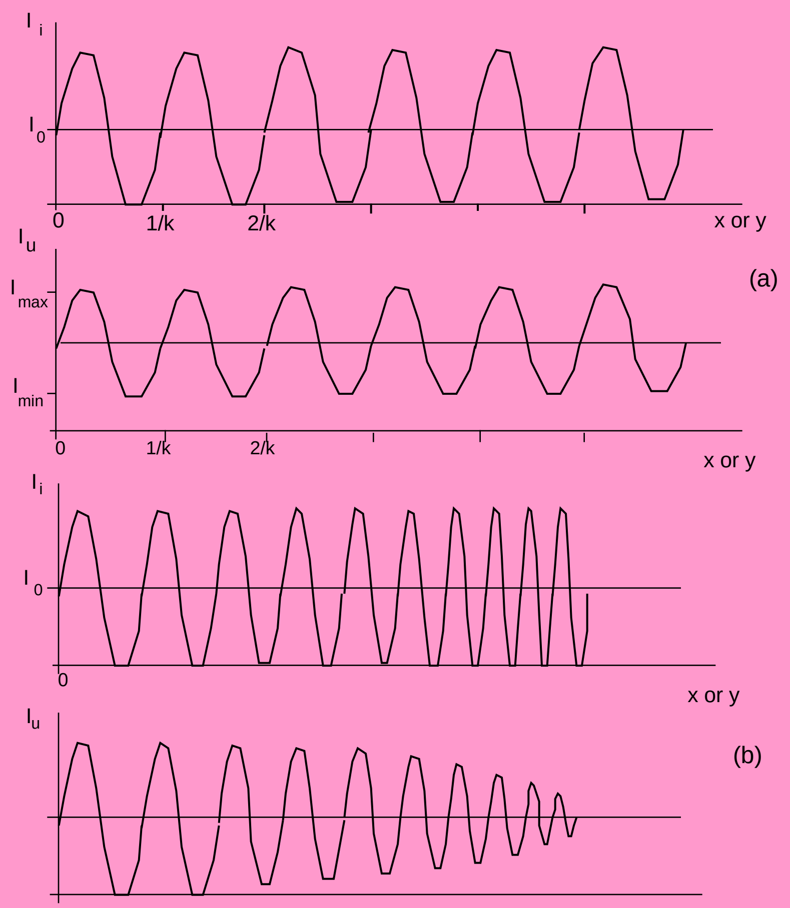
MTF is the ratio of the output-to-input signal content at frequency k, or, the ratio of output to input contrast C for an input of the type:

$$I(x) = I_0(1+ C \sin 2\pi kx)$$

Contrast C is defined as:

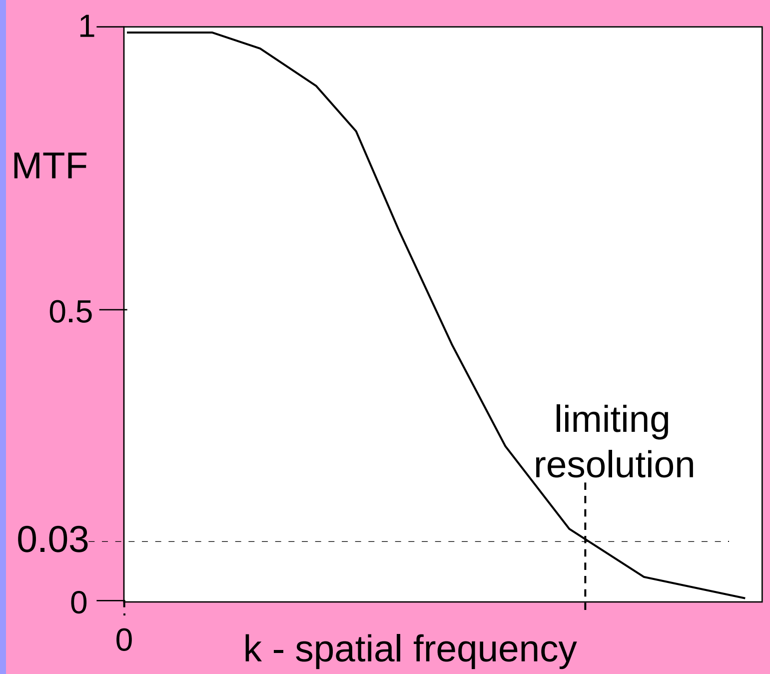
$$C = (I_{max}-I_{min}) / (I_{max}+I_{min})$$

# I/O contrast




A procedure to measure MTF readily follows: at system input a grating is presented with variable period and unity contrast C=1 .For each spatial frequency k, output contrast $C_U$ is computed; this $C_U$ is the MTF. The procedure can be carried out visually using resolution charts to find out limiting resolution.

# Limiting resolution




A generic MTF diagram always starts from MTF=1 at k≈ 0 (low spatial frequency), and gradually decreases to zero at increasing spatial frequencies.

The cutoff frequency is defined as that of eye perceptivity to contrast, $C_{lim}$=0.03, and the corresponding spatial frequency is called *limit spatial frequency* (or, limiting resolution).

## MTF of cascaded systems

To analyze the cascade of several image systems, we can write the output contrast from the n-th system as:

$$C_n=(C_n/C_{n-1})(C_{n-1}/C_{n-2})\ldots(C_1/C_0)C_0,$$

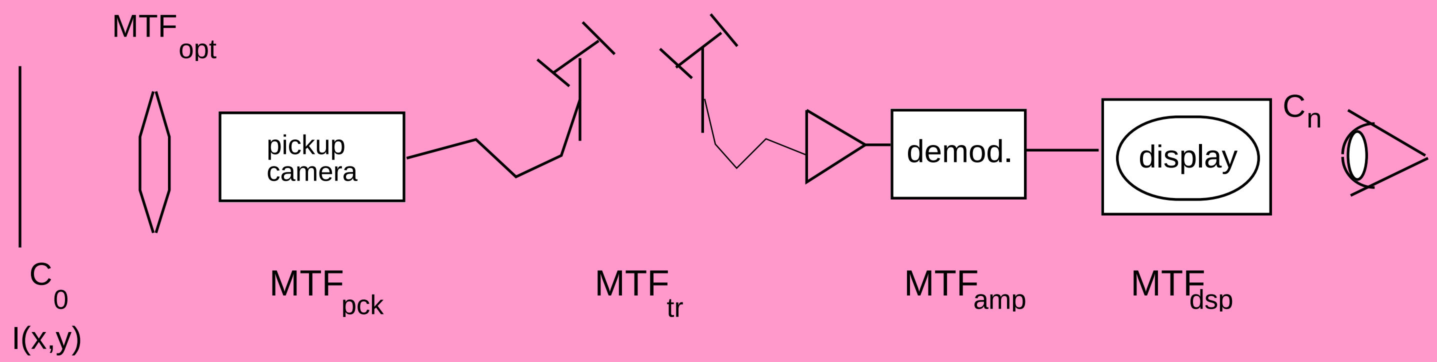
where $C_0$ is the input image contrast; the composition rule in product follows as:

$$MTF = MTF_1 \cdot MTF_2 \cdots MTF_n$$

To apply this rule, the spatial frequencies k of all individual terms shall be homogeneous.

Consider a TV broadcast of images, where we have: an objective lens, an image pickup device, a transmission, an amplifier and demodulator circuits, and a display, all described by an MTF.

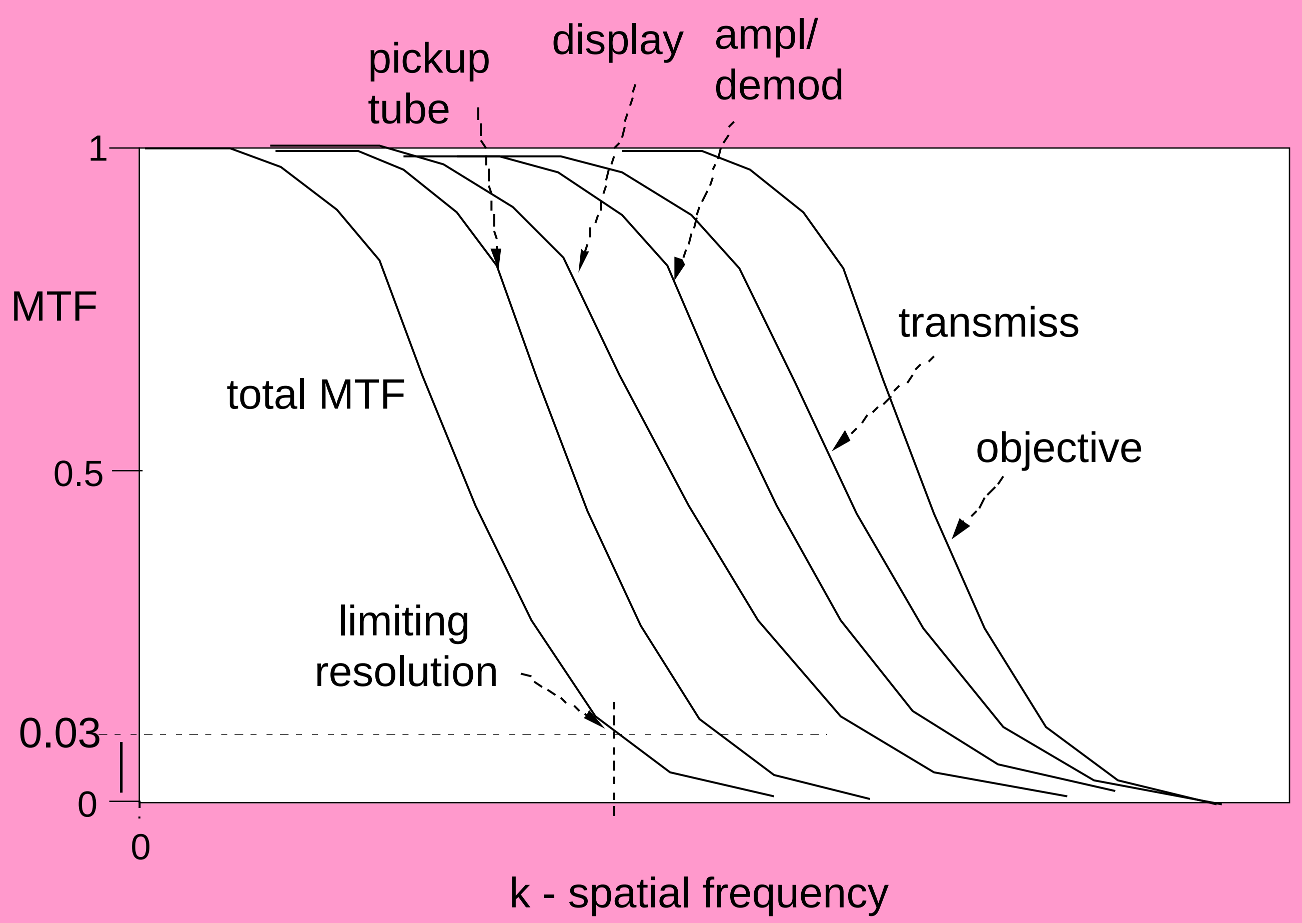
# Making MTFs homogeneous




Frequencies k or f are easily connected to each other, and if we choose to express them, all in terms e.g., of $k_x$ we get:

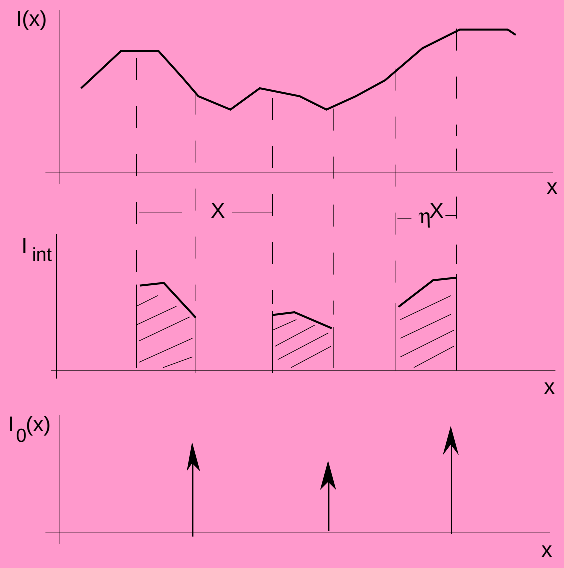
- $k_\theta = k_x F$, where F is the objective focal length;
- $f = k_x v$, where v is the horizontal scan speed of the image
- $k_\psi = k_{x'} D$, where D is the eye distance from display;
- $k_{x'} = M k_x$, where $M = d_{dsp}/d_{rip}$ is the linear magnification.

# Finding resolution




from:"Photodetectors", by S.Donati, Prentice Hall 2000

# Image sampling

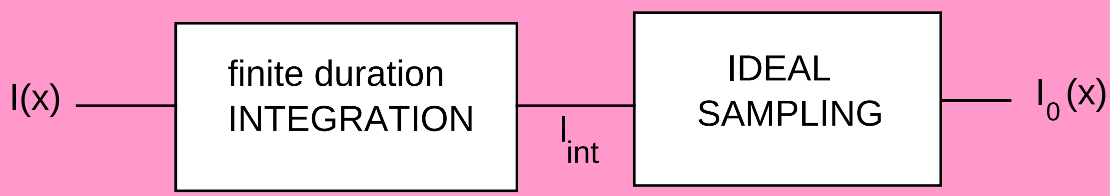



On an image element we find:

• integration on a finite duration (or,integrate-and-dump) with a δ-response $C(x)= \mathbf{1}(x)-\mathbf{1}(x-\eta X)$

• an ideal sampling equivalent to multiplying the result by: $\mathrm{comb}(x)=\Sigma_{m=-\infty,+\infty}\ \delta(x-mX)$

# Sampling schematization



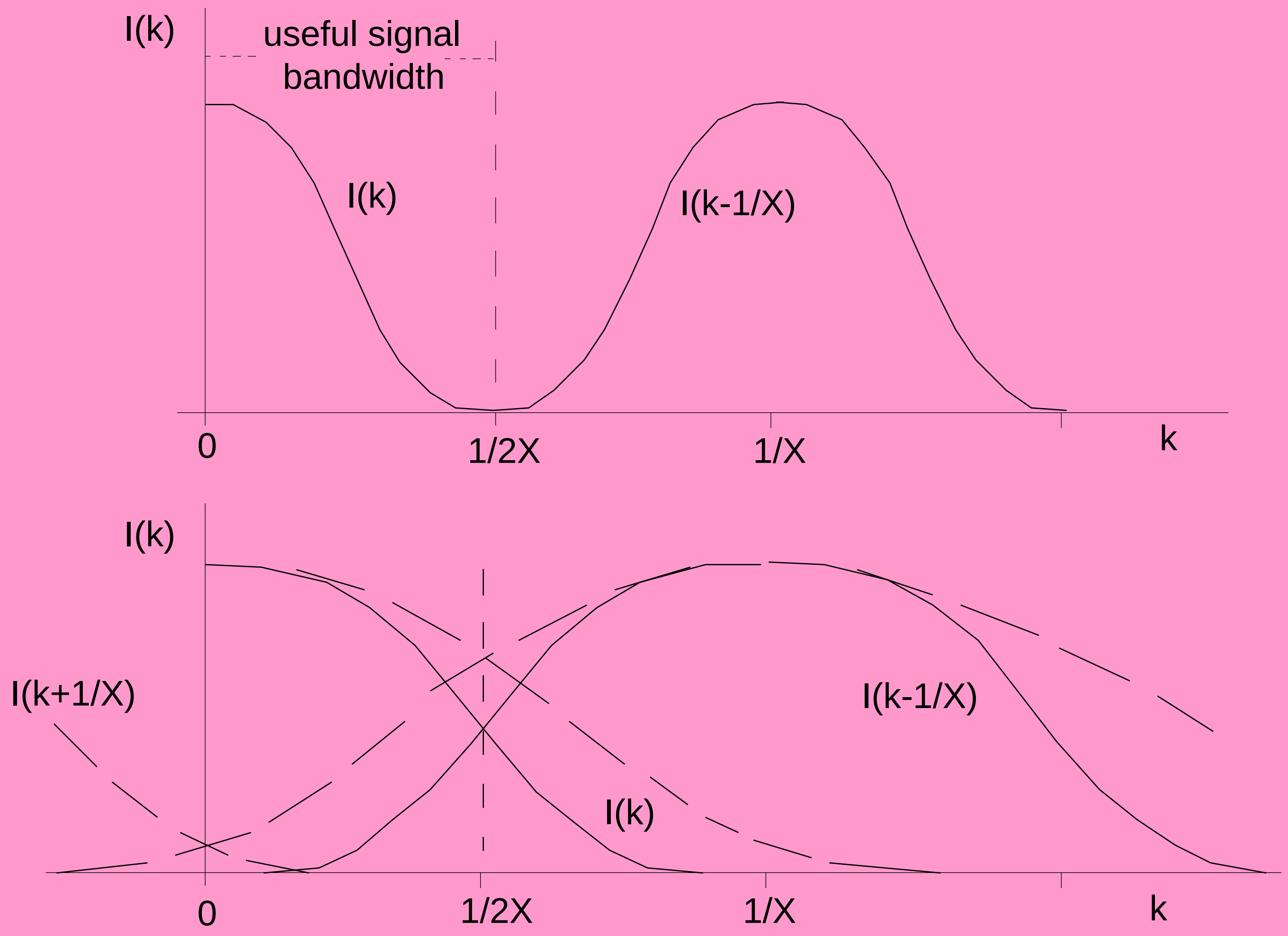

$$I_0(x) = [I(x) * C(x)] \cdot \mathrm{comb}(x)$$

$$\mathbf{I}_0(k) = [\mathbf{I}(k) \cdot \mathbf{C}(k)] * \mathrm{comb}(x)$$

$$= \Sigma_{m=-\infty\ +\infty}\ I(k-m/X)\ C(k-m/X)$$

input signal I(k) is repeated periodically every 1/X in spatial frequency, at integer multiples (positive and negative) of the sampling frequency. Integrate and dump has: $\mathbf{C}(k) = \sin(2\pi k\eta X/2) / (2\pi k\eta X/2)$

# Moire’ (or aliasing) effect




from:”Photodetectors”, by S.Donati, Prentice Hall 2000

# Measuring I(k) by Moirè

Before the image we place a sinusoidal grating with vertical bars having a sinusoidal transmission

$$T = T_0(1+\cos 2\pi k_x x),$$

and collect the transmitted radiation at a single PD. Current is:

$$i = \int_x I(x,y)\, T_0(1+\cos 2\pi k_x x)\, dx$$

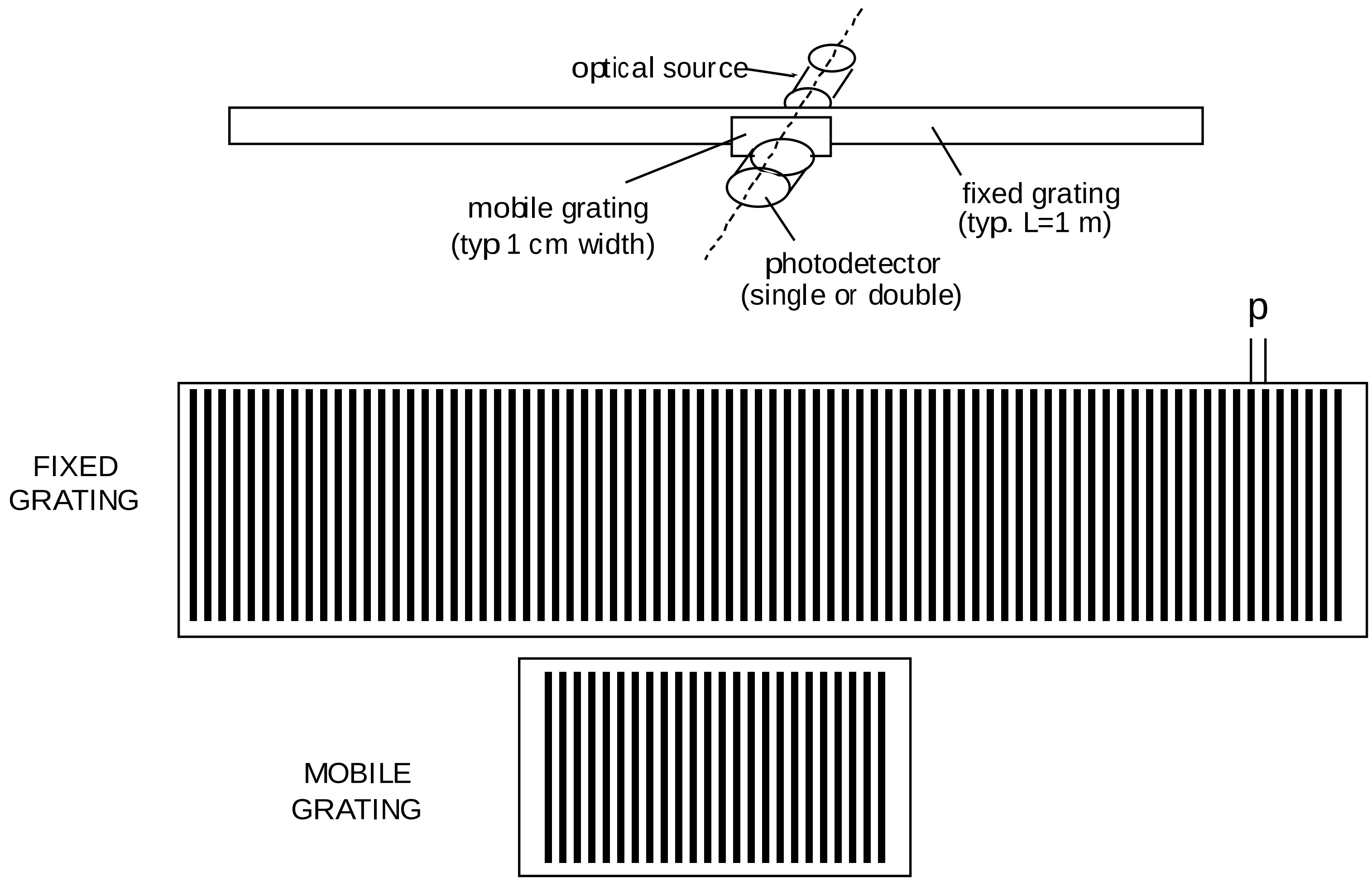
and, recalling that I = *FT* (**I**):

$$i = T_0\, [\mathbf{I}(0,0) + \mathbf{I}(k_x,0)\,]$$

or, we get the Fourier transform component in $k_x$ (plus a dc term).

Similarly, with a horizontal grating, we get **I**($k_y$,0).

# Optical rule

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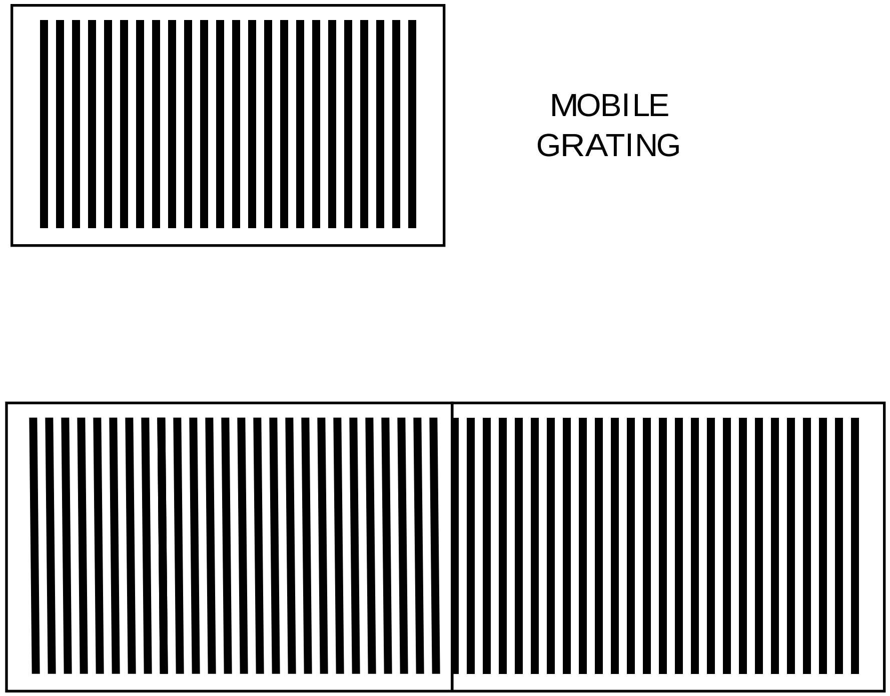
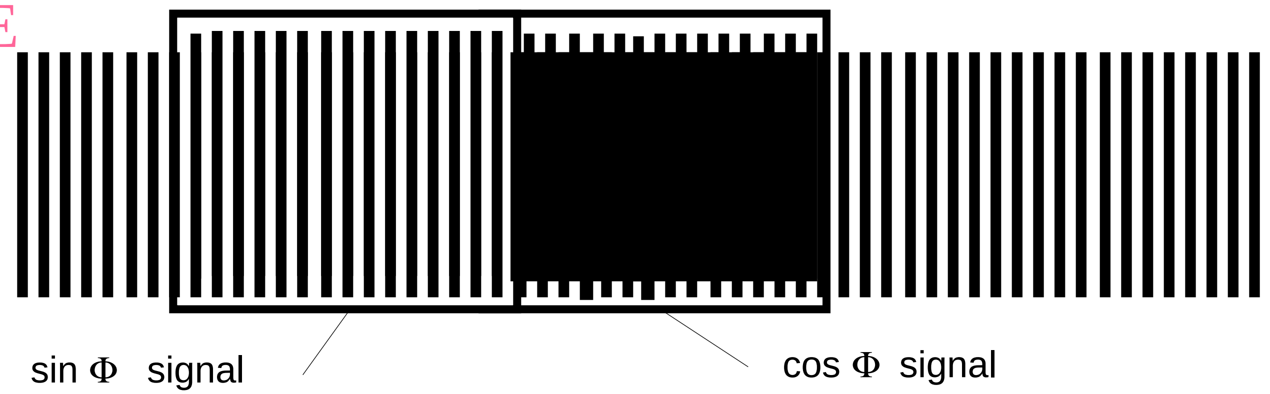
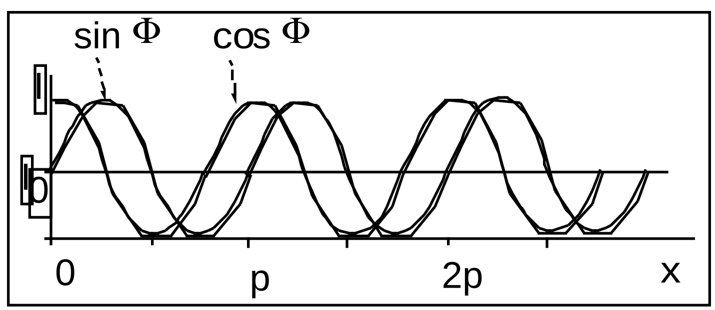



MOB LE GRAT ING with a p/4
PHASE JUMP at CENTER

# Optical rule circuits

RIFARE

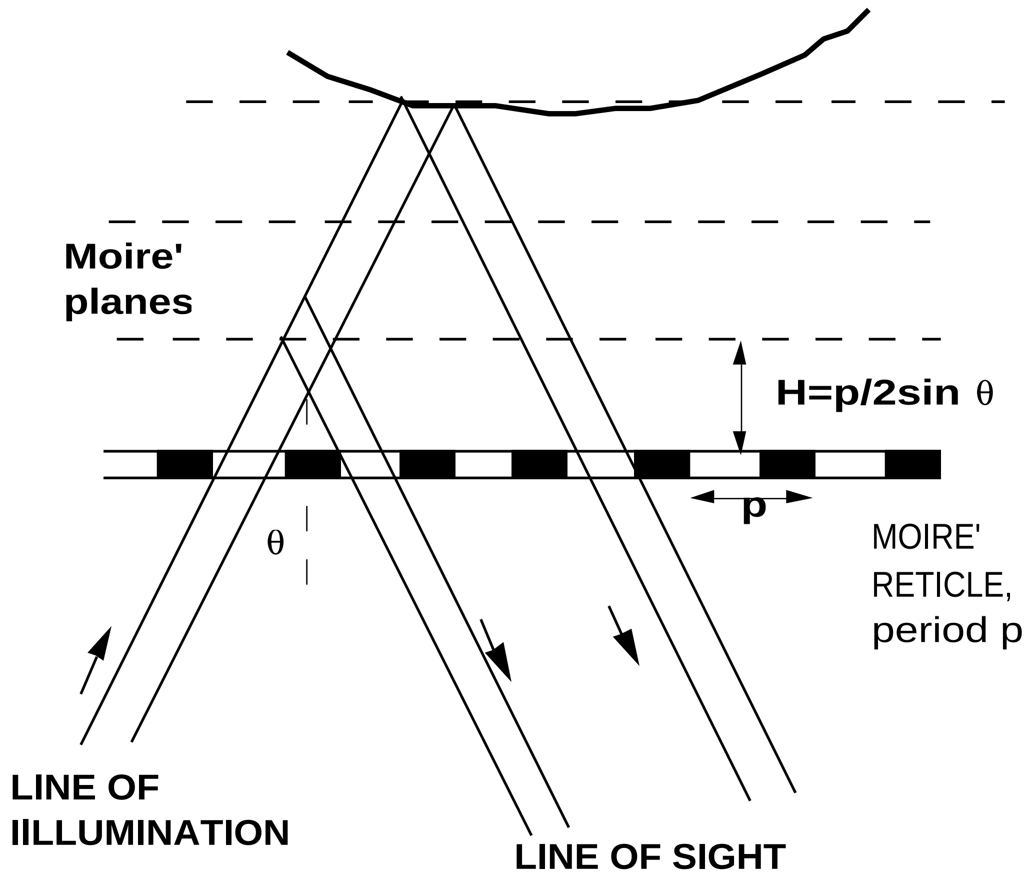



By counting the sin Φ and cos Φ crossings of mean level $I_0$, (4 per period) displacement is measured with p/4 resolution (and sign)

from:"Photodetectors", by S.Donati, Prentice Hall 2000

# 3D Contouring

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## Moiré contouring

Aiming to devise a simple technique for checking scoliosis,

Takasaki has reported (Applied Optics vol. 12 p.845) this example of 3-D contouring on a small statue 50-cm tall