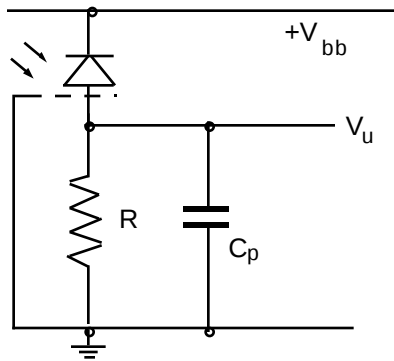
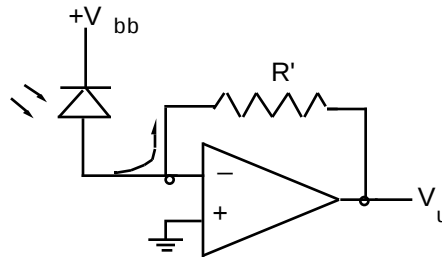


Overview



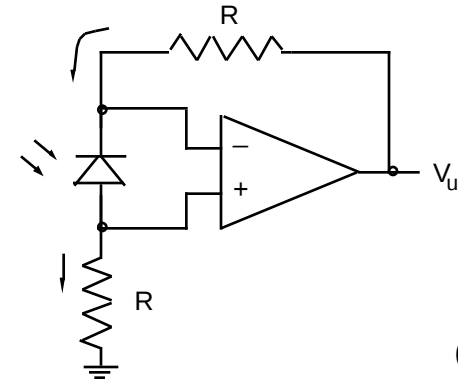
(a)

basic circuit



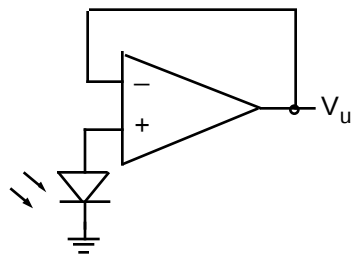
(b)

transimpedance



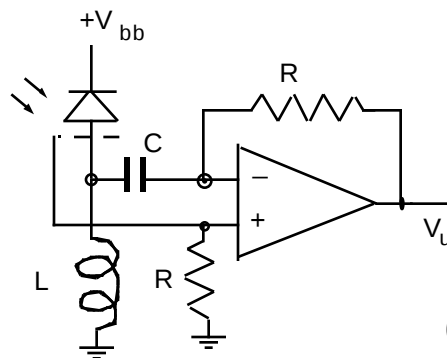
(c)

dark current cancellation



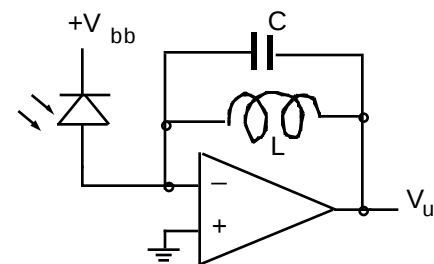
(d)

logarithmic



(e)

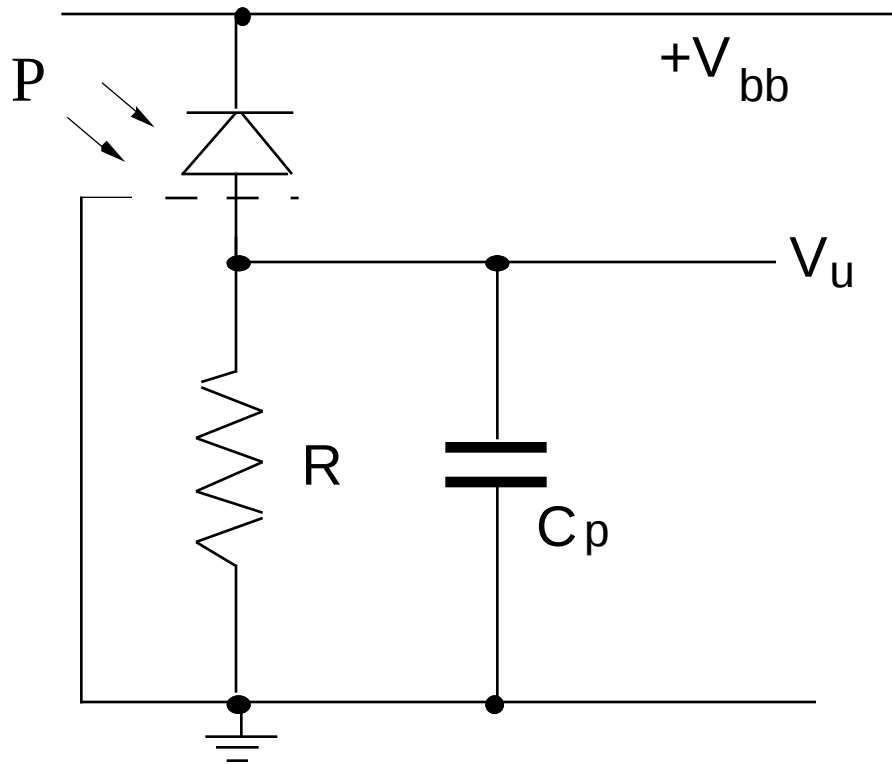
dc cancellation



(f)

narrow-band

Basic Scheme



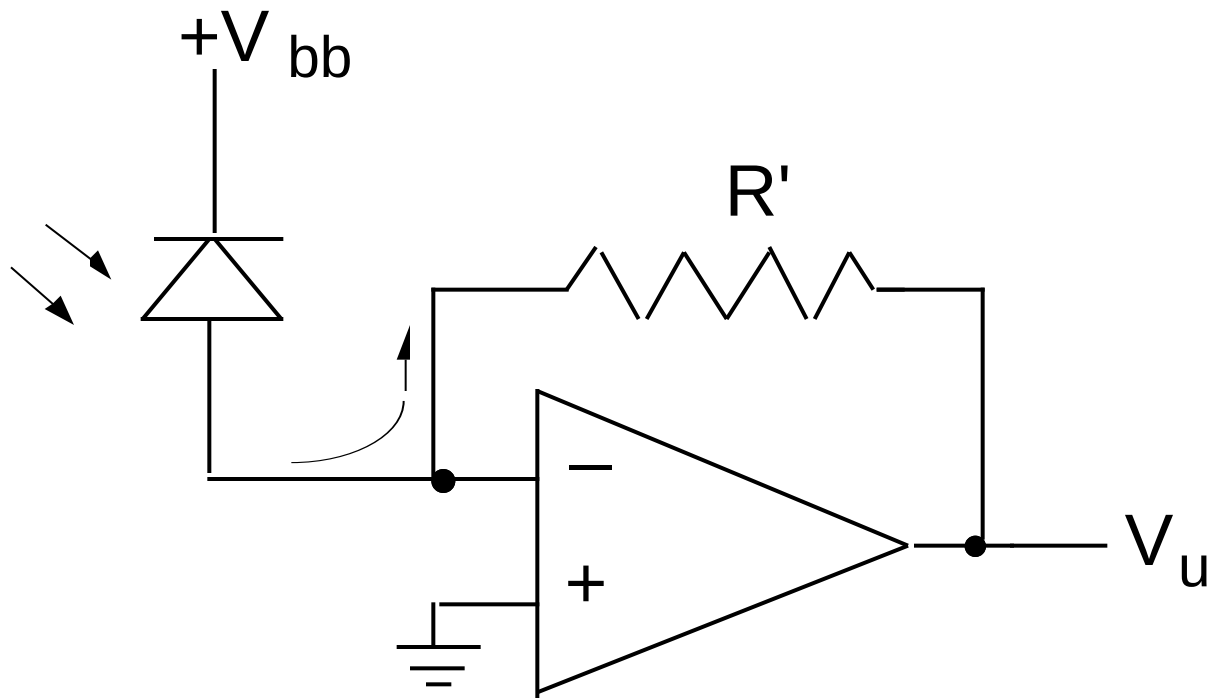
$$V_u = V_{u0} + v_u,$$

$$V_{u0} = I_o R \quad v_u = \sigma P R,$$

$$v_{nu}^2 = 4kTBR + 2e(\sigma P + I_o)R^2B$$

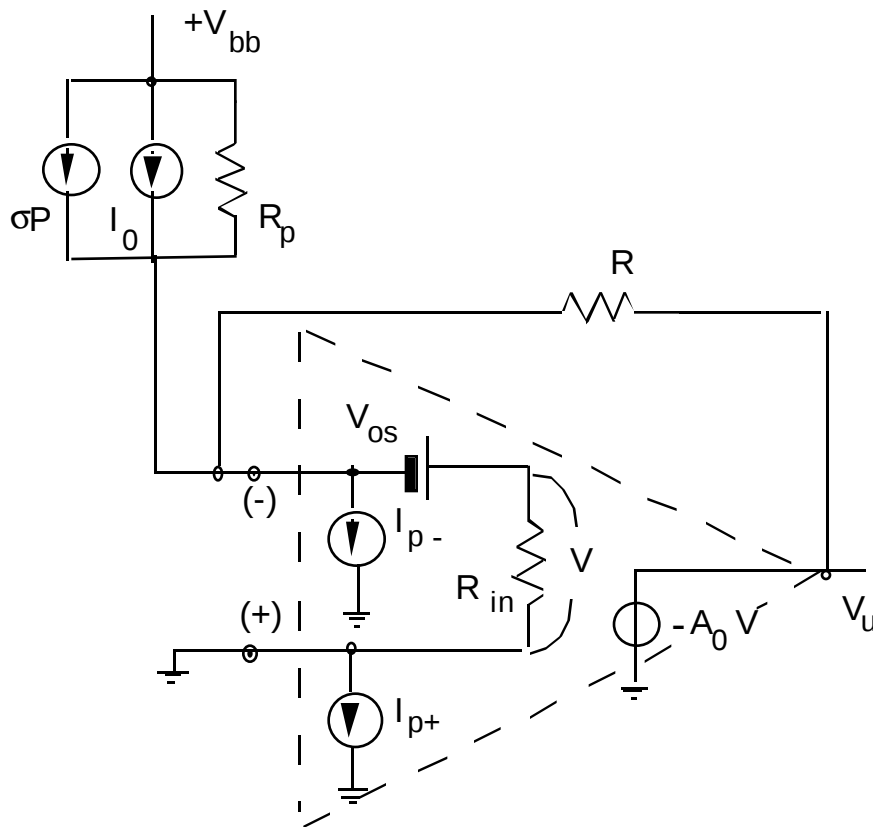
$$f_2 = 1/2\pi RC_p, \quad C_p = C_b + C_{ext}$$

Cold-resistance (or Transimpedance) Scheme



As PD sees a much smaller $R = R'/(1 + A)$, this *cold resistance* preamplifier has the same noise and a much larger bandwidth than the basic circuit

Transimpedance in dc



$$V_u = -\sigma P R \gamma + V_{u0}$$

$$V_{u0} = - (I_o - I_{p-}) R \gamma + V_{os} (1 + R/R_p) \gamma$$

$$\text{where } \gamma = [1 + (1 + R/R_i)/A_o]^{-1}$$

$$\text{and } R_i = R_p // R_{in}$$

for $\gamma \approx 1$ (or, $A_o \gg R/R_i$):

$$V_{u0} = -(I_o - I_{p-}) R + V_{os} (1 + R/R_p) V_{u0} \\ \approx V_{os} \text{ for small } R, \text{ and is dominated by term } I_o - I_{p-} \text{ at large } R; \text{ break-point value is: } R = V_{os}/(I_o - I_{p-})$$

in typ. Op-Amps:

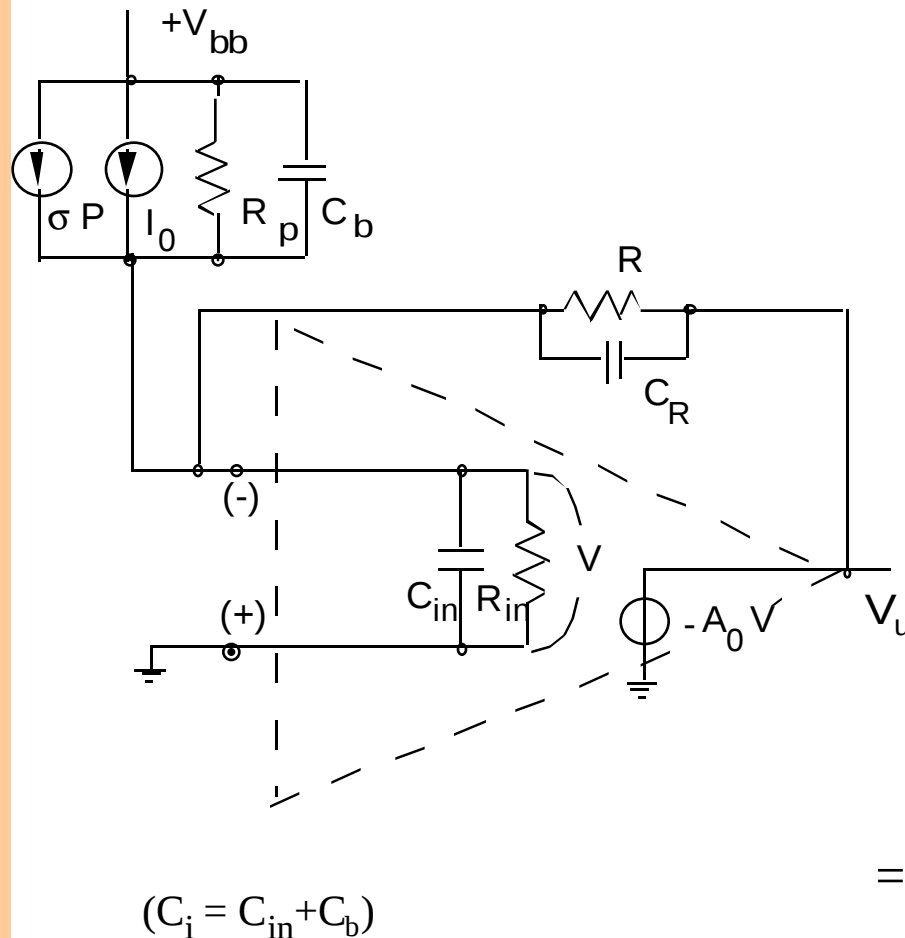
$$V_{os}/I_p = 0.5\text{mV}/0.2\mu\text{A} = 2.5 \text{ k}\Omega$$

with a bipolar input stage, and :

$$V_{os}/I_p = 3\text{mV}/200\text{pA} = 1.5 \text{ M}\Omega$$

with a FET input stage

Transimpedance in ac



$$v_u(0) = -\sigma P R \gamma \approx -\sigma P R$$

$$v_u(s) = -\sigma P Z / [1 + (1 + Z/Z_i)/A]$$

transfer function is:

$$v_u(s)/v_u(0) = \{1 + s[RC_R + (1 + R/R_i)/\omega_o A_o + R(C_i + C_R)/A_o] + s^2 R(C_i + C_R)/\omega_o A_o\}^{-1}$$

or, is of the type: $\{1 + s\chi/\omega_2 + s^2/\omega_2^2\}^{-1}$,

a 2nd-order response, with **cutoff frequency**:

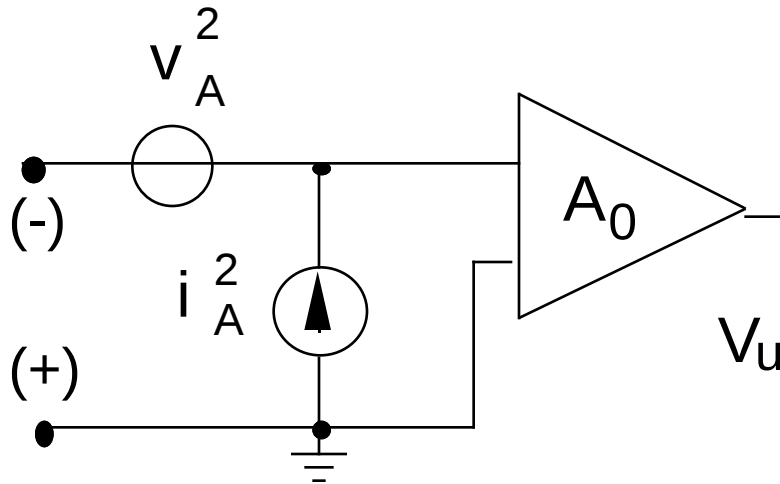
$$f_2 = [2\pi R(C_i + C_R)/\omega_o A_o]^{-1/2} = [f_R GB_A]^{1/2},$$

(the geometrical mean of f_R and GB_A)

and **damping**:

$$\chi = \frac{1}{2} f_2 (1/f_R + 1/GB_A) = \frac{1}{2} \{ [C_R/(C_i + C_R)] [GB_A/f_R]^{1/2} + [f_R/GB_A]^{1/2} \}$$

Transimpedance noise



typ FET Op-Amp (356):
 $[dv_A^2/df]^{1/2} = 15 \text{ nV}/\sqrt{\text{Hz}}$
 $[di_A^2/df]^{1/2} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$

output noise spectral density:

$$dv_{nu}^2/df = [4kT/(R_i // R) + 2e(\sigma P + I_o) + di_A^2/df] (Z\gamma)^2 + dv_A^2/df (1 + \gamma Z/Z_i)^2$$

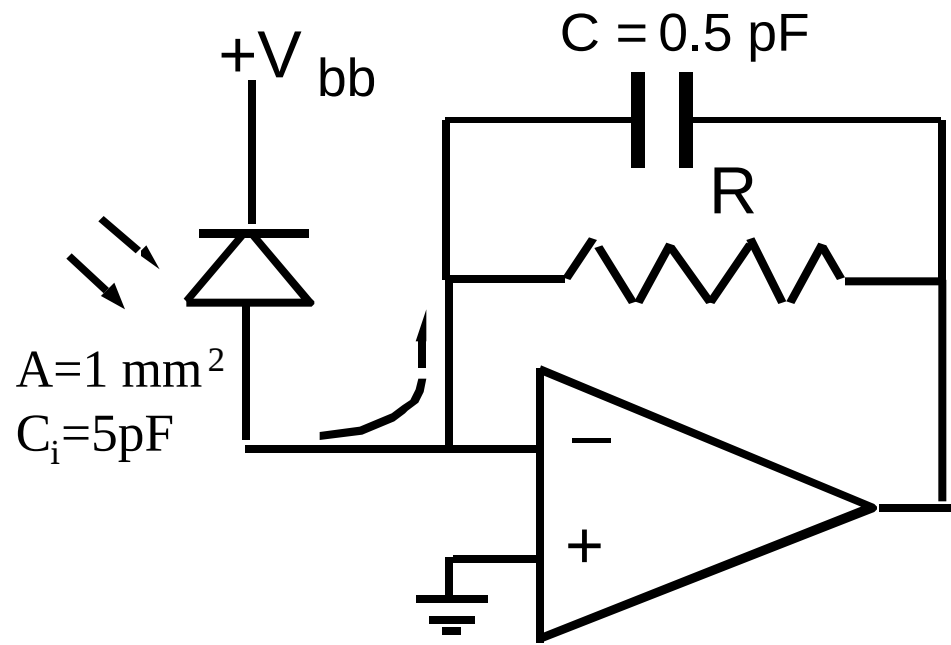
in the passband and for $\gamma \approx 1$, $R \ll R_i$, this becomes:

$$dv_{nu}^2/df = [4kT/R + 2e(\sigma P + I_o) + di_A^2/df] R^2 + dv_A^2/df$$

integrating on f in 0–B, total output noise for a measurement bandwidth B is:

$$\begin{aligned} v_{nu}^2 &= 4kTBR + [2e(\sigma P + I_o)B + i_A^2] R^2 + v_A^2 \\ &= 4kTBR + 2e[\sigma P + I_o + I_A(1 + R_A^2/R^2)] BR^2 \end{aligned}$$

Example: transimpedance with an IC Op-Amp



op-amp 741:

$GB_A = 10 \text{ MHz}$, $R_i > 10 \text{ M}\Omega$

Example: transimpedance with an IC Op-Amp

for feedback resistances:

$R=10k, 100k, 1M, 10M\Omega$ giving

$$f_R = 2.9M, 290k, 29k, 2.9kHz$$

high frequency cutoff $\sqrt{GB_A f_R}$ is:

$$f_2 = 5.4M, 1.7M, 540k, 170kHz$$

and damping factor is:

$$\chi = 0.35, 0.46, 0.87, 2.6$$

(overshoot) 5 3 0 0 dB

$$\text{noise: } i_{nu} = \sqrt{[4kTB/R + 2e(\sigma P + I_o)B + i_A^2 + v_A^2/R^2]} = 2eI_{pho}B$$

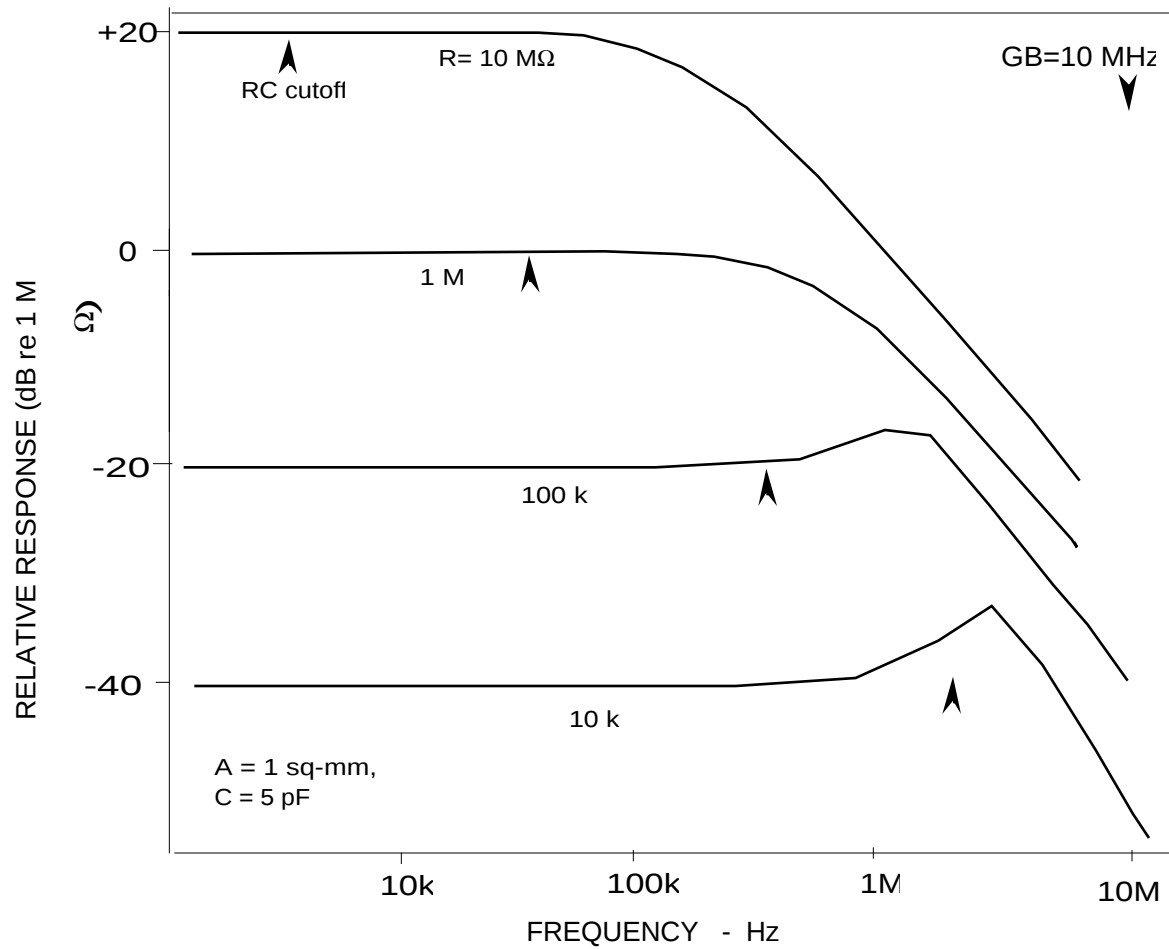
(FET-input)

$$I_{pho} = 12\mu A, 0.57\mu A, 51nA, 5.3nA$$

(bipolar-input)

$$= 12\mu A, 0.6\mu A, 210nA, 200nA$$

IC Op-Amp transimpedance: performance



from: "Photodetectors", by S. Donati, Prentice Hall 2000

noise definitions and parameters

(i) input equivalent *current noise density*

$$di_{\text{neq}}^2/df = (dv_{\text{nu}}^2/df) / [Z\gamma]^2,$$

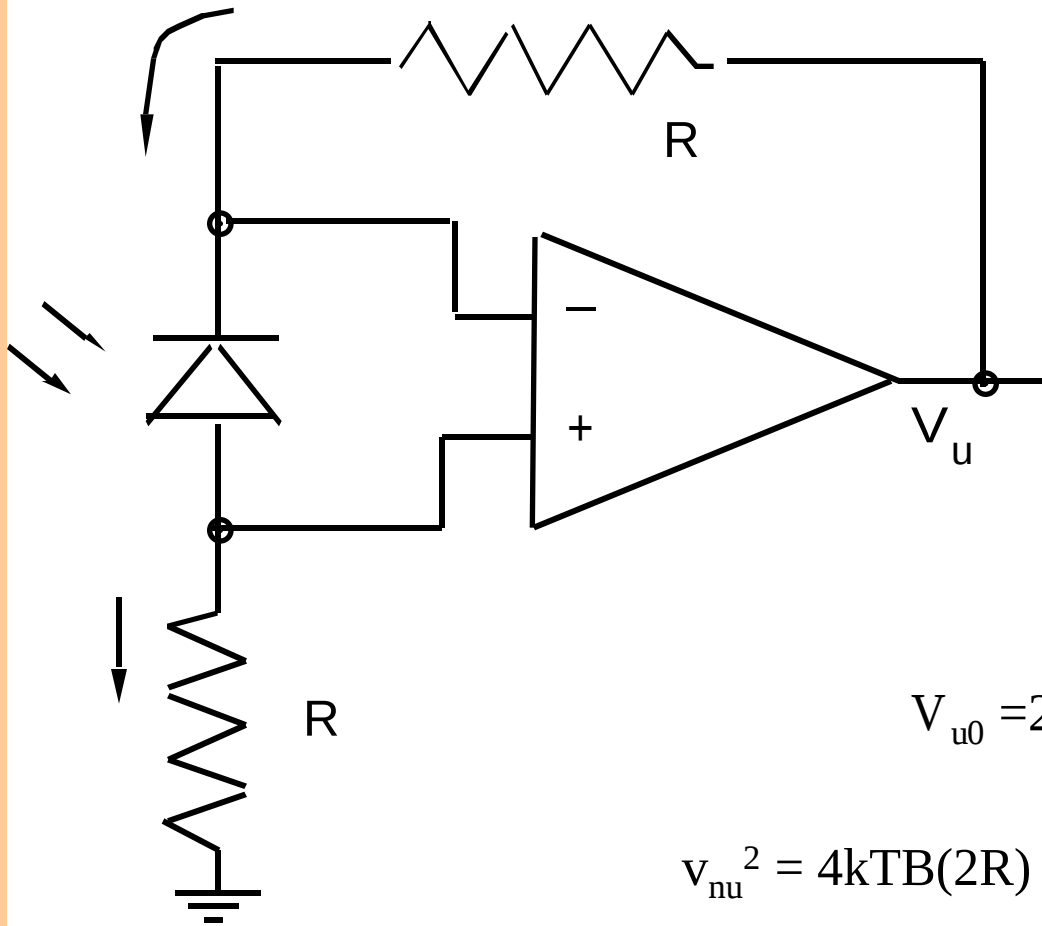
$$\begin{aligned} di_{\text{neq}}^2/df &= 4kT/(R_i // R) + 2e(\sigma P + I_o) + di_A^2/df + dv_A^2/df(1/\gamma Z + 1/Z_i)^2 \\ &= 4kT/(R_i // R) + 2e(\sigma P + I_o) + 2eI_A[1 + (R_A/\gamma Z // Z_i)^2] \end{aligned}$$

(ii) input equivalent *total noise current*, (by integration of di_{neq}^2/df)

(iii) the *noise figure* F_A ratio of the total input noise to that already contained in detected signal and dark current $[2e(\sigma P + I_o)B]$:

$$\begin{aligned} F_A^2 &= 1 + \{4kTB/R + i_A^2 + v_A^2/R^2\} / 2e(\sigma P + I_o)B \\ &= 1 + [(2kT/e)/R + I_A(1 + R_A^2/R^2)] / (\sigma P + I_o) \end{aligned}$$

Dark-cancellation Op-Amp circuit



$$I = I_0 [\exp V/(kT/e) - 1] - I_{ph}$$

$$\text{for } V = V_{os} \ll (kT/e)$$

$$I = I_0 V_{os}/(kT/e) - I_{ph}$$

$$V_u = V_{u0} + v_u$$

$$v_u = 2\sigma P R$$

$$V_{u0} = 2R I_0 [V_{os}/(kT/e)] - R\Delta I_p + V_{os}$$

$$v_{nu}^2 = 4kTB(2R) + [2e(\sigma P + 2I_0)B + i_A^2](2R)^2 + v_A^2$$

Dark-cancellation: an example

$$v_u = 2\sigma P R \quad V_{u0} = 2R I_o [V_{os}/(kT/e)] - R\Delta I_p + V_{os}$$

$$v_{nu}^2 = 4kTB(2R) + [2e(\sigma P + 2I_o)B + i_A^2](2R)^2 + v_A^2$$

Example: using a FET op-amp (356): • $V_{os}=3\text{mV}$, $I_p=200\text{pA}$, $\Delta I_p=5\text{pA}$

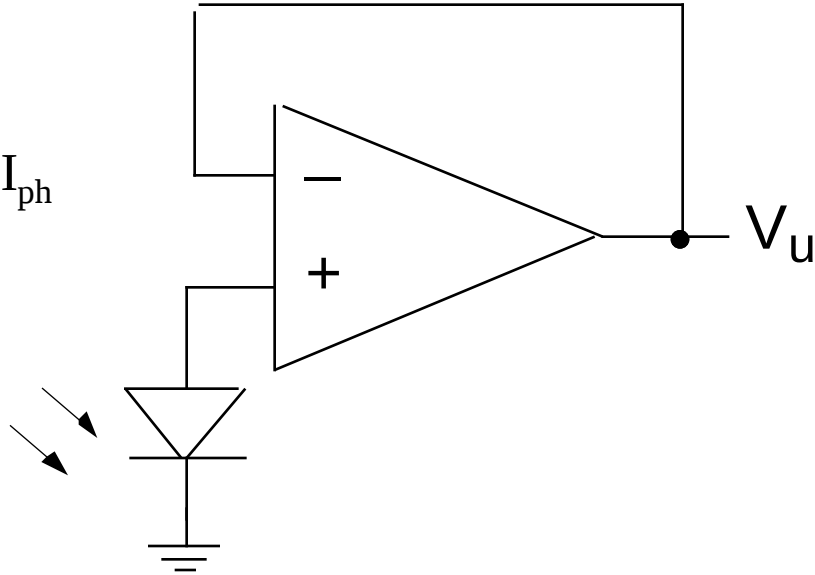
- after trimming of fset to zero, residual error may be $V_{os}=25\mu\text{V}$
- dark current term is reduced to $(25\mu\text{V}/25\text{mV})1\text{nA}=1\text{pA}$
- second tem [...] dominates for $R>25\mu\text{V}/5\text{pA}=5\text{M}\Omega$
- using $R=500\text{M}\Omega$, output offset is $V_{u0}=500\text{M}\Omega\cdot(2\cdot 1\text{pA}+5\text{pA})=3.5\text{mV}$.
- minimum detectable power in dc follows:

$$P = 3.5\text{mV}/(2\sigma 500\text{M}\Omega) = 3.5\text{pW} \quad (\text{at } \sigma=1\text{A/W})$$

- circuit responsivity is $v_u/P=1\text{V/pW}$
- noise voltage is $v_{nu}=13\mu\text{V}$ ($B=1\text{Hz}$), whence $p_{ni}=13\text{fW}$ (or $\approx -110\text{ dBm}$ equivalent noise power)

Logarithmic conversion circuit

$$I = I_o [\exp (eV / nkT) - 1] - I_{ph}$$



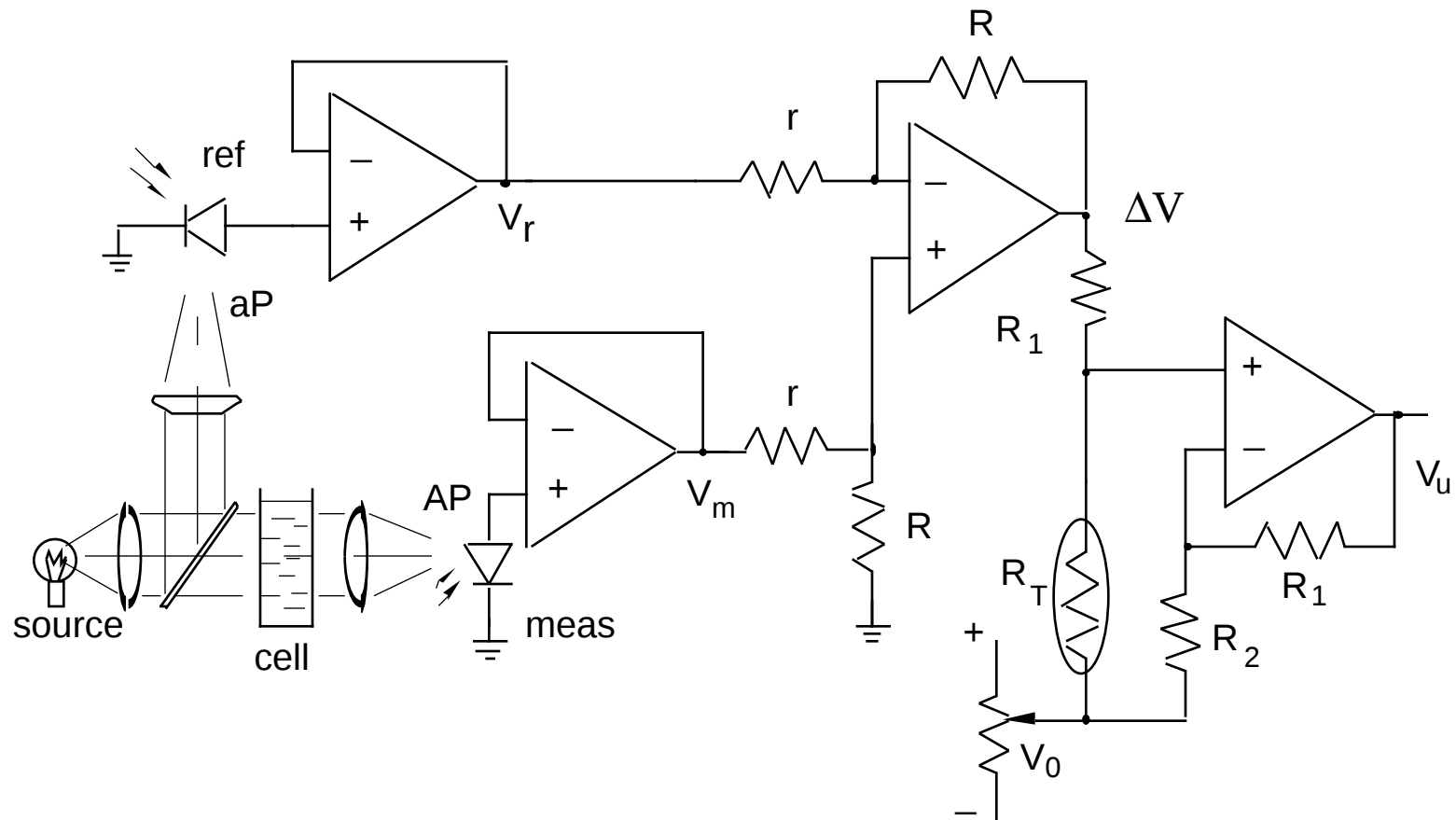
$$V_{u0} = V_{os} + (nkT/e) \ln [(I_{ph} - I_p + I_o)/I_o]$$

at $T=300K$, $n=1$, and for a signal $I_{ph} \gg I_o$, I_p :

$$V_{u0} = V_{os} + [59.6 \text{ mV}] \text{Log}_{10}(I_{ph}/I_o)$$

both scale factor and added constant vary with T

Log-scheme for 2-Ch attenuation measurements



Log-scheme for 2-Ch attenuation measurements (cont'n)

Signals from (meas) and (ref) channels:

$$V_m = V_{osm} + (nkT/e) \ln [(\sigma_m AP - I_{pm} + I_{om})/I_{om}]$$

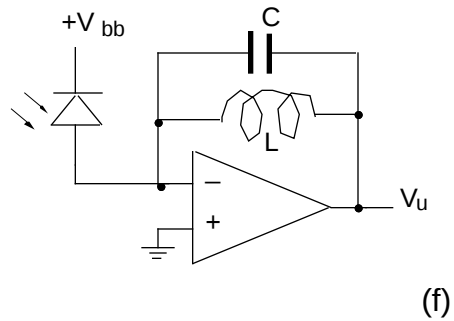
$$V_r = V_{osr} + (nkT/e) \ln [(\sigma_r aP - I_{pr} + I_{or})/I_{or}]$$

Subtracting in 2nd stage, for $\sigma_m AP, \sigma_m aP \gg I_{pm}, I_{om}, I_{pr}, I_{or}, n=1$:

$$\begin{aligned} \Delta V &= (kT/e) \ln A + V_{osm} - V_{osr} + (kT/e) \ln(\sigma_m I_{or} / a \sigma_r I_{om}) \\ &= [59.6 \text{mV}] \text{Log}_{10} A + \text{const} \end{aligned}$$

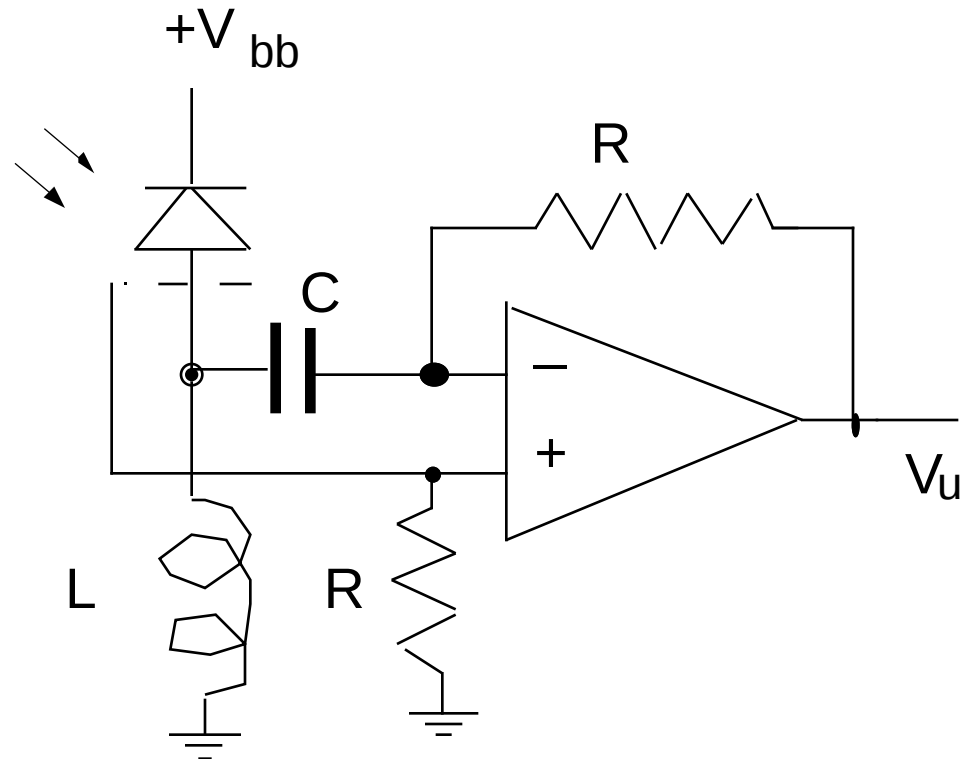
Scale factor drifts with T. NTC thermistor bridge corrects this tempco $\alpha_A = -\alpha_T R_1 / (R_1 + R_2)$ to residual error of $\pm 0.005\%/^{\circ}\text{C}$, and also zero dc level $V_{u(dc)} = \text{const} = [(e/kT)(V_{osm} - V_{osr}) + \ln(\sigma_m I_{or} / a \sigma_r I_{om})] \cdot 1 \text{V}$ against all tempcos [typ.: $\alpha_{Vos} = -4 \text{mV}/^{\circ}\text{C}$, $\alpha_{\sigma} = \pm 5 \text{mV}/^{\circ}\text{C}$, $\alpha_{Io} = +140 \text{mV}/^{\circ}\text{C}$]
to: $v_{nu} = A_2 \cdot 10 \mu\text{V}/\sqrt{\text{Hz}}$ at $I_{ph} = 1 \mu\text{A}$ or $10 \mu\text{AU}/\sqrt{\text{Hz}}$

dc cancellation scheme

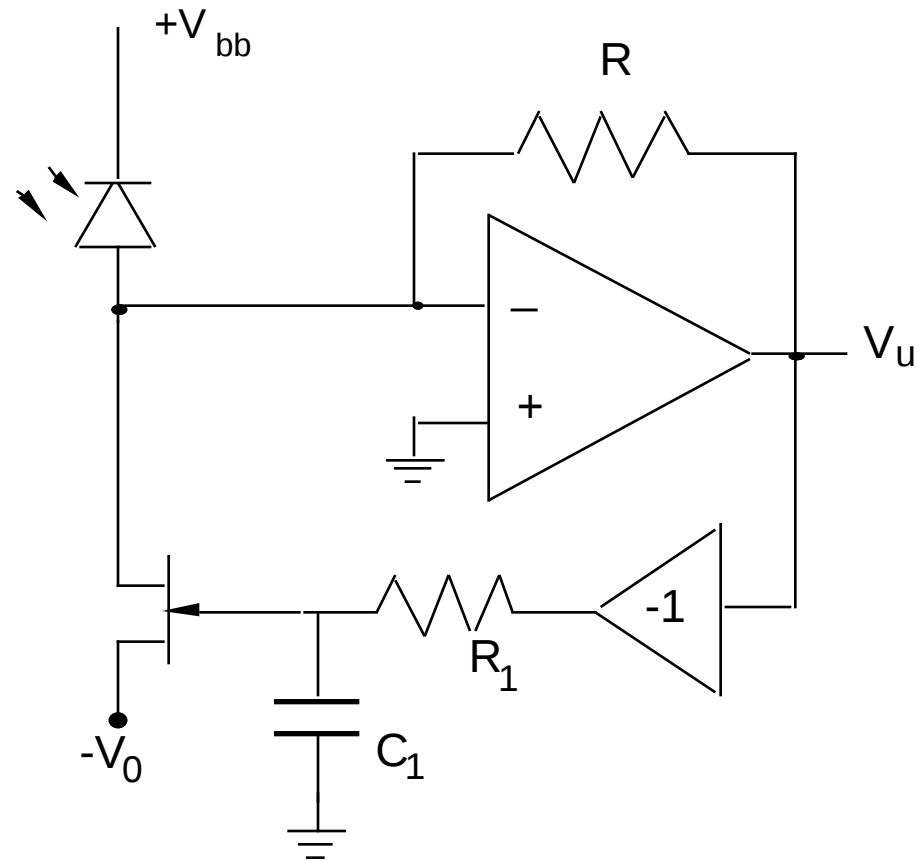


$$f_{\text{pass}} = 1/2\pi\sqrt{LC},$$

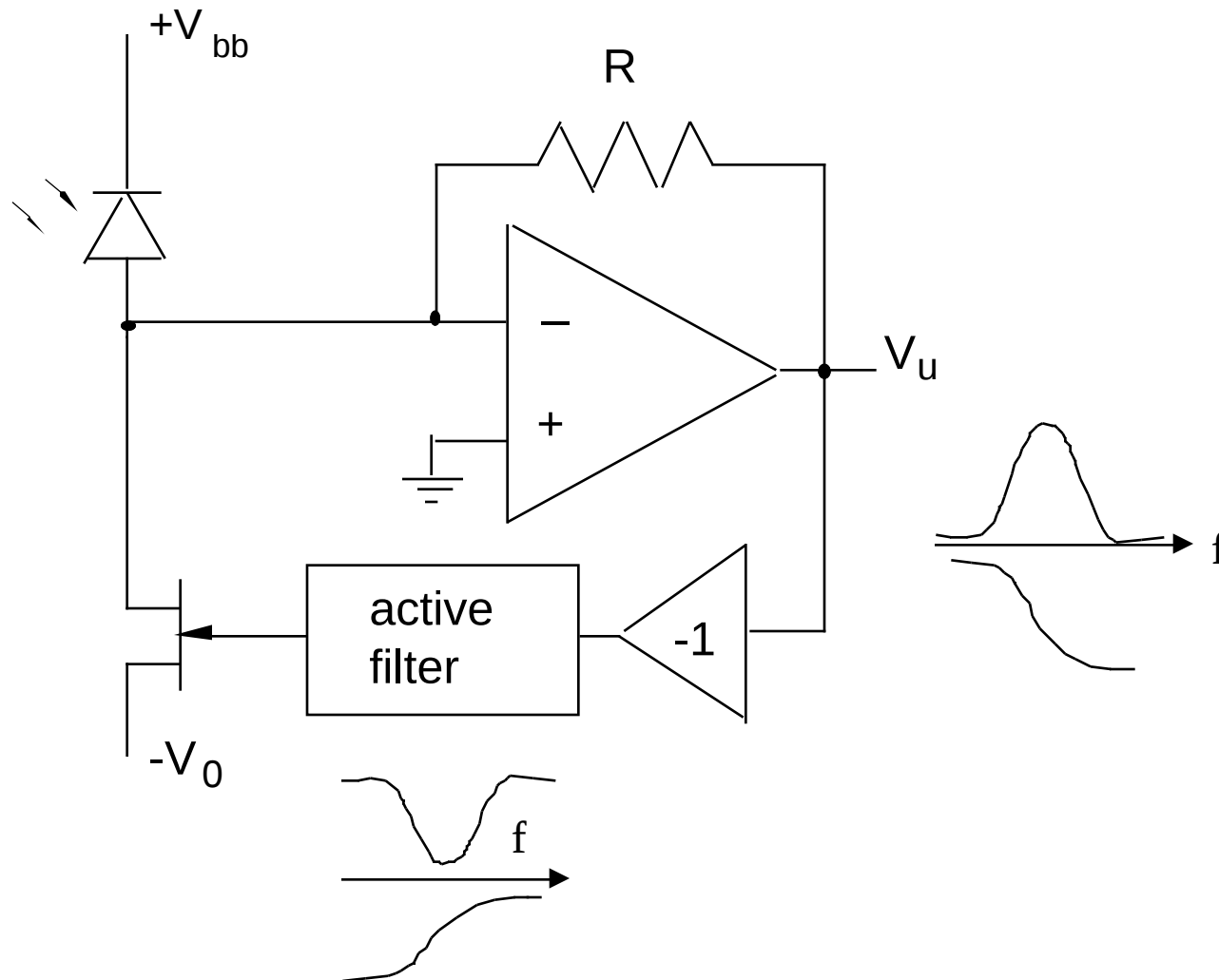
$$\chi = 1/2(\sqrt{C/L})R/(1+A)$$



dc cancellation scheme (2)

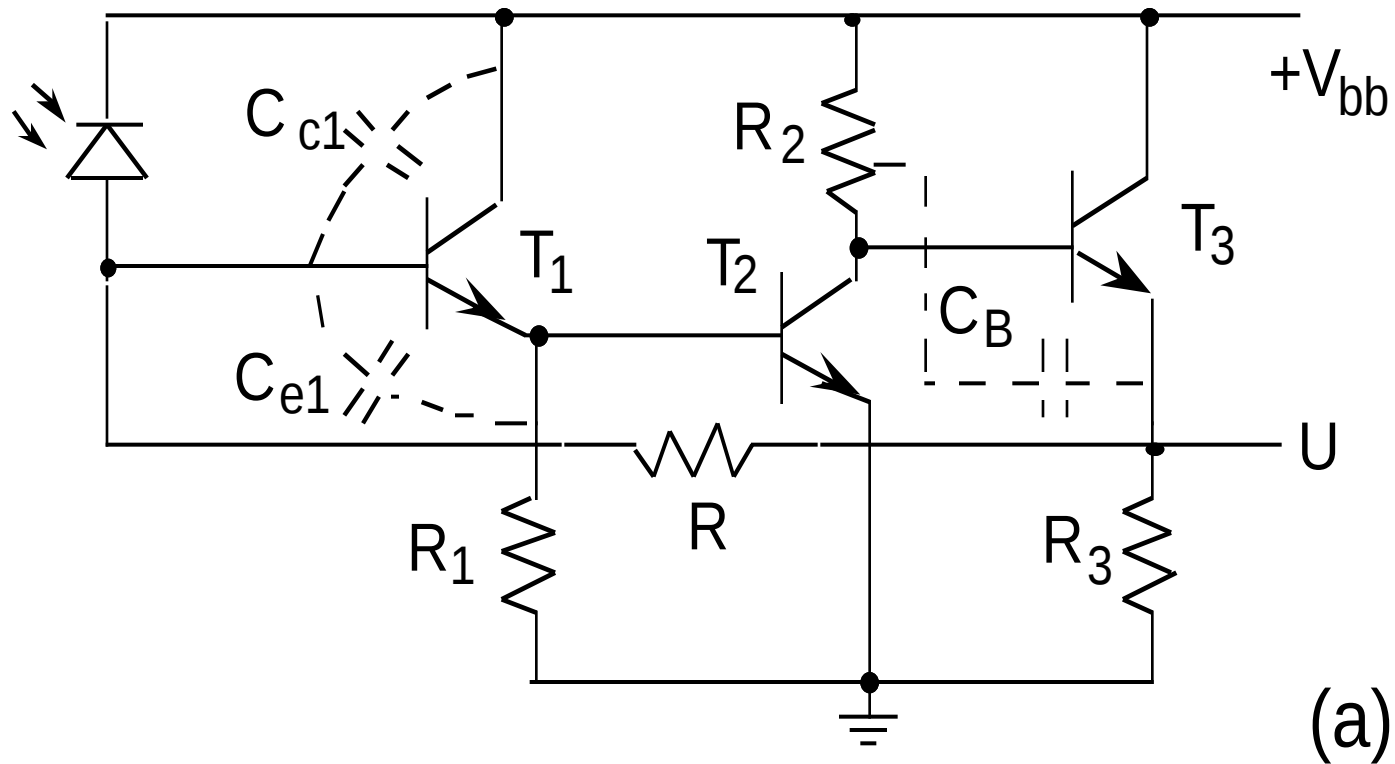


Selective (or synthetic response) scheme

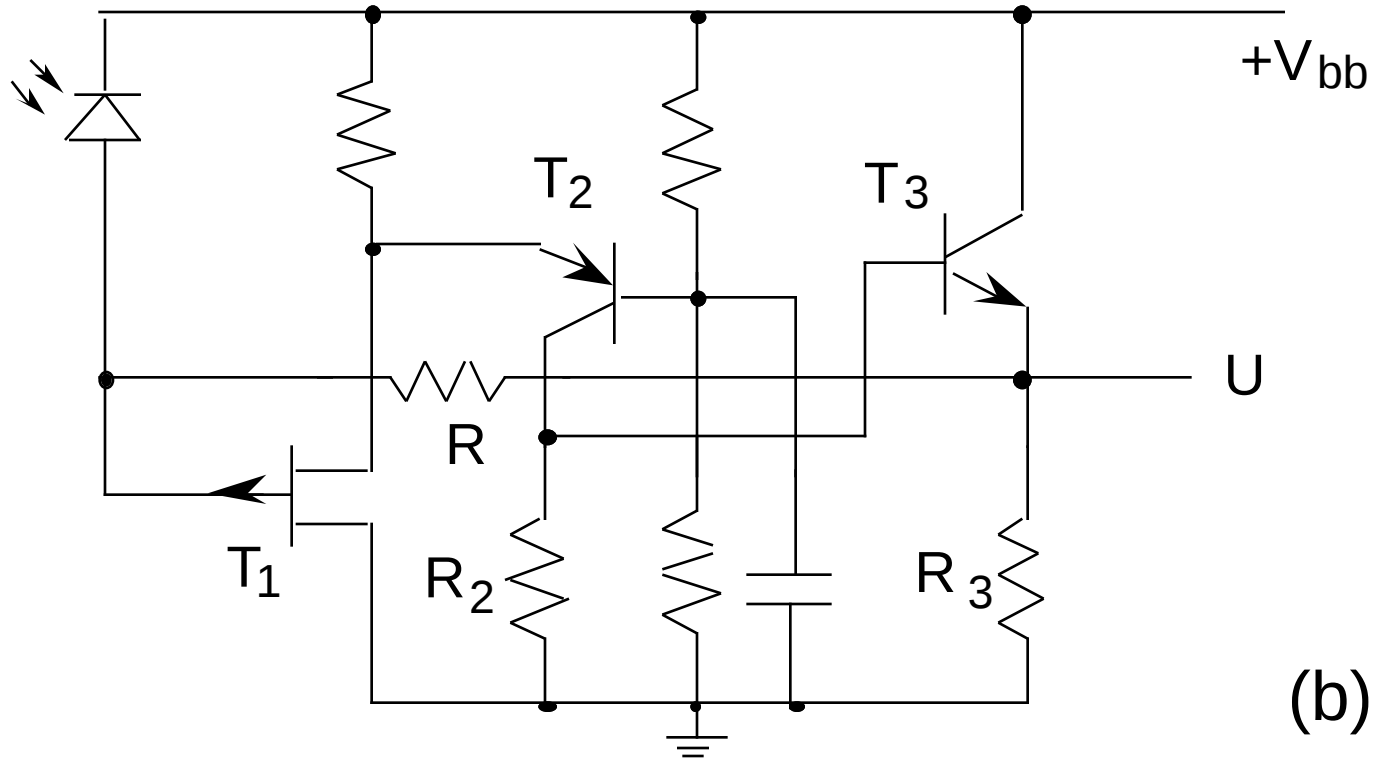


from: "Photodetectors", by S. Donati, Prentice Hall 2000

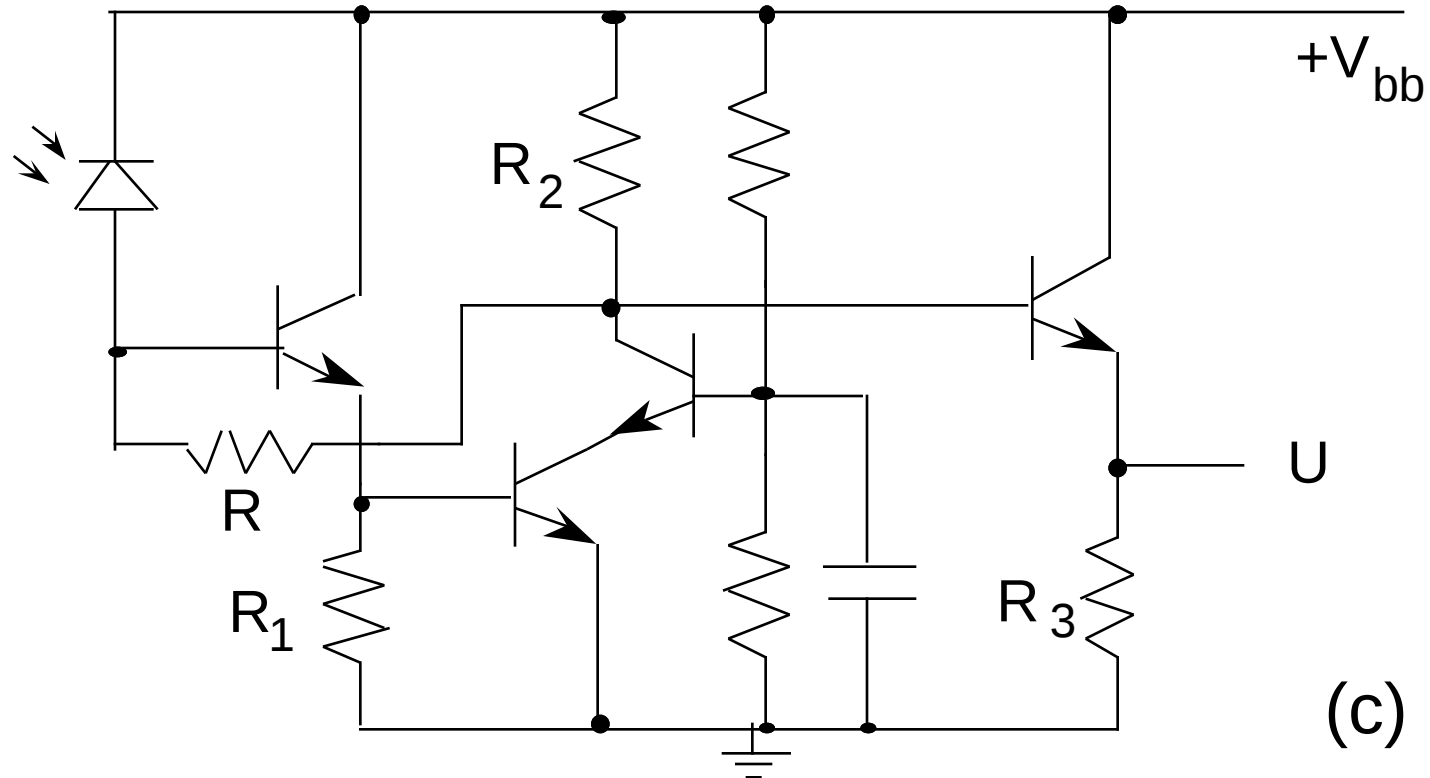
Fast-pulses and High-frequency Transimpedance



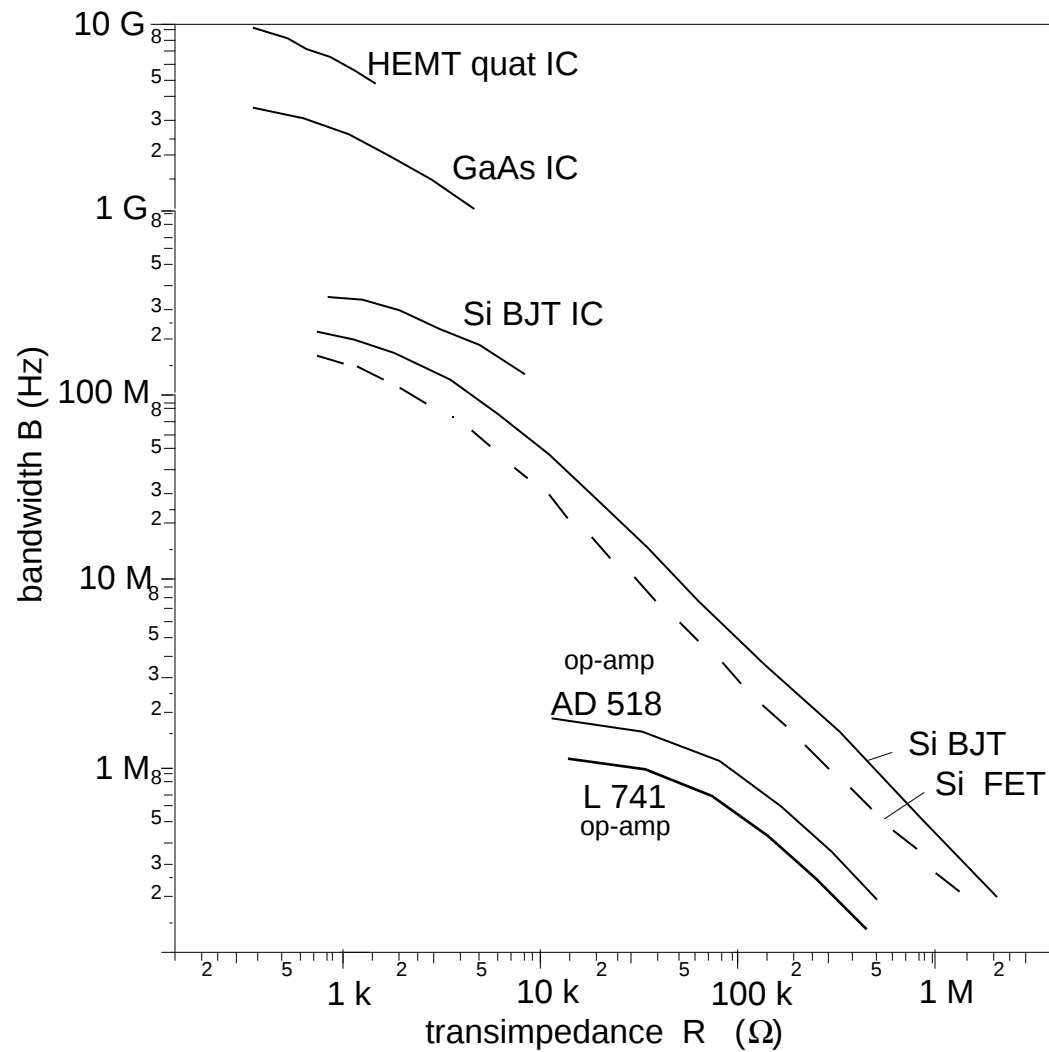
Fast-pulses and High-frequency Transimpedance (2)



Fast-pulses and High-frequency Transimpedance (3)



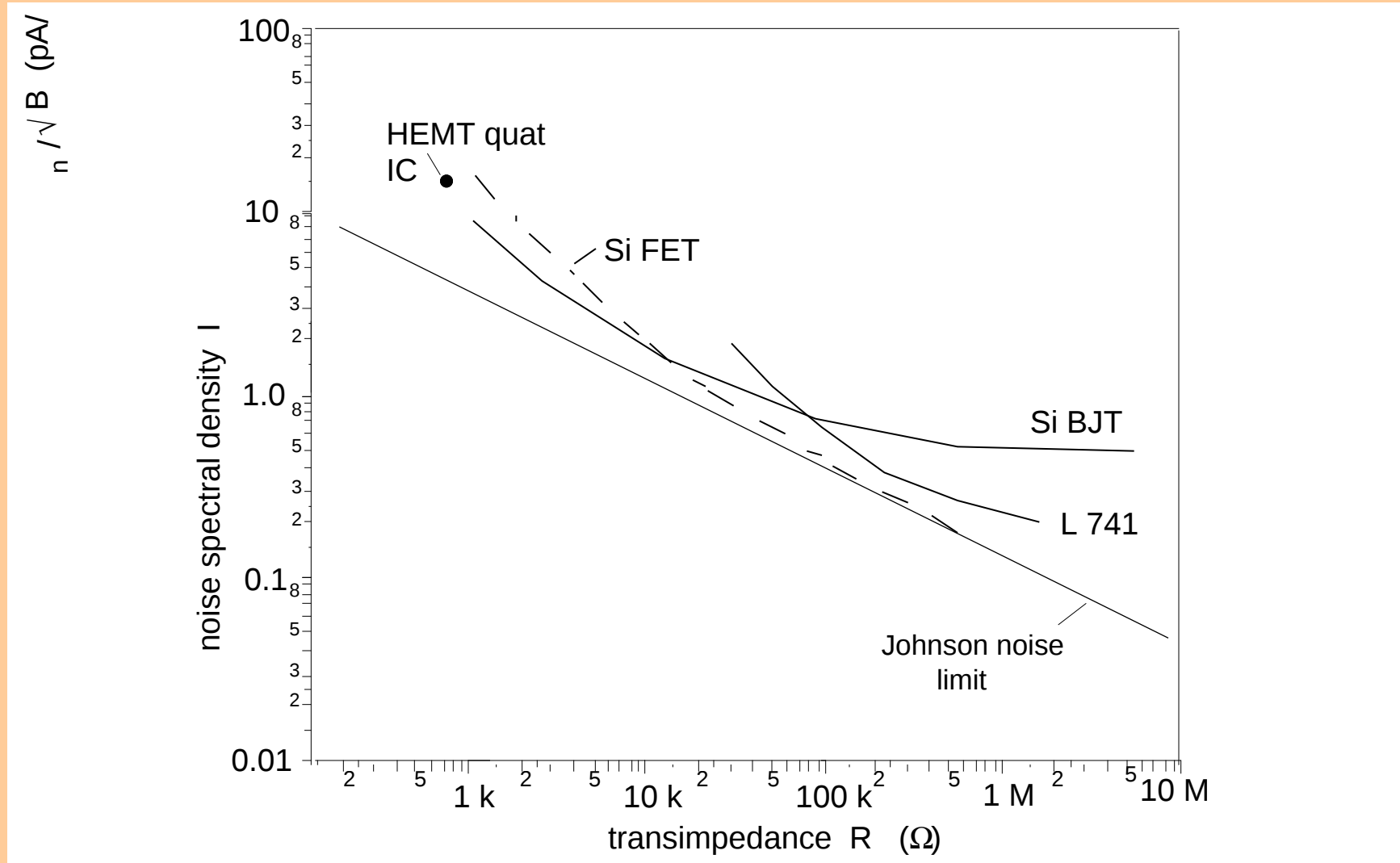
High-frequency Transimpedance performance (bandwidth)



from: "Photodetectors", by S. Donati, Prentice Hall 2000

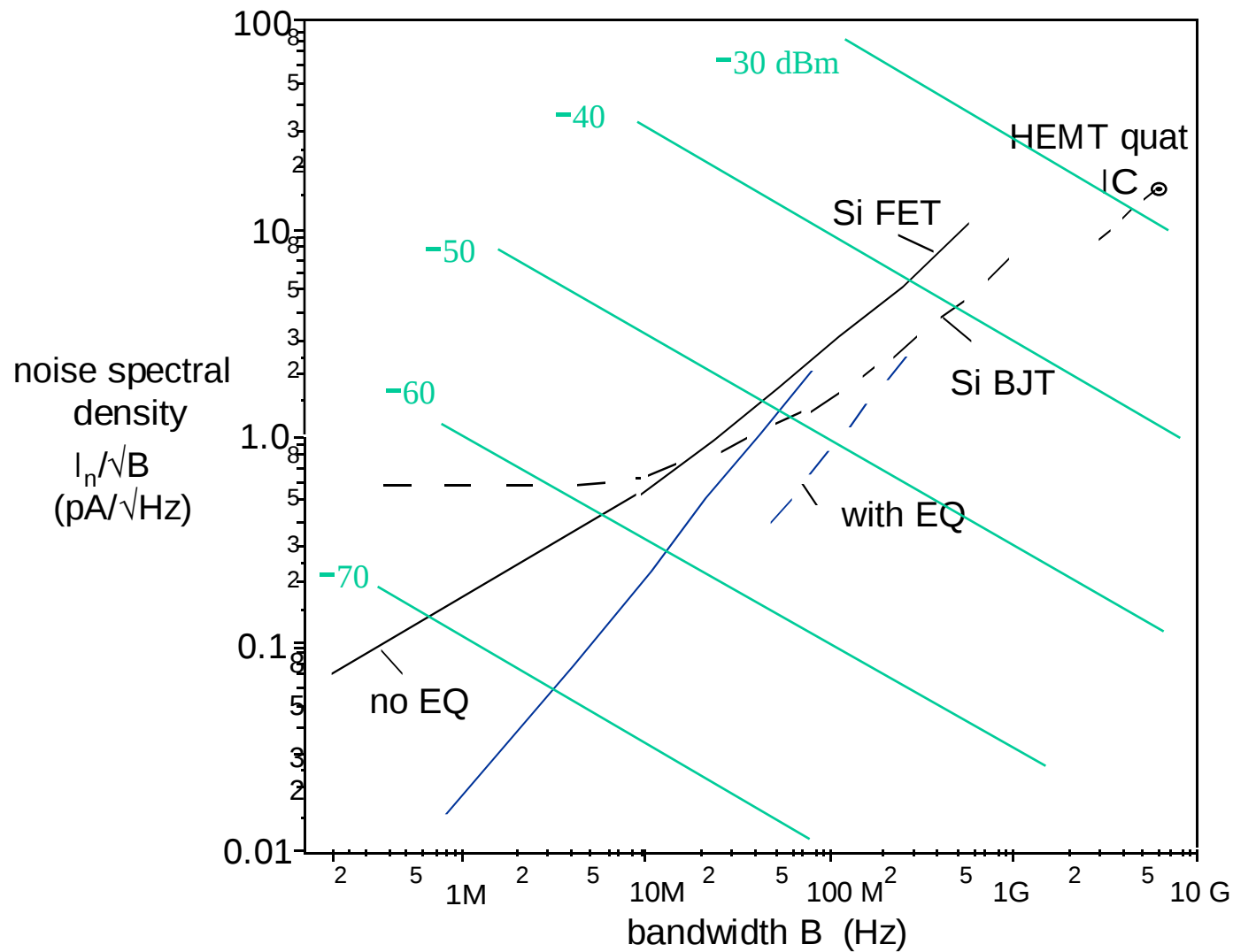
$\sqrt{\text{Hz}}$

High-frequency Transimpedance performance (noise spectral density vs R)



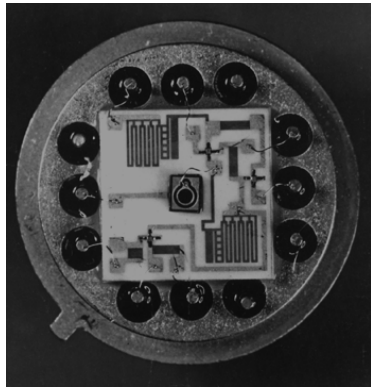
from: "Photodetectors", by S. Donati, Prentice Hall 2000

High-frequency Transimpedance performance (noise spectral density vs B)

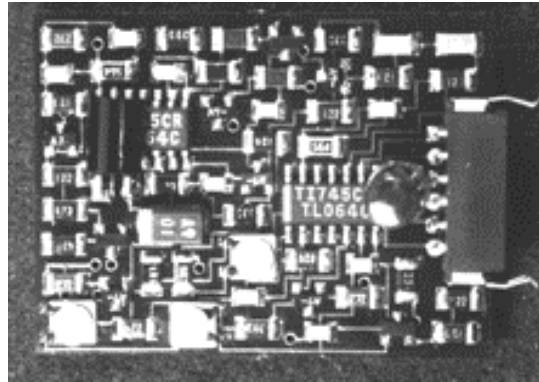


from: "Photodetectors", by S. Donati, Prentice Hall 2000

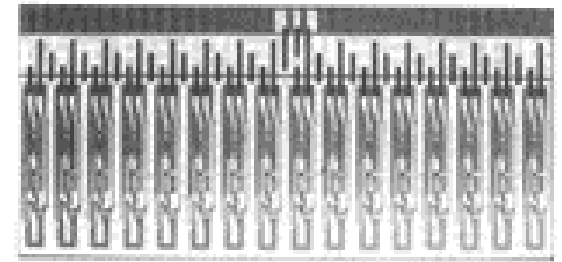
Transimpedance integration technologies



HTF
hybrid-thick-film
(100MHz)

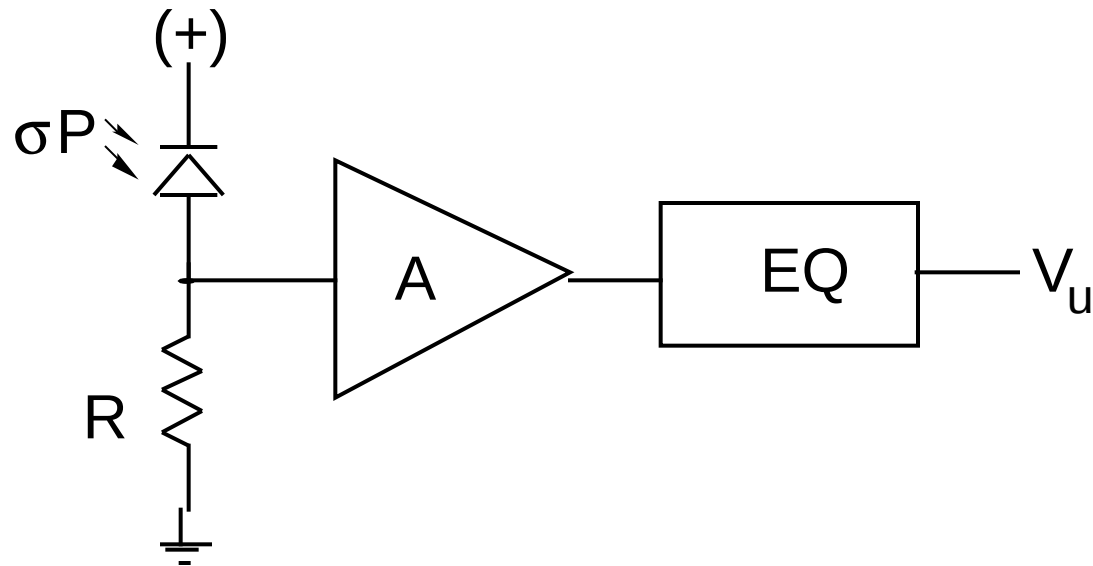


SMD
surface-mount-device
(200 MHz)



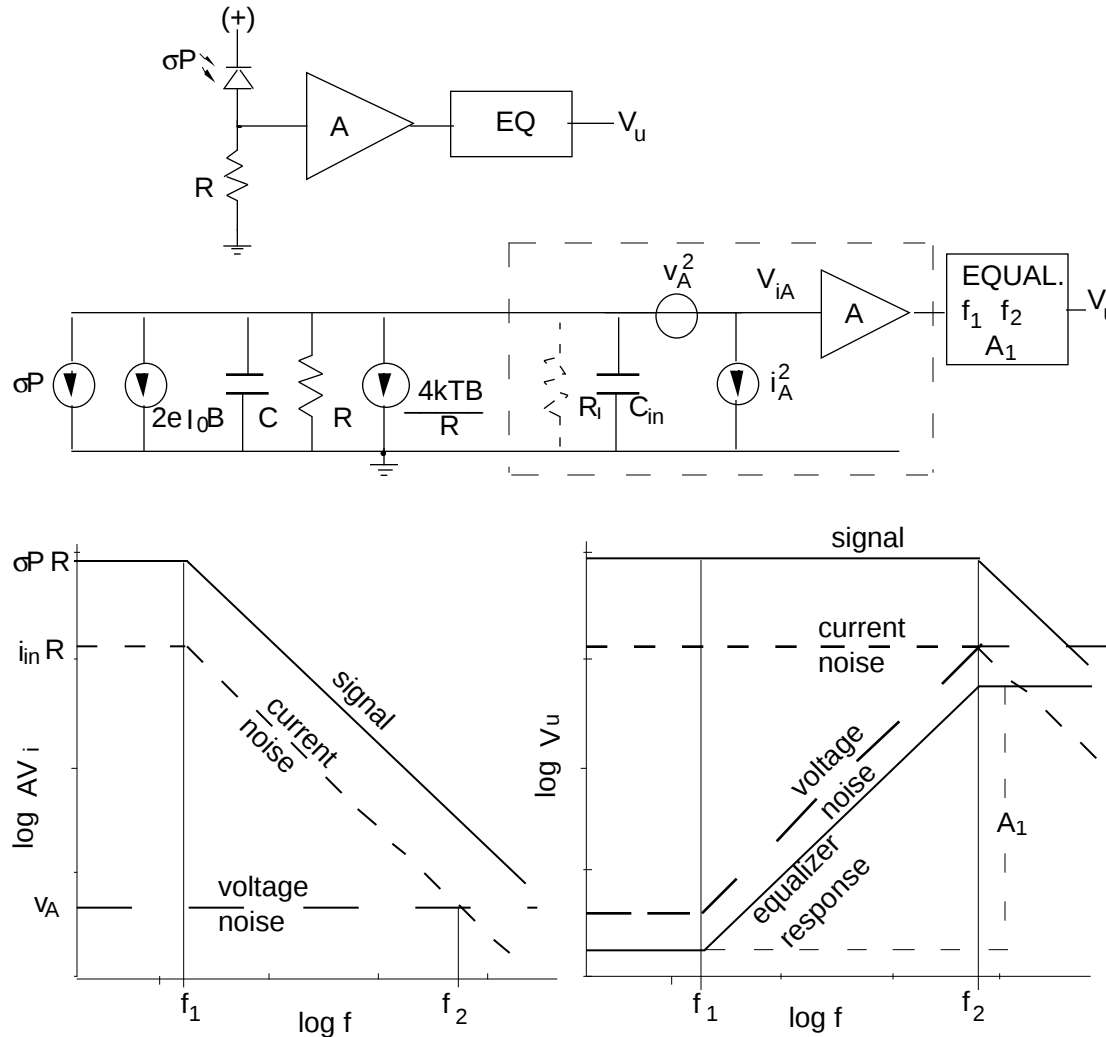
16-channel
In P IC
(11GHz)

Equalization



$$4kTB/R \leq 2eI_0B + i_A^2 \quad \text{or} \\ \leq 2e(I_0 + \sigma P)B.$$

Frequency response



Analysis

We ask that $4kTB/R \leq 2eI_0B + i_A^2$ [or $\leq 2e(I_0 + \sigma P)B$].

In a good amplifier, noise $i_A^2 \ll$ shot noise $2eI_0B$:

let $i_A^2 = \kappa 2eI_0B$ with $\kappa < 1$,

With the = sign above, $R = (2kT/e)/I_0(1+\kappa)$ so that

$$i_{in}^2 = 4kTB/R + 2eI_0B + i_A^2 = i_A^2 2(1+\kappa), \text{ and } f_1 = 1/2\pi RC_i.$$

Max. frequency f_2 with no S/N degradation is when the amplified v_A equals current noise i_{in} :

$$i_{in} R = v_A(f_2/f_1) = v_A(f_2 2\pi RC_i)$$

whence $f_2 = 1/2\pi RC_i (v_A/i_{in}) = [2(1+\kappa)]^{1/2}/2\pi C_i (v_A/i_A)$

new cutoff is that of a resistance $R_A = v_A/i_A$ (typ. 50)

bandwidth improvement is $f_2/f_1 = [2(1+\kappa)]^{1/2} R/R_A$

yet noise remains: $i_{in}^2 = 2eI_0B 2\kappa(1+\kappa)$

Equalization: an example

For a PD with $I_0=500\text{pA}$, and $\kappa=0.5$, the required load is:

$$R = 50\text{mV}/300\text{pA} \cdot 1.5 = 110 \text{ M}\Omega$$

In a FET-input op-amp (LF356), typical noises are:

$$i_A/\sqrt{B}=10 \text{ fA}/\sqrt{\text{Hz}}, \quad v_A/\sqrt{B}=15 \text{ nV}/\sqrt{\text{Hz}}$$

so that: $R_A=15\text{nV}/10\text{fA} = 1.5 \text{ M}\Omega$

Bandwidth improvement obtained with equalization is

$$\sqrt{[2(1+1/\kappa)]} R/R_A = \sqrt{6} \cdot 110/1.5 = 180$$

Noise spectral density corresponding to the dark current I_0 is $\sqrt{(2eI_0)} = 14 \text{ fA}/\sqrt{\text{Hz}}$, whence the correctness of $\kappa \approx 0.5$.

With $C_i=5\text{pF}$, input cutoff frequency is: $f_1= 21\text{kHz}$

and equalized cutoff frequency is $f_2= 3.8 \text{ MHz}$;

current noise (0- f_2 frequency band): $i_{in}/\sqrt{B}=25 \text{ fA}/\sqrt{\text{Hz}}$

Equalization beyond f_2

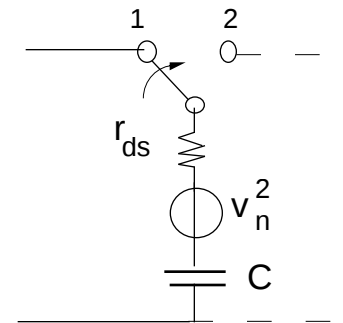
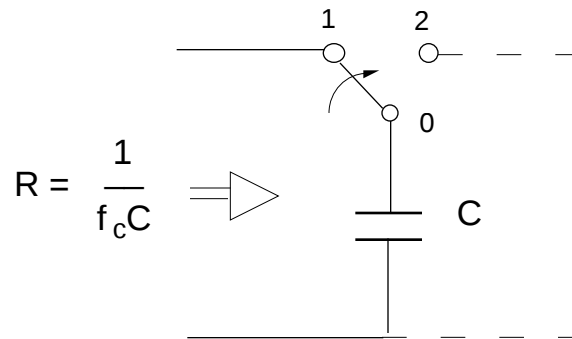
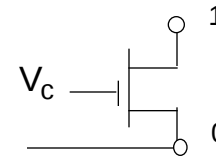
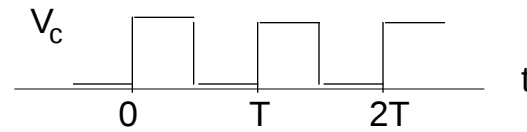
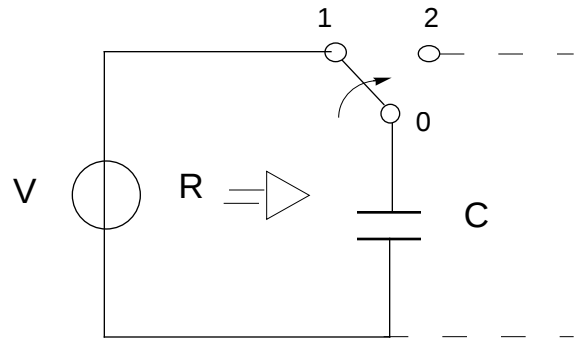
We may also go beyond f_2 with the equalization, but then noise worsens because v_A dominates at $f > f_2$. If $f_3 > f_2$ is the desired new cutoff, noise is increased by $1 + (f_3/3f_2)^2$ respect to optimum value, or, it becomes

$$i_{in}^2 = 2eI_0B \cdot 2\kappa(1 + \kappa)[1 + (f_3/3f_2)^2]$$

Limitation of equalization: the equalizer stage requires a gain $A = f_2/f_1$ at the equalized frequency f_2 , thus the active device shall have f_T at least equal to Af_2 . When f_2 approaches f_T no reserve of gain is available for equalization.

Thus, the technique is good (and widely applied) for $f_2 \ll f_T$, but at the highest frequencies $f_2 \approx f_T$ where it would be most challenging, its improvement becomes marginal.

Switched capacitor (basics)



$$R = VT/Q = 1/f_c C, \quad I_{ph0} = (50 \text{ mV}) f_c C$$

Example on SC

In a $5 \times 5 \text{ } (\mu\text{m})^2$ MOS-switch, it is typ. $r_{ds} \approx 2\text{-}3\text{ k}\Omega/\text{sq}$, and the strays are $\approx 0.05 \text{ pF}$. Choose $C = 2\text{ pF}$ ($\gg C_{\text{stray}}$) and make the switch short ($L/W=0.3$) to lower r_{ds} , so as $r_{ds}=0.8 \text{ k}\Omega$ (typ). Then, the min. clock period is $\tau=(3\div 5) Cr_{ds} = 5\div 8 \text{ ns}$.

At the clock frequency $f_c = 1, 10, 100, 1000 \text{ kHz}$

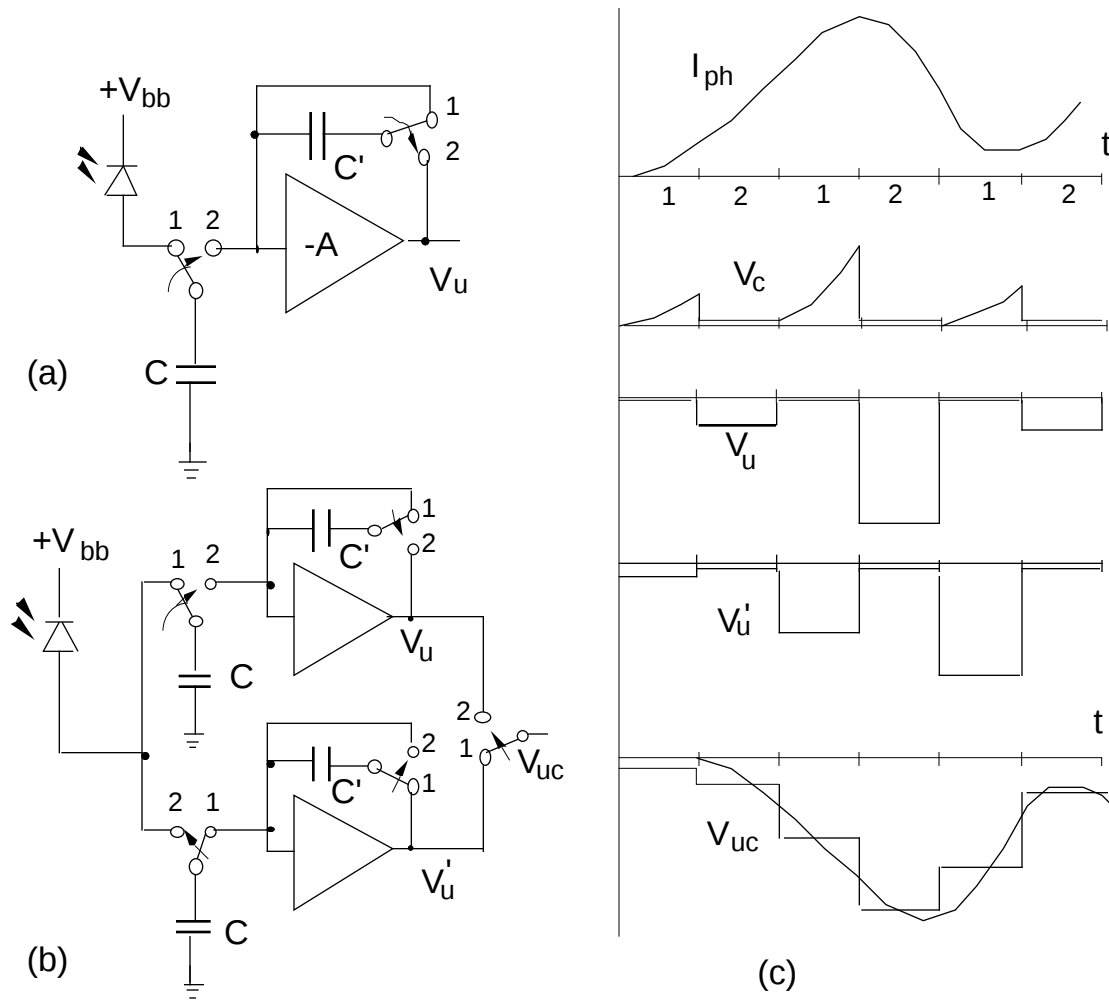
the SC resistance is $R = 500, 50, 5, 0.5 \text{ M}\Omega$

and quantum regime current is $I_{ph0} = 0.1, 1, 10, 100 \text{ nA}$.

Performance is good at low f , but worsens approaching the MHz range.

Values above are typ. of IC-switches in SSI MOS (CD4066). With JFETs SSI (LF13333), r_{ds} is smaller ($100 \text{ }\Omega$) but parasitic is higher (2 pF). Taking $C=50\text{ pF}$, one can have, at the above freq. an SC $R = 20, 2, 0.2, 0.02 \text{ M}\Omega$, still adequate up to the MHz range.

SC Transimpedance



SC transimpedance with chopped output

