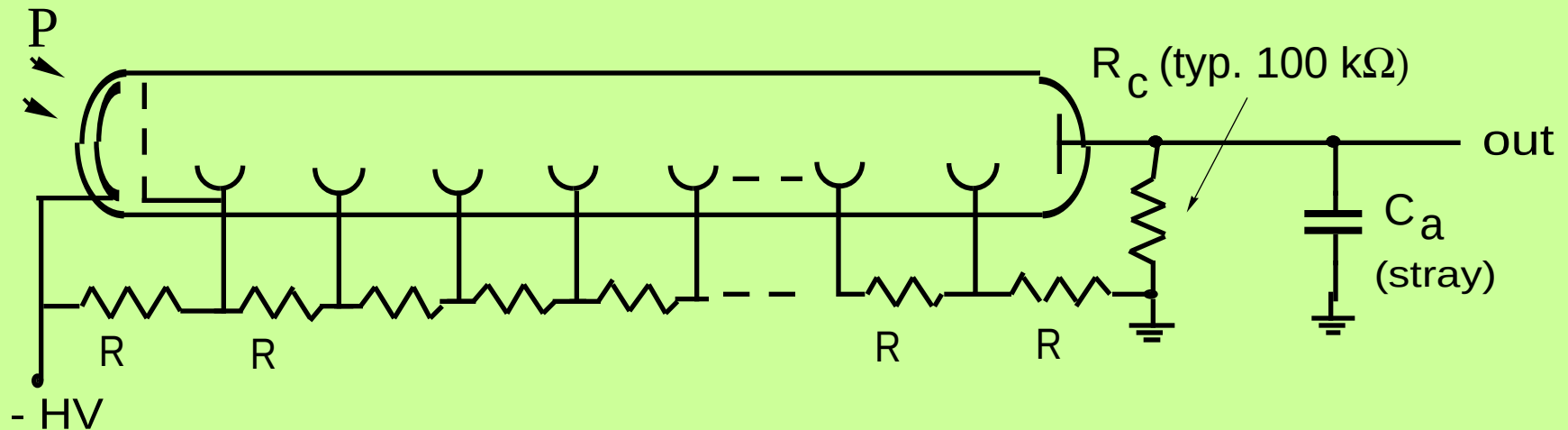


Resume'

Categorization of PMTs applications:

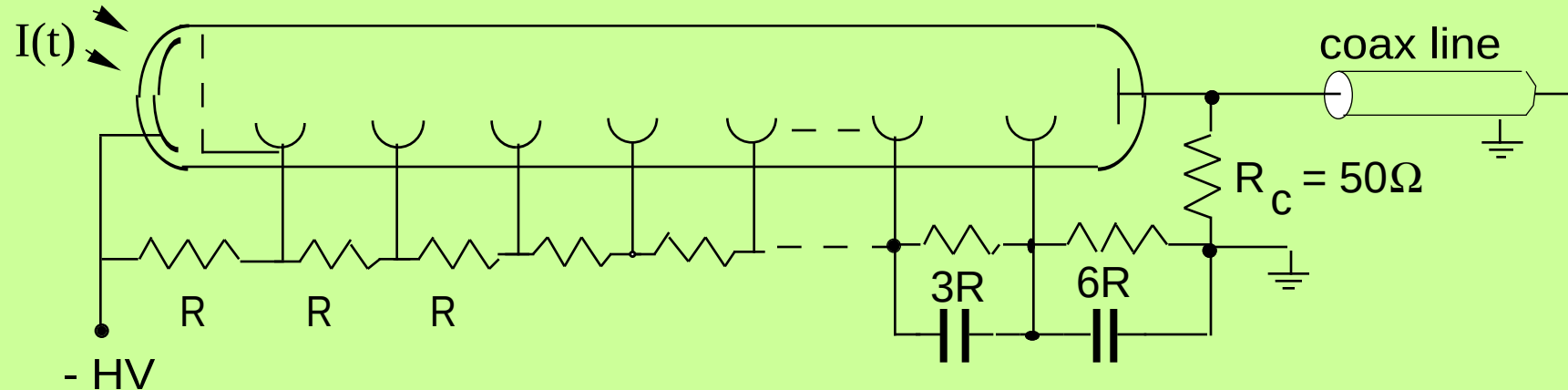
- weak signals, moderate bandwidth
- fast waveforms (and weak signals)
- time measurements and sorting
 - single-photon counting
 - scintillation counting
- dating with radionuclides

Weak signals with moderate bandwidth



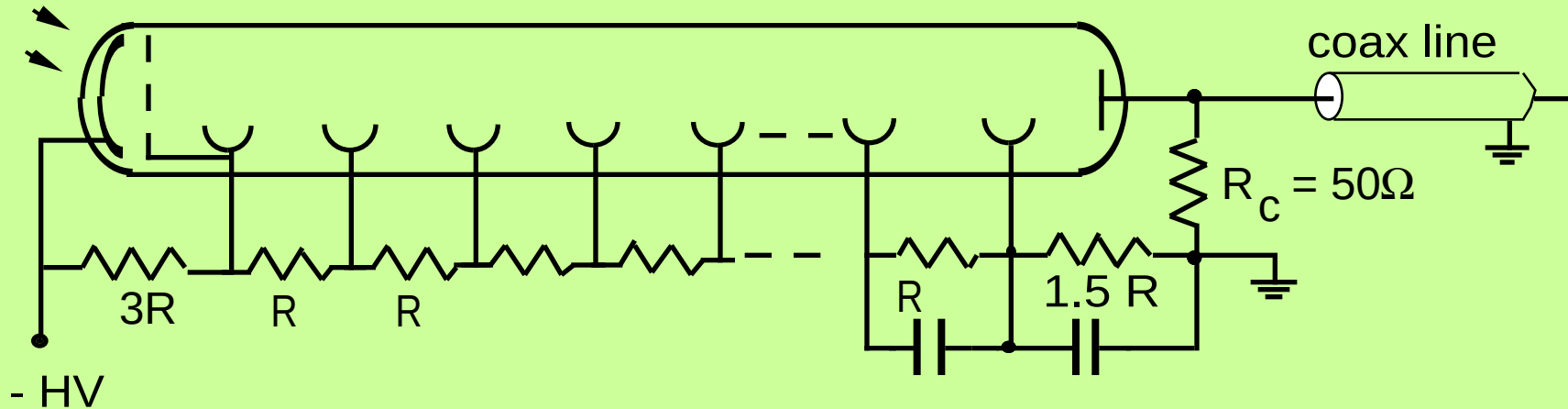
- best with $n=6-9$ stages ($G=10^4 - 10^6$)
- choose the largest possible R_c for desired bandwidth,
 $R_c = 1/[2\pi C_a B]$ (e.g., $=100 \text{ k}\Omega$ for $B=160 \text{ kHz}$ and $C_a=10 \text{ pF}$)
and have: $V_{\text{out}} = R_c I_a = R_c \sigma G P$, dynamic range: $10^3 - 10^5$,
noise figure: $NF^2 = 2.5-2.7$

Fast waveforms



- best with $n=12$ stages ($G=10^7 - 10^8$) for weak signals
 - output terminated on $R_c=50\Omega$ for best bandwidth, last dynodes bypassed and with more voltage ($3R, 6R$)
- impulse response: SER-limited (typ. duration $\Delta\tau = 2\text{ ns}$),
 bandwidth: $B_{[3\text{-dB}]} = 0.31/\Delta\tau$ dynamic range: 10^2 (typ.),
 noise figure: $NF^2 = 2.5$ (typ.)
 output signal waveform: $S_{\text{out}}(t) = I(t) * \text{SER}(t)$

Time measurements

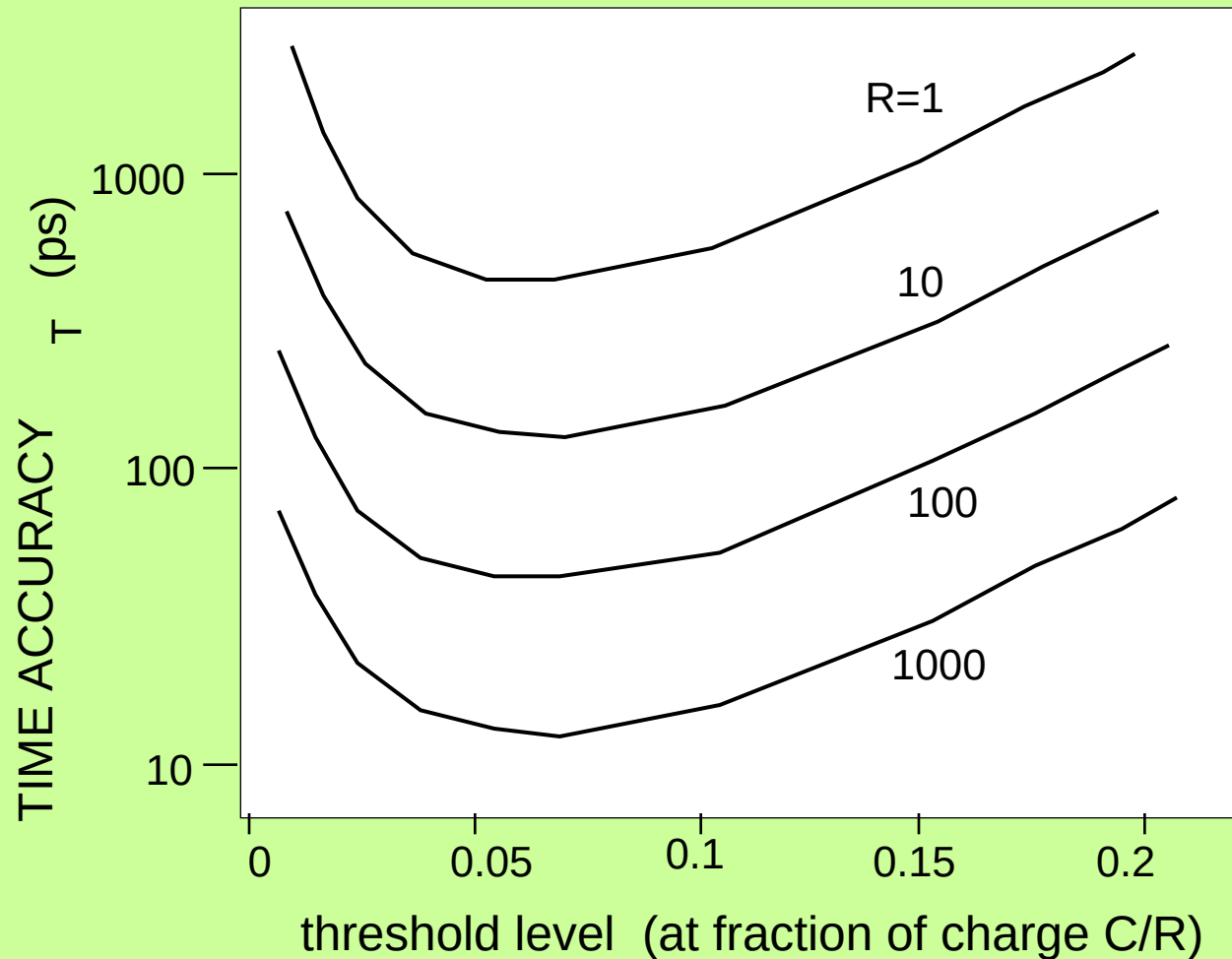


- best with $n=12$ stages ($G=10^7 - 10^8$) for weak signals
- output terminated on $R_c=50\ \Omega$ for best bandwidth, first (3R) and last dynodes with more voltage (3R, 6R)

impulse response: SER-limited (typ. duration $\Delta\tau = 2\ \text{ns}$),

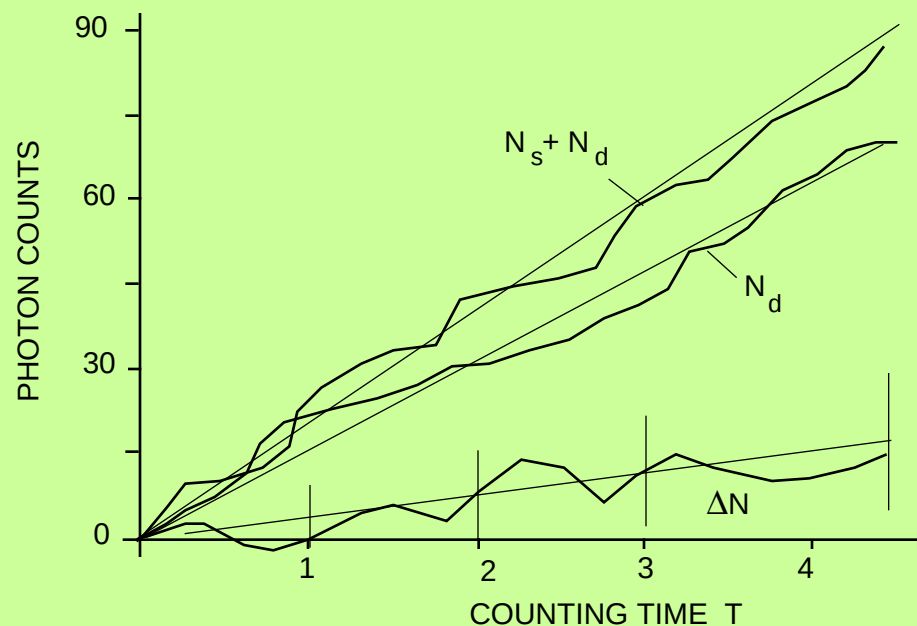
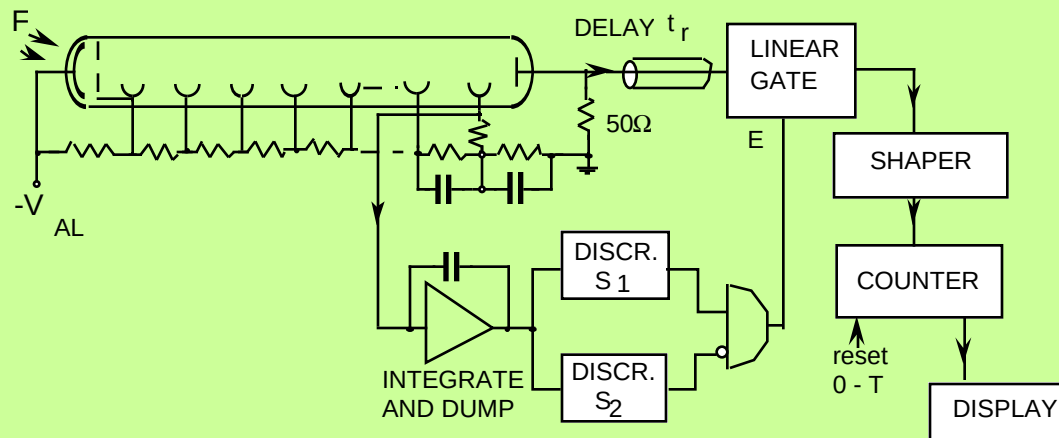
intrinsic limit of accuracy: $\sigma_T = \{\sigma_{t0}^2 + [g/(g-1)g_1]\sigma_{ti}^2\}^{1/2}$
 (typ.) = 0.58 ns for 1 photoelectron

Time measurements



Time resolution of a typical 12-stage PMT with $\Delta\tau=3$ ns followed by a constant fraction timing (CFT). Data for a $I(t)=\delta(t)$ light pulse; timing threshold is set at a fractional level $S_0 = C/R$ of the total collected charge R .

Photon counting



basic functional scheme of photon counting with PMTs (top), and an example of a measurement, showing evolution of signal plus dark, dark only, and result after dark subtraction (bottom); vertical bars on ΔN indicate the $\pm 0.5\sigma_{\Delta n}$ standard deviation confidence intervals

Photon counting (cont'd)

Requirements for PMTs in a photocounting regime:

- a SER amplitude in the mA range (to get $\approx 100\text{mV}$ in circuits), i.e., $G \approx 10^7$ - 10^8 and $n \approx 12$ dynodes
- a high first dynode gain for a good discrimination efficiency
- a voltage divider adequate to have a short $\Delta\tau$ of SER.

- maximum photon rate acquired for the photocounting:

$$F = 1/T_r \quad (\text{set by the integrator recovery time, } T_r = 3\text{-}10 \Delta\tau \text{ typ.})$$

- dynamic range (in power):

$$P = e/T_r \sigma \quad (\text{for } \sigma = 20\text{mA/W, } T_r = 10\text{ns it is } P = 0.8 \text{ nW})$$

Analysis of the photon counting

Total counting $N = N_s + N_d$, (signal plus dark) has a mean value:

$$\langle N \rangle = \langle N_s \rangle + \langle N_d \rangle = \eta \cdot \eta_p F T + (\eta_d I_d / e) T$$

and, following Poisson statistics, variance is:

$$\sigma_N^2 = \langle N_s \rangle + \langle N_d \rangle$$

whence $(S/N)^2 = \langle N_s \rangle^2 / [\langle N_s \rangle + \langle N_d \rangle]$

noise figure NF^2 of the photocounting process:

$$NF^2 = (S/N)_{N_d=0}^2 / (S/N)^2 = 1 + \langle N_d \rangle / \langle N_s \rangle = 1 + (\eta_d I_d / e) / \eta \cdot \eta_p F$$

The minimum measurable radiant power, at dark counting rates I_d/e of a few electrons/cm²·s, (at $\eta \cdot \eta_p = 0.1$, near the peak of photocathode response) is:

$$P = F h \nu / \eta \cdot \eta_p \approx 10^{-18} \text{ W}$$

a performance unsurpassed by any other kind of photodetector

Photon counting with dark subtraction

Subtracting dark from signal plus dark (with the same T) gives:

$$\Delta N = \langle N_1 \rangle - \langle N_2 \rangle = \langle N_s \rangle + \langle N_d \rangle - \langle N_d \rangle = \langle N_s \rangle$$

The variance of ΔN is the sum of the variances, so that:

$$\sigma_{\Delta N}^2 = \langle N_s \rangle + 2\langle N_d \rangle \quad (S/N)^2 = \langle N_s \rangle^2 / [\langle N_s \rangle + 2\langle N_d \rangle]$$

$$\text{and} \quad NF^2 = 1 + 2\langle N_d \rangle / \langle N_s \rangle = 1 + (2\eta_d I_d / e) / \eta \cdot \eta_p F$$

letting $S/N=1$ and for weak signals ($\langle N_s \rangle \ll \langle N_d \rangle$), it is:

$$\langle N_s \rangle_{\min} = \sqrt{2\langle N_d \rangle} = \sqrt{(2\eta_d I_d T / e)}$$

or, the minimum detectable signal, being $\langle N_s \rangle = \eta \eta_p F T$, is:

$$F_{\min} = \sqrt{(2\eta_d I_d / e T) / \eta \eta_p}$$

Photon counting with dark subtraction: an example

With a PMT having:

- a S-24 response, with $\eta=0.35$ at $\lambda=400\text{nm}$,
- a 1cm^2 area, with a dark current rate $I_d/e=1\text{cm}^{-2}$
- a first dynode gain $g_1=3$
- thresholds $Q_1/Ge=0.8$ and $Q_2/Ge=2.5$ ($\eta_p=0.7$, $\eta_d=0.8$)

the minimum detectable rate is: $F_{\min}=\sqrt{(2\cdot 0.7\cdot 1/T)/0.35\cdot 0.7}$
 $=4.83/\sqrt{T}$ photon/s.

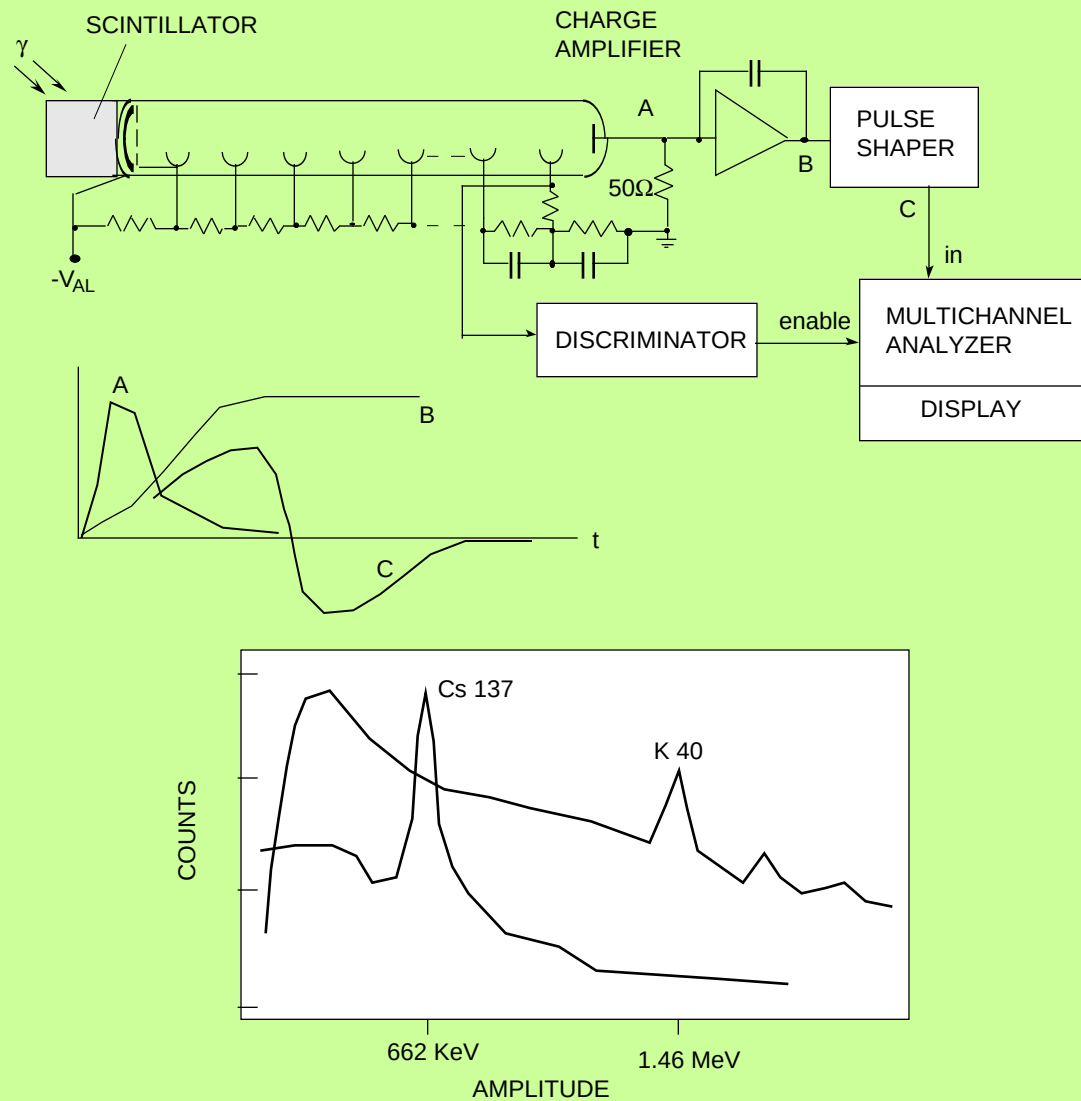
for a 8-hour integration period this yields:

$$F_{\min}=4.83/\sqrt{28800}=0.028 \text{ photon/s, or}$$

$$P_{\min}=h\nu F_{\min}=1.3\cdot 10^{-20} \text{ W,}$$

i.e., the power collected from a $m=28$ th magnitude star with a 1m^2 telescope aperture.

Nuclear spectrometry with scintillation counters



functional scheme of energy spectrometry measurements (top) and waveforms (middle). The typical energy spectrum obtained with the scintillation detector (bottom) reveals radionuclides species (energy signature) and their concentrations (counts intensity)

Dating with scintillation counters

Carbon isotope ^{14}C , initially absorbed from atmosphere during specimen life, decays with a half-life $T_{1/2}=5700$ years emitting 160 keV electrons.

Dating is performed by looking at the detection rate of 160 keV electrons (peak amplitude of the MCA+ scintillator spectrum), which is proportional to the residual content of ^{14}C in it.

Measurable concentrations are down to $c=10^{-3}$ of the initial value c_0 , according to the simple expression

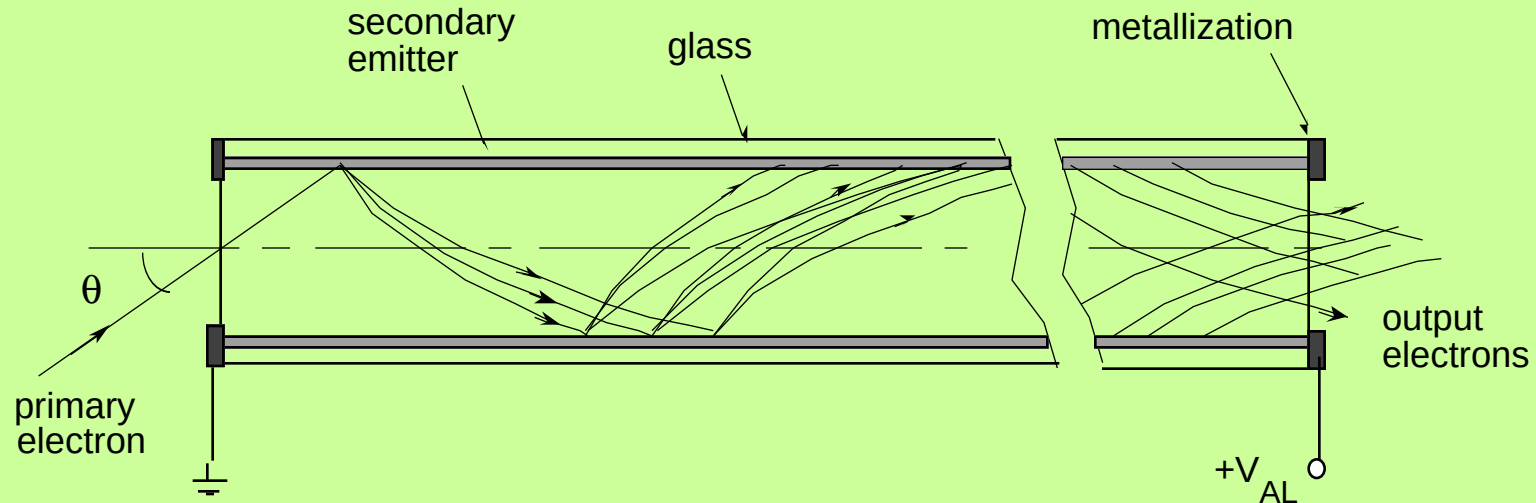
$$T=T_{1/2} \log_2 (c/c_0),$$

with a practical limit in dating to about 40 000 years.

Other radionuclides, such as ^{87}Rb , ^{232}Th , ^{235}U and ^{40}K , are used to cover much larger time spans (up to 10^6 - 10^9 years) and in inorganic samples where ^{14}C is absent

Fundamentals

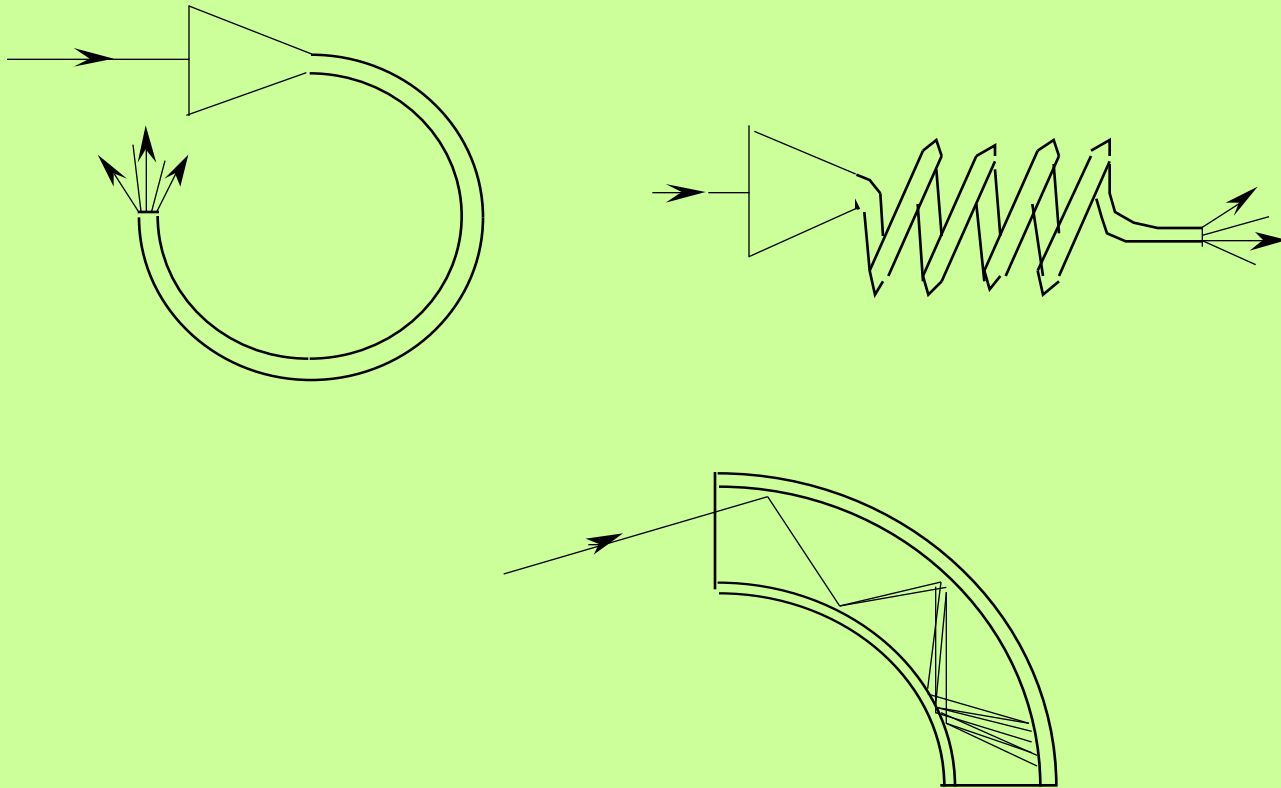
The **microchannel (or channeltron)** is a secondary emission electron multiplier equivalent of a continuous dynode chain



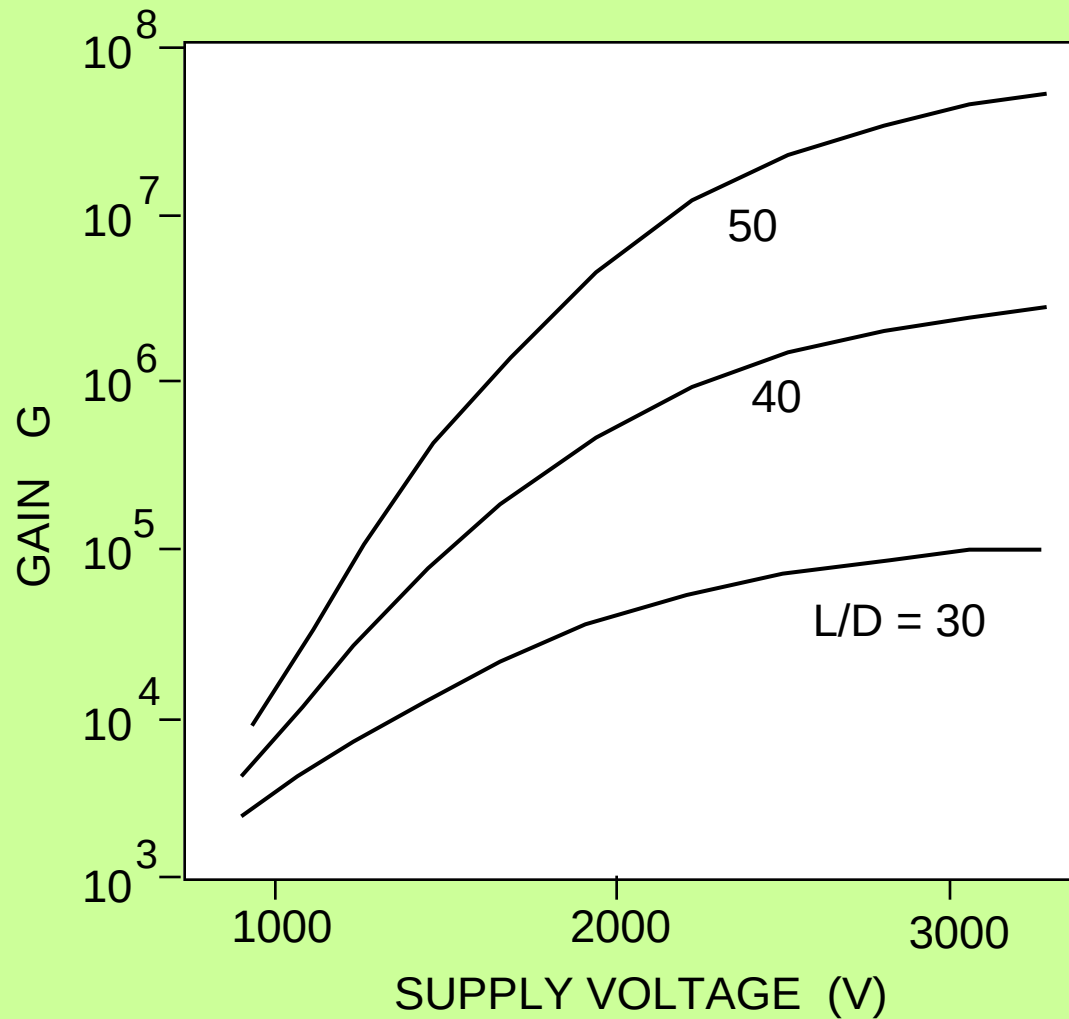
It is a small tube, with a **secondary emitter** material deposited on the internal walls. Because of the field established by supply voltage, an input electron hits the surface and multiplies, and so do the secondaries. The primary electron impinges at an angle θ [typ. 5-30°] respect to microchannel axis so as to obtain the first impact at a distance $l \approx D/\tan\theta$.

Fundamentals (cont'd)

For use as a stand-alone device, ring and spiral MC shapes are preferred, as here electron paths and the mean free path l_c are less dependent on E_s and θ_s , and both dispersion in n and excess variance are smaller.

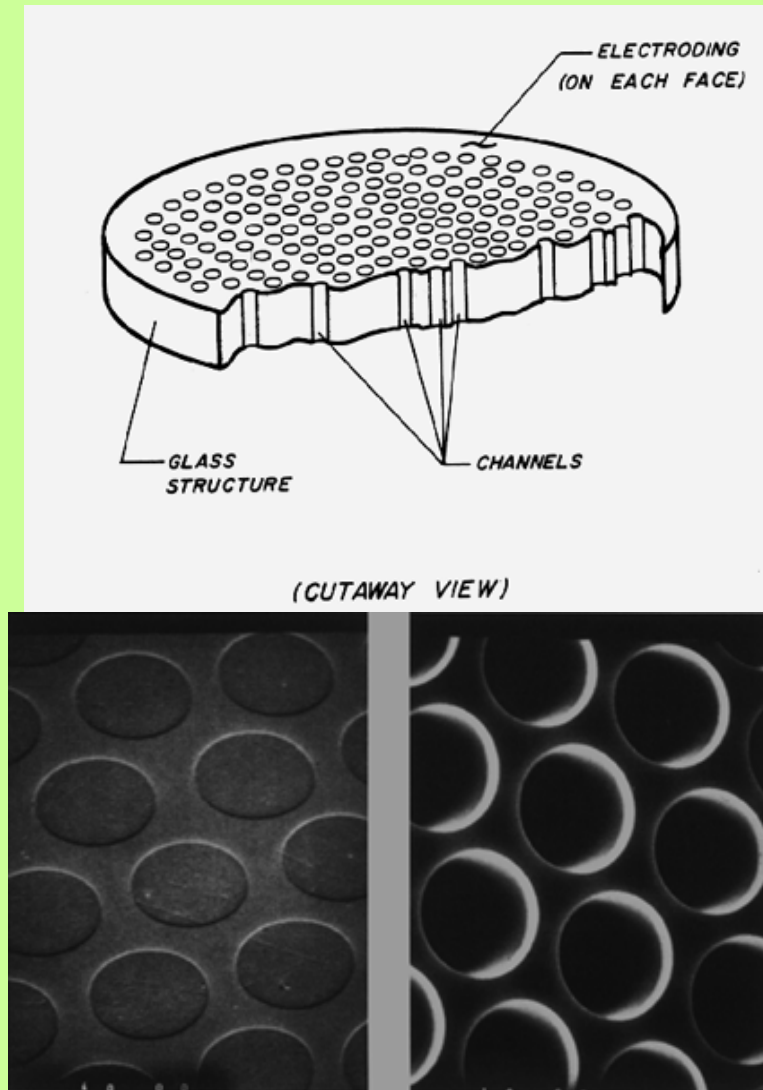


Fundamentals (cont'd)



typical trend of the
MC gain as a
function of supply
voltage and aspect
ratio L/D

Fundamentals (cont'd)



fabricating several microchannels side by side, we get a Multi-Channel-Plate (or MCP).

MCP plates with diameter (or side) of 25-80 mm and 0.5-2 mm thickness, comprising 10^6 - 10^8 individual 8-15 μm -diameter microchannels are used for charge- amplification in image intensifier tubes, the devices for which this technology was originally developed, and CRTs.

Gain of the MC

secondary gain: $g = (AV)^\beta$

for n-stages: $g^n = [AV_{al}/n]^\beta \therefore G$

number of stages: $n = L/l_c$

path-length: $l_c = \frac{1}{2} [eV_{al}/Lm] t^2$

time between multipl.: $t = D/v_s$

perpendicular velocity: $v_s = v \cos \theta_s$

and $E_s = \frac{1}{2} mv^2 = \frac{1}{2} mv_s^2 / \cos^2 \theta_s$

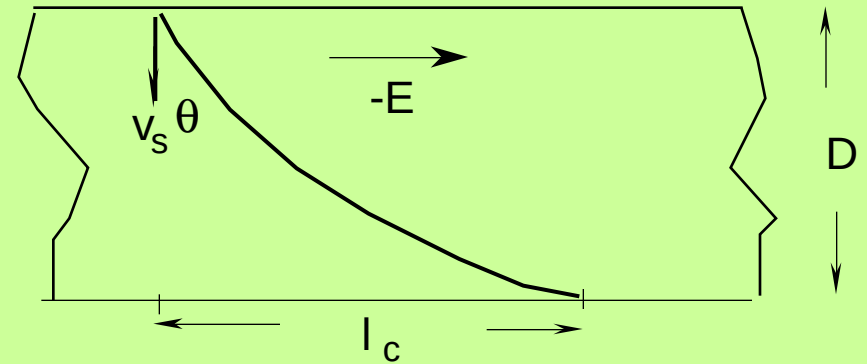
so that $\langle \frac{1}{2} mv_s^2 \rangle = E_s \langle \cos^2 \theta_s \rangle = E_s / 2 \therefore eV_s$

solving for v_s (rms value): $v_s = \sqrt{(2eV_s / m)}$

then, $l_c = \frac{1}{2} [eV_{al}/Lm] [D/v_s]^2 = \frac{1}{4} [D^2 V_{al}/L V_s]$

$n = 4 [L^2 V_s / D^2 V_{al}]$ and

$G = [AV_{al}^2 D^2 / 4 L^2 V_s]^{4\beta} L^2 V_s / D^2 V_{al}$



Example: for AgMgO with $r=L/D=50$, $V_{al}=1000V$, $V_s=2V$, it is: $n=20$, $g=1.75$ and $G=(0.061 \cdot 1000/20)^{20 \cdot 0.5} = 7 \cdot 10^4$. With Cs_3Sb , (same values) $g=2.4$ and $G=(0.07 \cdot 1000/20)^{20 \cdot 0.7} = 4 \cdot 10^7$.

Charge response

For a fixed-n PMT we have:

$$\langle N \rangle = g^n$$
$$\sigma_N^2 = \langle N \rangle^2 / (g-1) = \langle N \rangle^2 \epsilon_A^2$$

Letting n fluctuate by Δn , the mean fluctuation is:

$$\Delta \langle N \rangle = g^n (\ln g) \Delta n$$

or, $\sigma_{\langle N \rangle}^2 = \langle [\Delta \langle N \rangle]^2 \rangle = [g^n \ln g]^2 \sigma_n^2$

this is the extra contribution to variance, $\langle \Delta N_{\text{stat}}^2 \rangle + \langle \Delta N_{\text{nst}}^2 \rangle$,
so for the M CP we have

$$\sigma_{N_{\text{mc}}}^2 = \langle N \rangle^2 \epsilon_A^2 + [g^n \ln g]^2 \sigma_n^2 = \langle N \rangle^2 \{ \epsilon_A^2 + [\ln g]^2 \sigma_n^2 \}$$

where, from Montecarlo simulations, $\sigma_n^2 = k \langle n \rangle$ (and $k \approx 0.02-0.1$ for a curved geometry and $k \approx 0.2-0.4$ for a linear geometry).

Charge response (cont'd)

By combining it is:

$$\sigma_{N_{mc}}^2 / \langle N \rangle^2 = \epsilon_A^2 + [\ln g]^2 k \langle n \rangle = \epsilon_{Amc}^2$$

Example: for a linear MCP with $g=2.4$, $n=20$ and $k=0.3$, it is:

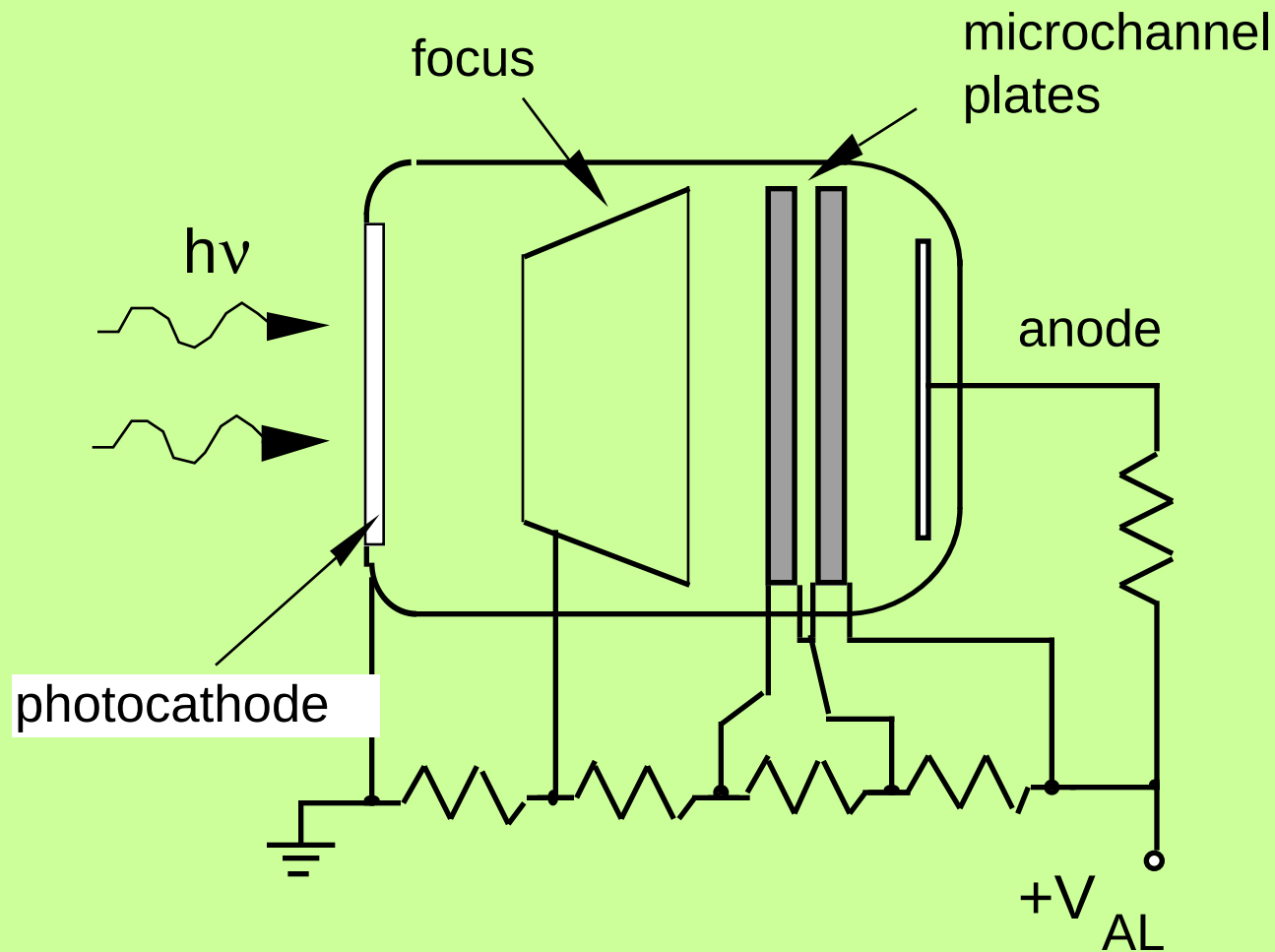
$\epsilon_{Amc}^2 = 0.71 + 0.77 \cdot 0.3 \cdot 20 = 4.59$, instead of $\epsilon_A^2 = 0.71$ for fixed n .

Thus, the variance worsens by a factor $\epsilon_{Amc}^2 / \epsilon_A^2 = 4.59 / 0.71 = 6.46$.

The MCP starts from a worse statistics than the PMT. However, exactly as in PMT, the statistics improves with increasing number of input electrons R . In this case:

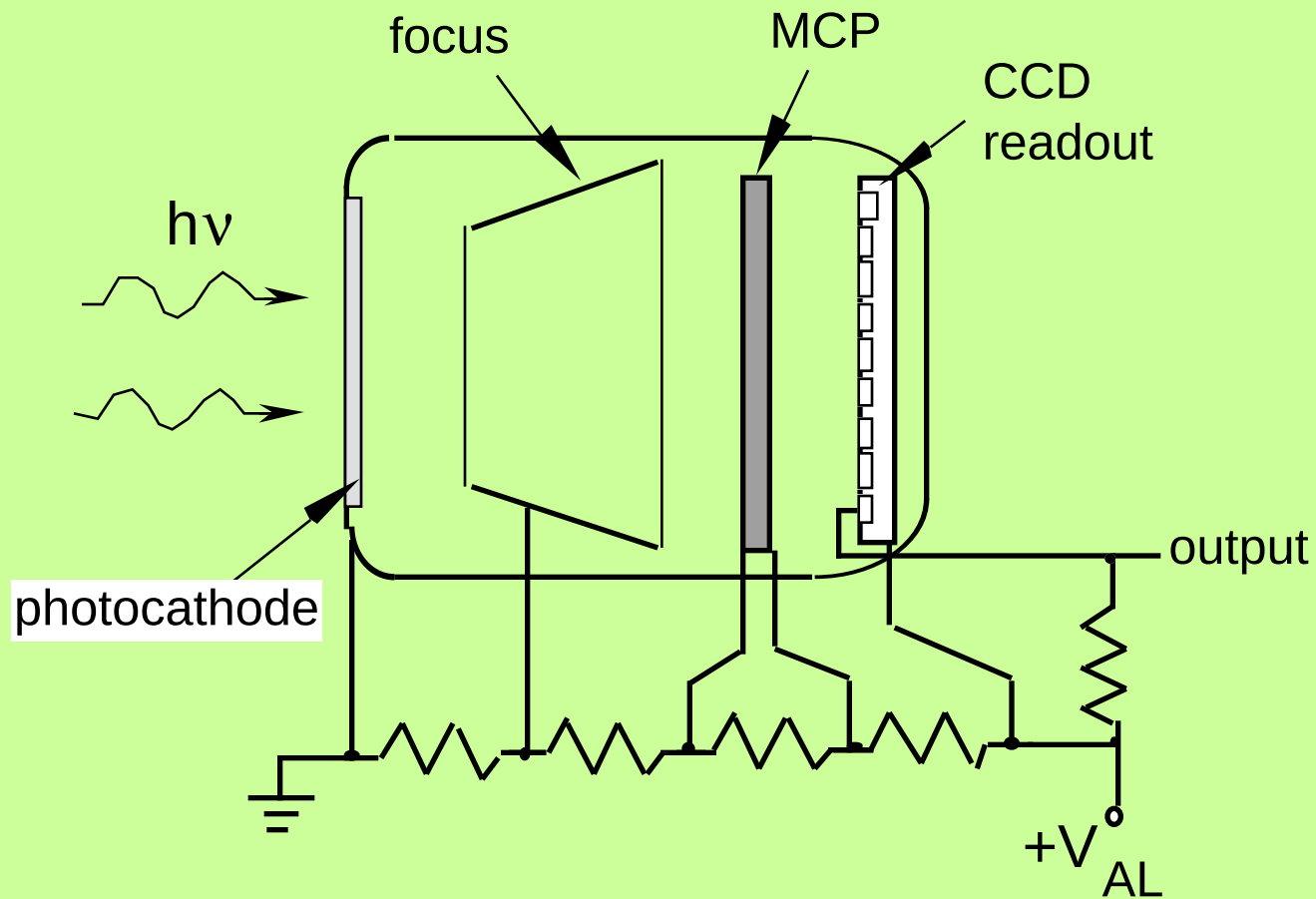
$$\sigma_{R_{tot}}^2 / \langle N_R \rangle^2 = \{ \epsilon_A^2 + [\ln g]^2 k \langle n \rangle \} / R = \epsilon_{Amc}^2 / R$$

MCP Photomultipliers



A two-stage MCP PMT

Fundamentals



A position-sensitive MCP-PMT with a charge-coupled device (CCD) readout