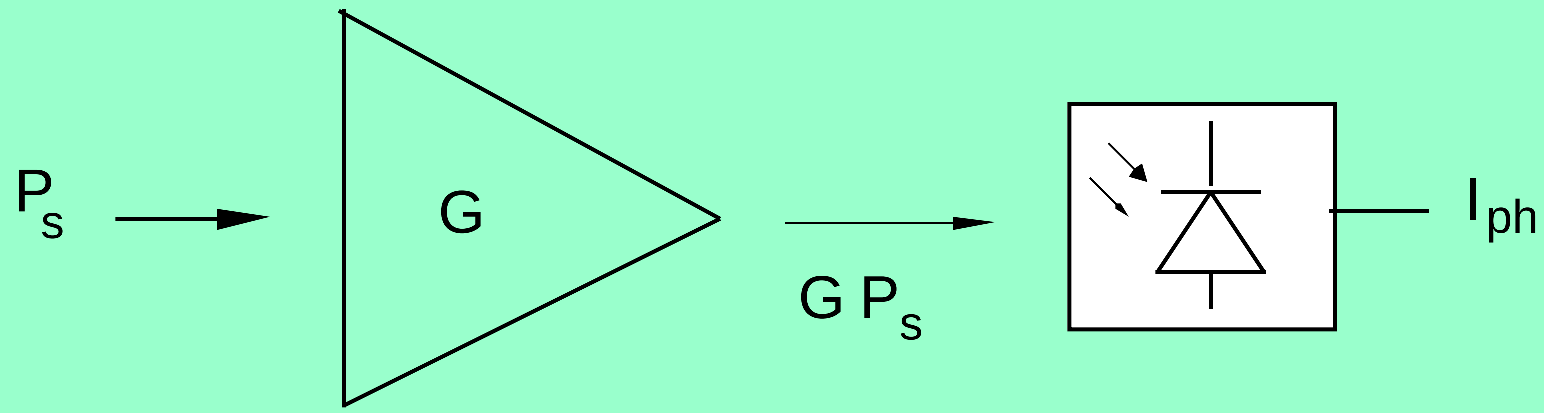
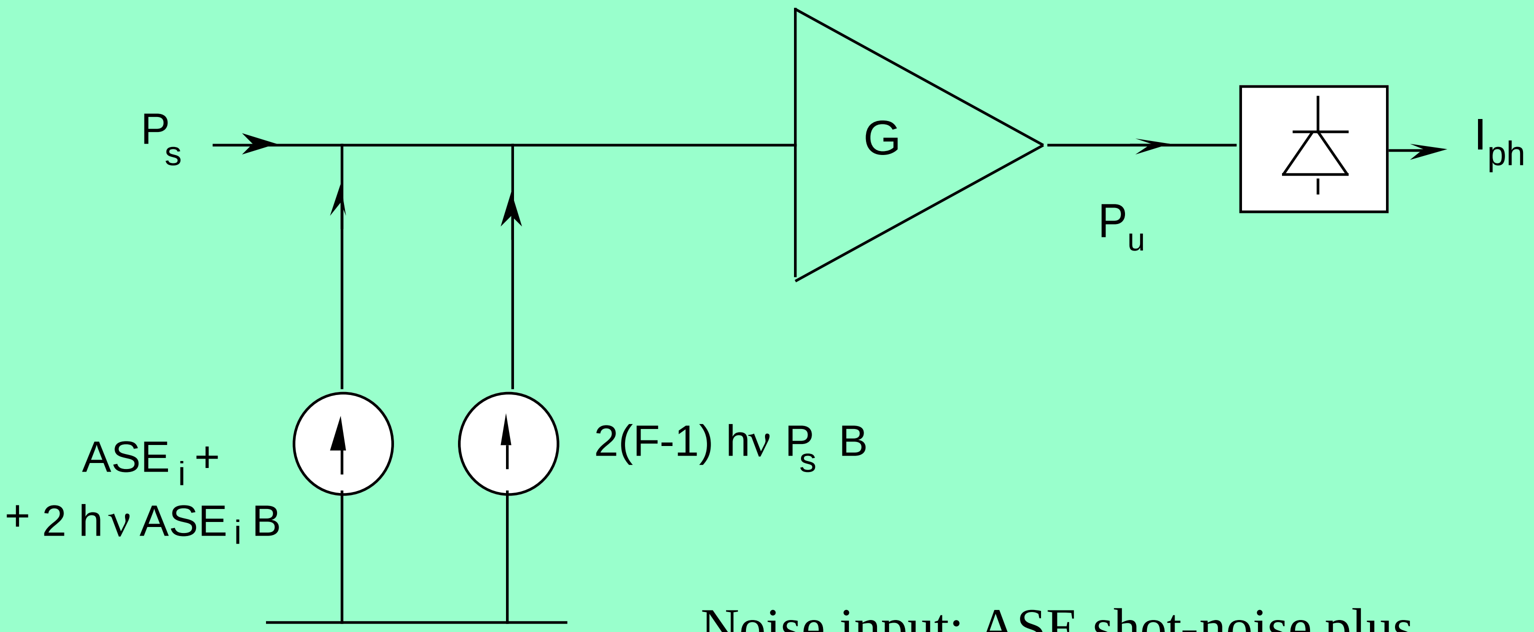
Optical preamplification



Optical power gain is G. At output, added to amplified signal GP_s a dc power due to amplified spontaneous emission is found:

$$ASE_{out} = n_{sp}(G-1)h\nu\Delta\nu_0$$

OA noise equivalent circuit

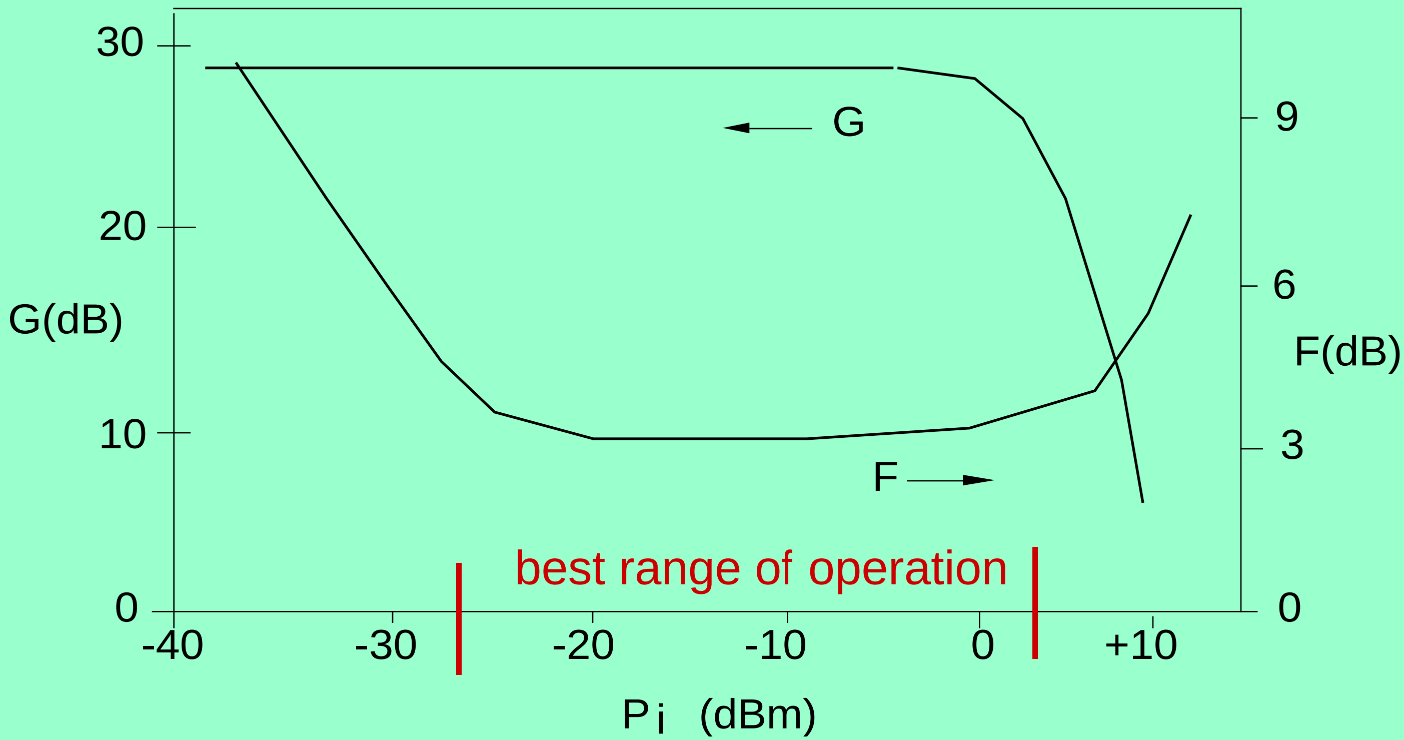


Noise input: ASE shot-noise plus excess noise (factor F)

$$P_u = GP_s + GASE_i$$

$$\sigma^2_{Pu} = 2F\,G^2\,h\nu\,P_s B + 2\,h\nu\,G^2 ASE_i\,B$$

AO performance

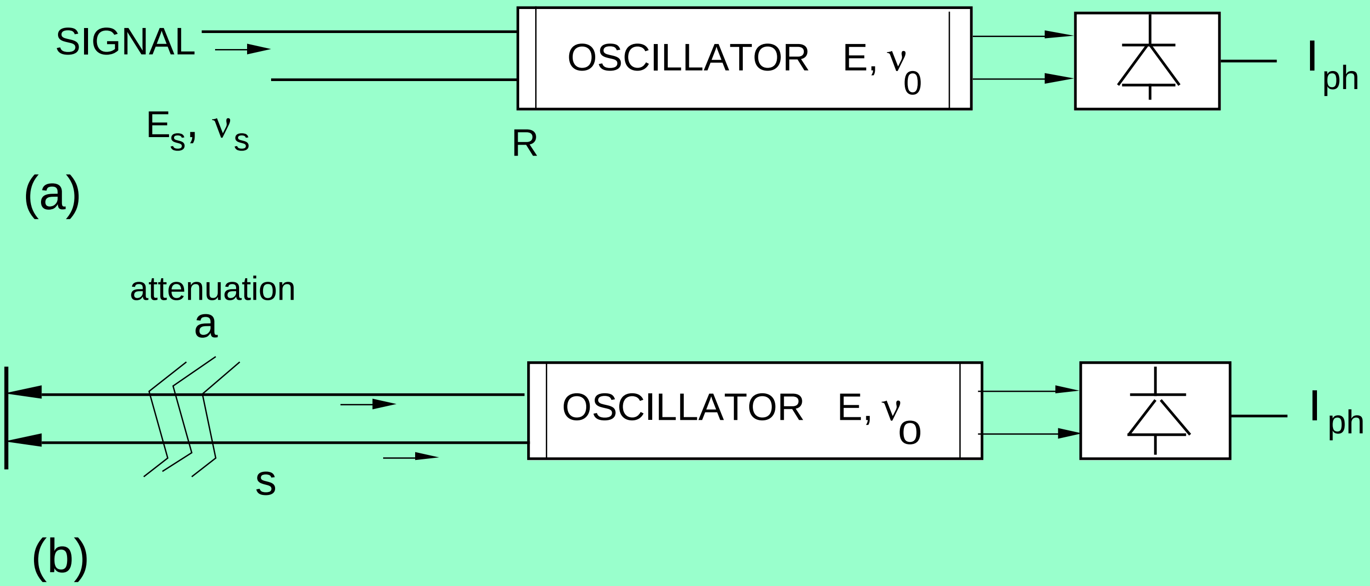


Performance is typical for EDFAs, 1480-1540 nm

Requirements for OA preamplification

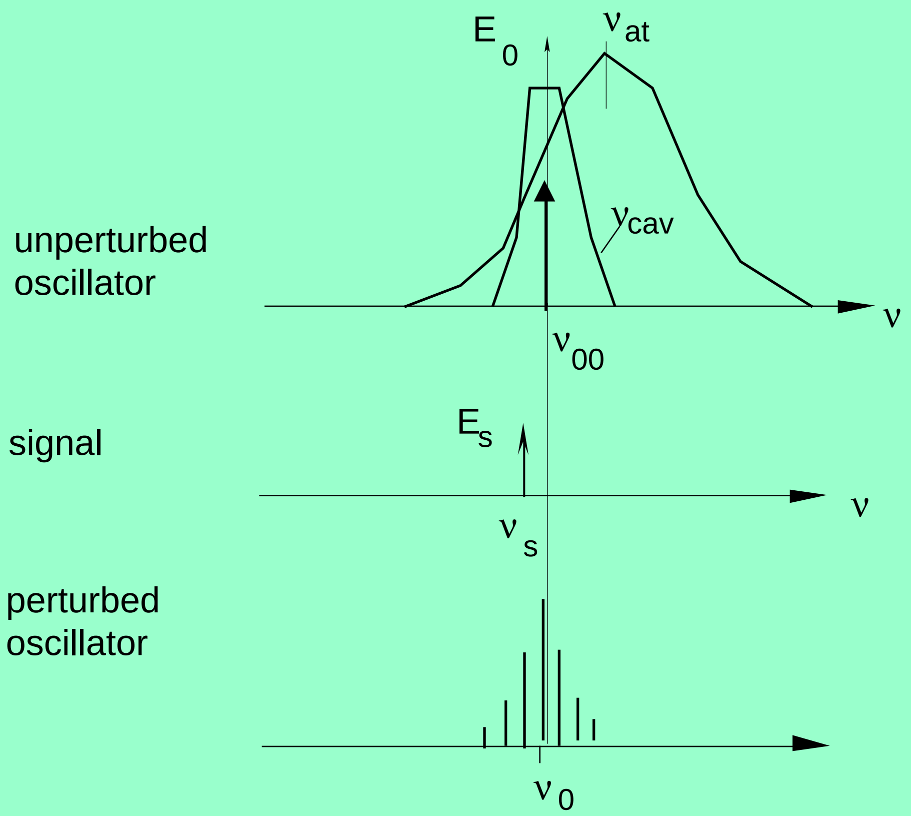
- SIGNAL LEVELS: ASE_i limits minimum signal amplitudes P_i=1÷10μW (or -30÷-20 dBm). Onset of saturationis at about 1-10 mW
- WAVELENGTHS: a few available, in correspondence to laser lines (e.g., 1500, 1300, 1060, 850 nm)
- SIGNAL MODE: a single spatial mode is required, or the low coupling η to DFAfiber would frustrate any amplification
- Large PIXEL #: extension theoretically feasible but not yet demonstrated: problem is that, in AO with *N* modes, ASE increases *N* times becoming very high for images with $N = 10^5..10^6$ pixels

Injection detection



Signal is sent directly into the local oscillator with the conditions of phase, spatial and polarization matching of coherent detection

Injection gain



gain:

$$G = \mu\ \kappa(\nu)\ 2\sqrt{(P_0/P_s)}$$

where $\kappa(\nu) \approx 1$ inside ν_{cav}, and $\mu\ 2\sqrt{(P_0/P_s)}$ is the normal coherent gain

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Calculation of injection gain

signal field $\mathbf{E}_s$ (outside input mirror of the laser) and the laser cavity field $\mathbf{E}_0$ are represented as rotating vectors:

$$\mathbf{E}_s = E_s \exp i\varphi_s \quad \text{and} \quad \mathbf{E}_0 = E_0 \exp i\varphi_0$$

$$\text{with: } \varphi_s = \Omega_s t+\psi_s=2\pi\nu_s t+\psi_s \quad \text{and} \quad \varphi_0 = \Omega_0 t+\psi_0=2\pi\nu_0+\psi_0$$

Lamb equations, modified to take account of the external injected signal read:

$$(d/dt)E_0 = [(\alpha-\beta E_0^2)- \Gamma]\, E_0 + (c/2L)\, TE_s \cos(\varphi_s-\varphi_0+\psi)$$

$$(d/dt)\varphi_0 = \Omega_0 + \zeta\,(\alpha-\beta E_0^2) +(c/2L)T(E_s/E_0) \sin(\varphi_s-\varphi_0+\psi)$$

where: $\alpha = \lambda^2 c(n_2-n_1)/8\pi\tau_{21}\Delta\nu_{at}$ is gain rate, β= gain saturation, $\Gamma=\nu_0/2Q$ cavity loss-rate, L= cavity length, and c/2L=mode spacing, $T = \sqrt{(1-R)}$ field transmission of the input mirror, ψ = field coupling phaseshift, $\zeta = (\nu_0-\nu_{at})/\Delta\nu_{at}$ is frequency offset respect to gain line center ν_{at}, $\Delta\nu_{at}$ = (atomic) gain linewidth

Calculation (2)

In semiconductor lasers, we shall add another equation relating carrier concentration n to photon density (or E_0^2), as:

$$(d/dt)n = J/ed - n/\tau_p - (n-n_{th})(1- \Xi E_0^2)[1-(\beta/\alpha)E_0^2]\alpha^* E_0^2$$

where: J = pump current density, τ_p is the charge-carrier lifetime,
n_{th} is the carrier concentration at threshold, Ξ is the mode-confinement factor,
$\alpha^* = 2\alpha/(n_2-n_1)$.

This set of equations is the Lang and Kobayashi description. However, with no loss of generality, we will proceed with Lamb equations.

Calculation (3)

Letting $(d/dt)E_0=0$ and solving for E_0 and $(d/dt)\varphi_0$ give the steady-state values of field and frequency, or, the values E_{00} and ν_{00} of the unperturbed regime. Using a WKB method to solve Lamb eq., that is, a trial solution like:

$$E_0 = E_{00} + e_0 \qquad (\text{with } e_0 \ll E_{00})$$

we get: $$(d/dt)(\varphi_0 - \varphi_s) = A\,[1 + K \sin(\varphi_0 - \varphi_s + \psi)]$$

this is Adler equation describing the *frequency locking* or synchronization of an oscillator. For $K \geq 1$ solution is $\varphi_0 - \varphi_s = 0$ (locking in phase to signal), and for $K<1$ it can be solved with:

$$\varphi_0(t) - \varphi_s = -\psi - \arcsin\{[K - \sin\Phi(t)]/[1 - K\sin\Phi(t)]\}$$

where $\Phi(t) = (1-K^2)^{1/2}(\Omega_s - \Omega_{00})t + \arcsin K$ is the phase of a quasi-sinusoidal oscillation at the unperturbed freq. diff. $\Omega_s - \Omega_{00}$.

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Calculation (4)

K has the meaning of coupling factor and is given by:

$$K = [T(c/2L)/(\nu_s - \nu_{00})]\, E_s/E_{00}$$

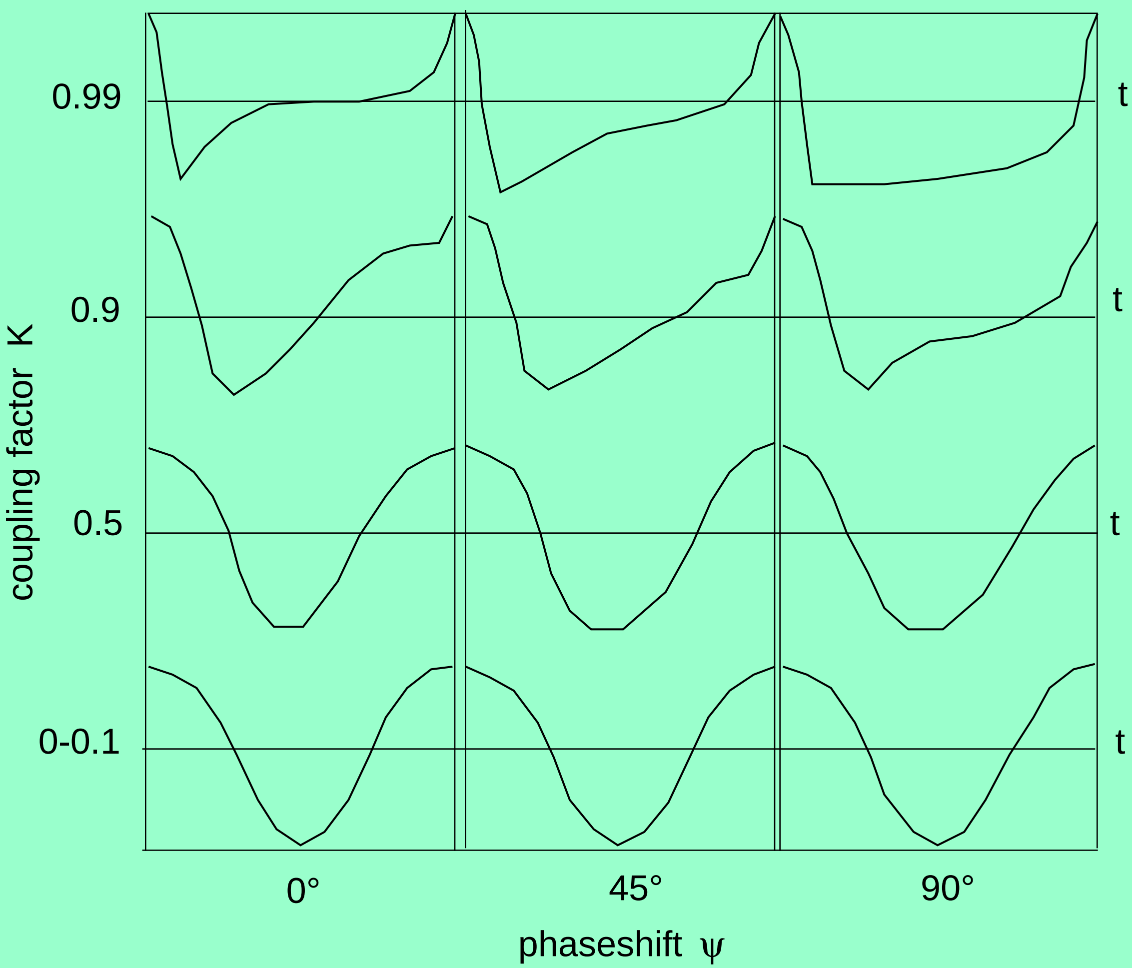
With this result, field amplitude is solved as:

$$E_0 = E_{00} + e_0 = E_{00} + [Tc/2L(\alpha - \Gamma)]\, E_s \cos[\varphi_0(t) - \varphi_s]$$

Thus, outside locking range ($K<1$), injection produces an A-M of the laser field with a modulation index proportional to the signal field E_s. F-M is also found because $\Omega_0(t)=(d/dt)\varphi_0(t)$ is not const. and depends on K and E_s . This explains the qualitative trend of the beating waveforms (next slide).

Thus we have, by injection heterodyning, a series of lateral sidebands impressed on the frequency spectrum of the laser; the carrier also has a minor frequency pulling to signal frequency.

Beating waveforms in injection



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Calculation (5)

Detecting the signal E_0, current is given by:

$$I_0 = (\sigma/2Z_0)\langle |E_{00}+e_0|^2\rangle = I_{00} + 2\sqrt{(I_{00}I_s)}\ [Tc/2L(\alpha-\Gamma)]\cos(\Omega_0-\Omega_s)t$$

On a dc term I_{00}, signal is at carrier at frequency $\Omega_{00}-\Omega_s$ and has an amplitude larger than with direct detection I_s by a factor G:

$$G = G_{coh}\ [Tc/2L(\alpha-\Gamma)] = G_{coh}\kappa(\nu)$$

$G_{coh}=2\sqrt{(I_{00}/I_s)}$ being the usual coherent gain. The quantity

$$\kappa(\nu)=Tc/2L(\alpha-\Gamma)$$

is usually <1 but is not <<1, so system performance is practically coincident to that of coherent detection. Bandwidth is however limited to about the laser cavity linewidth. Noise factor is $\approx$ 3 dB.

Calculation (6)

In *homodyne injection*, signal emitted by the laser re-enters the cavity after propagation to a target at distance s. Power attenuation is $A=a^2$. Injected term is now $E_s=aTE_0$ and has a phase $\psi = 2ks$

$$(d/dt)E_0 = [(\alpha-\beta E_0^2)- \Gamma]\, E_0 + (c/2L)\, aT^2E_0 \cos(\psi+2ks)$$

$$(d/dt)\varphi_0 = \Omega_0 + \zeta\,(\alpha-\beta E_0^2) + (c/2L)\, aT^2 \sin(\psi+2ks)$$

With the same perturbative method outlined above, we find that:

- in place of locking we have mode frequency-hop, that for $K=aT^2c/2L(\alpha-\Gamma)>1$ laser oscillates on the external cavity;
- at weak injection levels ($K\ll 1$), the cavity field is perturbed to:

$$E_0 = E_{00}\,\{1+ [aT^2c/2L(\alpha-\Gamma)] \cos 2ks\}$$

$$\nu_0 = \nu_{00} +(aT^2c/2L) \sin 2ks$$

Calculation (7)

Here, the role of relative signal amplitude E_s/E_{00} is replaced by the attenuation a of the field propagated external to the source.

Proportional to a , A-M and F-M modulations of the laser field are found, with a coherent gain for the amplitude a

⇒ coherent echo detectors (using the A-M signal)

A-M signal is easily detected at the photodetector as a photo-current variation given by:

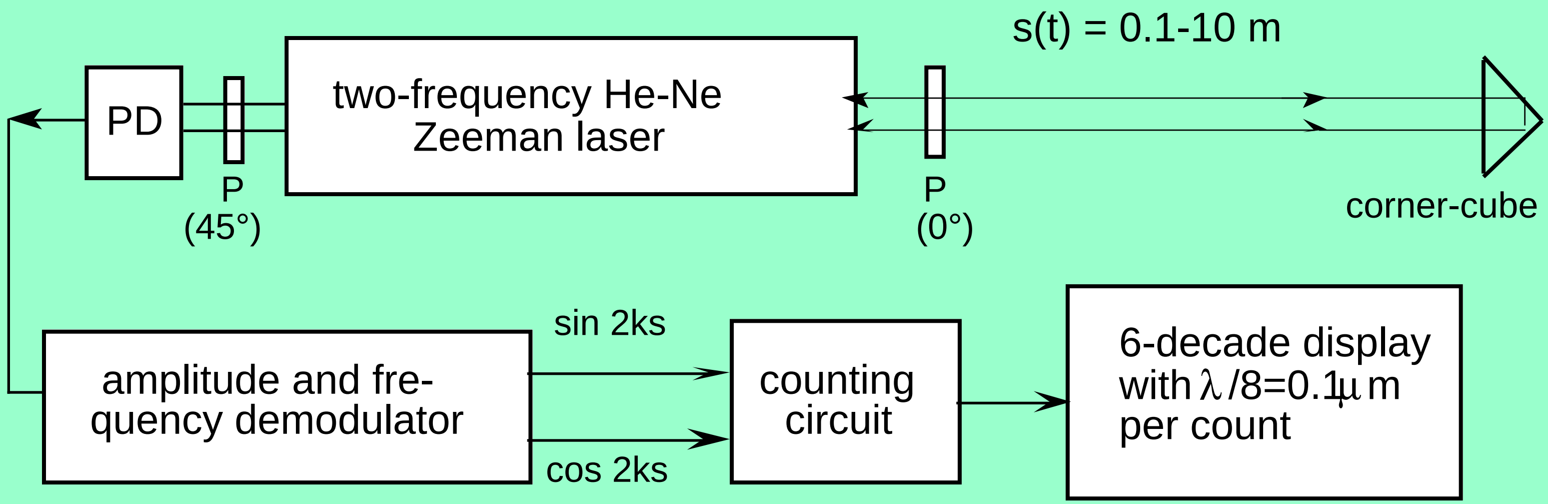
$$I_0 = (\sigma/2Z_0) \langle |E_{00}+e_0|^2 \rangle = I_{00}\{1+ 2[aT^2c/2L(\alpha-\Gamma)] \cos 2ks\}$$

F-M signal is: $\nu_0 = \nu_{00} + (aT^2c/2L) \sin 2ks$

and can be recovered by beating with a second oscillation in dual-mode (Zeeman) lasers

⇒ feedback interferometers, in which A-M and F-M provide the two quadrature signals cos2ks and sin 2ks.

Injection interferometer



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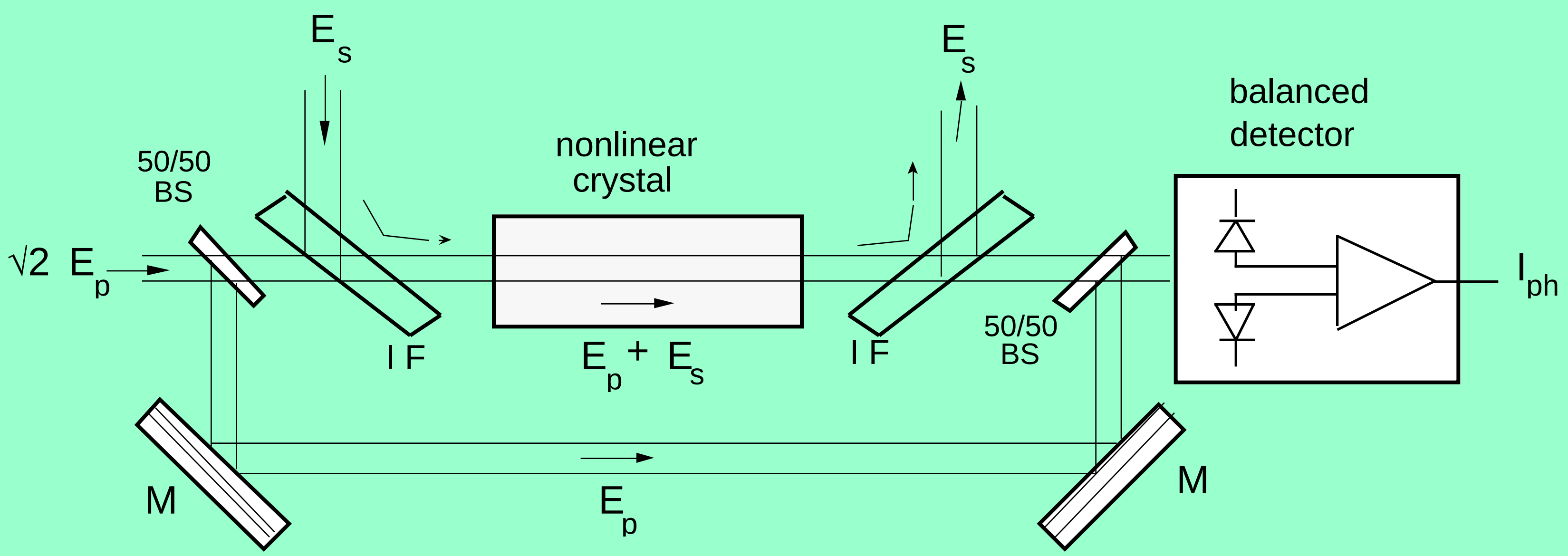
Quantum-Non-Demolitive detection

- QND is to detect photons without "demolishing" them (absorbing their energy) at the PD – photon can pass unaltered.
- To measure photons without absorbing, we need a *parametric* interaction with the detection system, giving a variation in a physical quantity measurable without energy exchange.
- A parametric medium may be one with an optical nonlinearity n_2 of the refraction index, in which a (weak) signal beam is superposed to a strong pump beam.
- Interaction of the two beams with nonlinearity generates an optical pathlength variation, proportional to signal power. This variation is read with a conventional interferometer, conveniently using the pump beam itself as the readout.

QND basic scheme



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QND signal

In the crystal of length L, the phase delay Φ=nkL is:

$$\Phi = nkL = kL\,[n_0+(n_2/2Z_0)\langle\,|E_p \exp i\varphi_p + E_s \exp i\varphi_s|^2\rangle\,]$$

$$= kL\,[n_0+(n_2/2Z_0)\,E_p^{\,2}] + (kLn_2/Z_0)E_pE_s\langle\cos(\varphi_p-\varphi_s)\rangle$$

$$= \Phi_0 + \Delta\Phi\,\mu$$

first term is an unessential fixed phase Φ_0,

second term contains the signal field amplitude E_s,

$$\Delta\Phi\,\mu = (kLn_2/Z_0)\,E_p\,E_s\,\mu,$$

proportional to the coherence factor $\mu=\langle\cos(\varphi_p-\varphi_s)\rangle$ containing the frequency difference $\varphi_p-\varphi_s=(\omega_p-\omega_s)t$. As in hetherodyne detection, it has a rms value $1/\sqrt{2}$.

Readout of $\Delta\Phi$ is performed dividing the pump beam in the two interferometer legs, ending in the balanced detector.

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QND signal (2)

Then, PDs currents are:

$$I_{ph1,2} = (\sigma A/2Z_0)\{(1/2)E_p^{\,2}+(1/2)E_p^{\,2} \pm 2(1/2)\cdot E_p E_s \cos nk(L-L_{ref})\}$$
$$= I_p \pm I_p \cos\{\Phi_0+(1/\sqrt{2})\Delta\Phi-\Phi_{ref}\}$$

where $I_p=(\sigma A/2Z_0)E_p^{\,2}$ is the photocurrent due to pump power and $\Phi_{ref}=nkL_{ref}$ is the phase delay of the reference leg. Adjusting the interferometer the balanced detector signal is:

$$S = I_{ph2} - I_{ph} = 2\, I_p \sin(1/\sqrt{2})\Delta\Phi = \sqrt{2}\, I_p\, \Delta\Phi \qquad (\text{for } \Delta\Phi \ll 1)$$

So, the dc component I_p is canceled and a linear dependence from $\Delta\Phi$ is obtained. S can be written, in terms of I_s:

$$S = \sqrt{2}\,(2kLn_2 I_p/\sigma A)\,\sqrt{(I_p I_s)} = \kappa\, G\, I_s$$

$G=\sqrt{(I_p/I_s)}$ being the coherent gain and $\kappa=2\sqrt{2}kLn_2 I_p/\sigma A$ the extra QND factor. QND detection is not much different from coherent detection if $\kappa \approx 1$.

QND signal (*Example*)

Carrying out the QND experiment in a GaAs crystal at λ=1μm where we have:

- $n_2 = 1.5 \cdot 10^{-12}$ cm^2/W,
- with a L=2cm guide of section $A = 2x3(\mu m)^2 = 6 \cdot 10^{-8}$ cm^2,

to attain $\kappa=1$ we need a pump power (in each arm):

$$I_p/\sigma = \kappa A / 2\sqrt{2} k L n_2 = 6 \cdot 10^{-8} / 2.82 \cdot 6.28 \cdot 10^{4} \cdot 2 \cdot 1.5 \cdot 10^{-12}$$

$= 110$ mW, a high but reasonable value

QND detection is difficult to implement for competing nonlinear effects (Raman and Brillouin scattering, modulation instability, parametric gain). These effects prevent performing the experiment in a km-long optical fiber where the pump power could be decreased because of the long interaction path.

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QND signal (*Comment*)

Also, from an engineering standpoint, it is objectionable to use a powerful laser as the pump in a detection system, respect to using the same active chip as an optical amplifier boosting a small fraction of power tapped from the signal, or, alternatively, to detect and re-transmit the signal.

Thus, if developed into a practical architecture, it is likely that QND will fill application niches rather than being a general tool. An envisaged application for QND could be in very large local area networks, where the signal could travel through a huge number of users and be detected with no loss at each location. Also, QND is interesting as the sole example of a parametric detection technique – eventually to be re-invented with a different working principle in a simpler, more viable scheme. .

QND noise

Noise in QND: each signal $I_{ph1,2}$ has shot-noise $2eI_pB$, $\Delta\Phi$ has a variance $\sigma^2_{\Delta\Phi}$, and last, we add Johnson-noise of the load R:

$$\sigma^2_S = 2e\,(2I_p)\,B + 2I_p^{\,2}\sigma^2_{\Delta\Phi} + 4kTB/R$$

With simple error-propagation calculations, we find that:

$$\sigma^2_{\Delta\Phi} = (kLn_2/\sigma A)^2\,[(I_s/I_p)\,\sigma^2_{Ip} + (I_p/I_s)\,\sigma^2_{Is}]$$
$$= (kLn_2/\sigma A)^2\,2e(I_s+I_p)B$$

therefore, the variance is:

$$\sigma^2_S = 2e(2I_p)B + 2\kappa^2\,2e(I_p+I_s)B + 4kTB/R$$

As $S^2=\kappa^2 I_p I_s$, using $N^2=\sigma^2_S$, we get for the QND S/N ratio:

$$(S/N)^2 = \kappa^2 I_s/2eB[2+2\kappa^2(1+I_s/I_p) + I_R/I_p)]$$
$$\approx (I_s/4eB)\,\kappa^2/(1+\kappa^2) \qquad \text{(for } I_p \gg I_s, I_R\text{)}$$

For large κ, QND performance reaches the quantum limit of I_s.

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A final remark on QND

Are photons leaving the QND detection really unaltered?

For Heisenberg uncertainty principle (HP), conjugate variables like photon number N (N=PT/hν) and phase Φ, have an uncertainty product not less than $\Delta N\ \Delta\Phi \geq 1/4$.

Observing (or measuring) N, amounts in quantum mechanics to having reduced the photon uncertainty ΔN. Then, it is necessary that phase uncertainty $\Delta\Phi$ after the QND mesurement becomes larger than that before observation.

Source for this is actually found in QND scheme: shot noise of the pump power modulates n by the nonlinear effect, and signal optical pathlength is affected by a random phase modulation increasing $\Delta\Phi$. By an analysis of the problem, it may be found that the excess phase-noise exactly satisfies HP.

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Beyond the Quantum Limit ?

Can we go beyond the quantum limit (QL) in any special case ?

No, with the usual sources having:

- a Poisson statistics of counts (energy $h\nu$ or charge *e* quanta)
- a shot (or quantum) noise with white spectral density $2h\nu P$ or $2eI$ and variance $2h\nu PB$ or $2eIB$

These statements are consequences of each other and establish the S/N QL we are familiar with, expressed equivalently as:

- mean photon number per bit N, giving $(S/N)=\sqrt{N}$
- noise in photocurrent $N_I^2=2eI_{ph}B$
- detected power $N_P^2=2eh\nu PB$.

Note: quantum limit is traced back to Poisson photon-statistics - is not caused by the detector.

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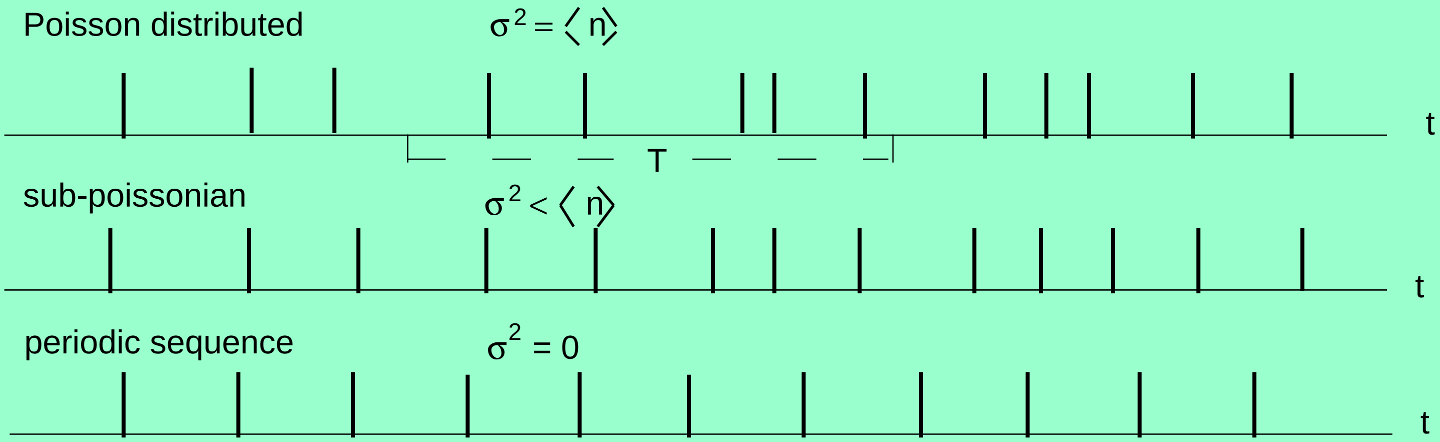
Squeezed-States: photons



To have a variance below QL, we should be able to build an unconventional radiation source with a sub-Poisson statistics of photons, i.e., a time distribution of emitted photon more regular than the Poissonian:

$$\sigma^2_n < \langle n \rangle \quad \text{(or } F<1\text{)}$$

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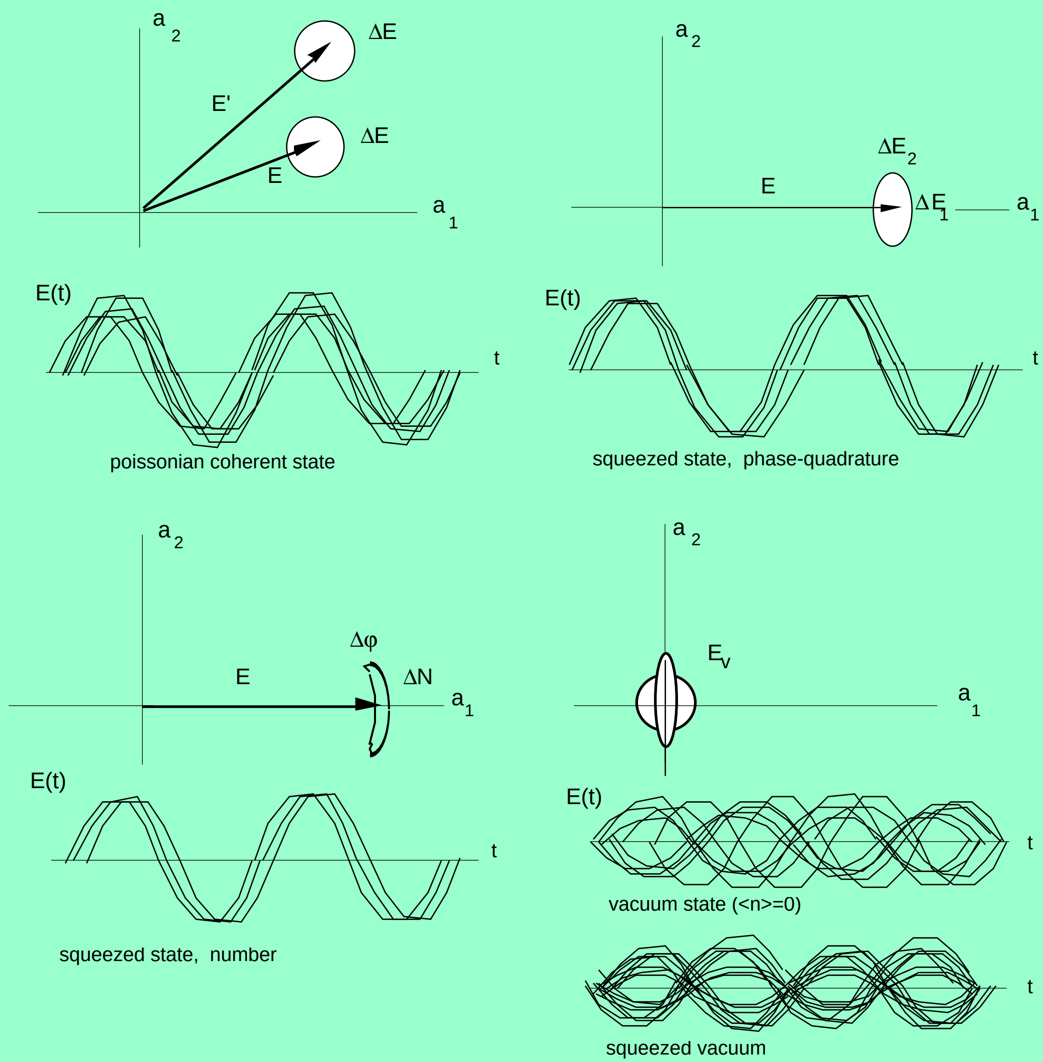
Squeezed-States: fields



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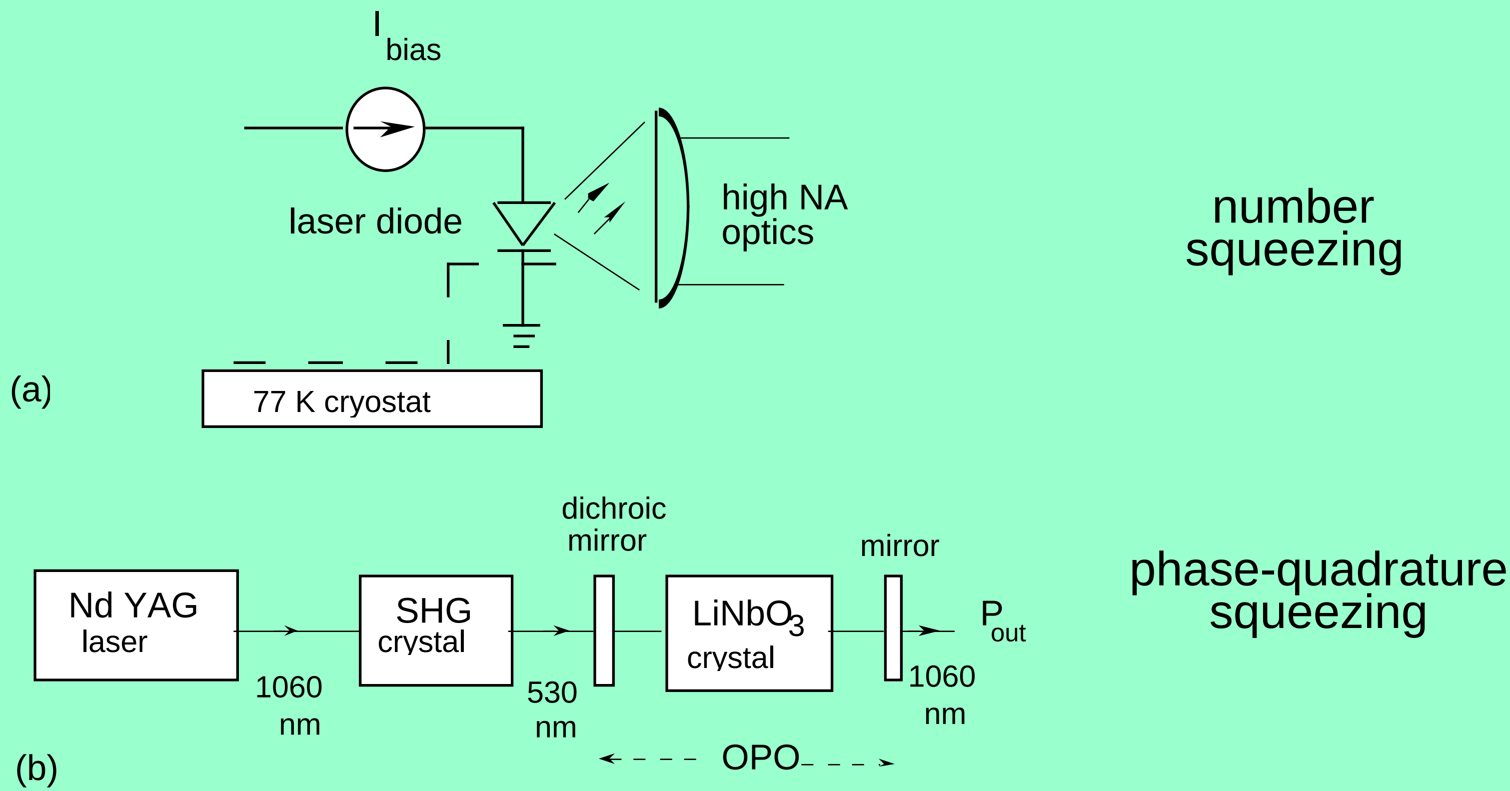
SS sources



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semiclassical view of vacuum fluctuations

power associated to $E+\Delta E$ is $P=(A/2Z_0)\langle|E+\Delta E|^2$. By developing the square we get:

$$P=(A/2Z_0)[|E|^2+2Re(E\Delta E^*)+|\Delta E^2|]$$

first term is the average power $P=(A/2Z_0)|E|^2$, last term can be neglected because small, and the second has a zero mean value and a variance given by:

$$\sigma^2_P=\langle\Delta P^2\rangle=(A/2Z_0)^2 4|E|^2\langle\Delta E^2\rangle = 4P(A/2Z_0)\langle\Delta E^2\rangle$$

If we want this expression coincides to shot noise, $\sigma^2_P=2h\nu PB$, we need $2(A/2Z_0)\langle\Delta E^2\rangle=h\nu B$, and, as the spectrum is white, we get:

$$d(\Delta E^2 A/2Z_0)/d\nu = {}^1/_2\, h\nu$$

In addition, letting $Z_0=\sqrt{(\mu_0/\varepsilon_0)}$ and $B=1/2T$ where $T=L/c$ is the time of observation and $L=V/A$, we readily get $\langle\Delta E^2\rangle = h\nu/2\varepsilon_0 V$. Again, $a^2=[(1/2)\varepsilon_0 V](E/h\nu)^2$ can be interpreted as the mean number of photons, and $\langle\Delta a^2\rangle$ its fluctuation, because $(1/2)\varepsilon_0 VE^2$ is the energy contained in the observation volume V.

semiclassical view of vacuum fluctuations 2

Thus, shot noise is explained as the beating of the mean field E and of the field fluctuation ΔE, with a power spectral density equal to half photon ($h\nu/2$) per Hertz of bandwidth.

Also, from ${}^{1}/{}_{2}\varepsilon_0 V\langle \Delta E^2\rangle /h\nu = {}^{1}/{}_{4}$, fluctuation in the number of observed photons is 1/2 photon and this comes from the ($h\nu/2$) uncertainty of energy $(1/2)\varepsilon_0 VE^2$ in the observation volume V.

Note that fluctuation ΔE is always the same amplitude, independent of the mean field E (or photon number N). This is true also for E=0 or n=0, that is, for the *zero-field* or vacuum state, the fluctuation ΔE is frequently referred to as the coherent-state or the vacuum-state fluctuation.

The phase fluctuation is expressed as the ratio of ΔE to mean value E, i.e., $\Delta\phi = \Delta E/E$. By squaring and averaging we get

$\langle \Delta\phi^2\rangle = \langle \Delta E^2\rangle / |E|^2 = (1/2)h\nu B/P = h\nu/4TP = 1/4N$,

where $N = \langle n\rangle = PT/h\nu$ is the mean number of photons.

SS in coherent detection

Detecting a SS radiation by direct-detection, one has

$$(S/N)^2 = \langle n \rangle^2/\sigma^2_n = N / F$$

But, if SS is attenuated by ε, squeezing factor is degraded to:

$$F' = 1 - \varepsilon (1-F)$$

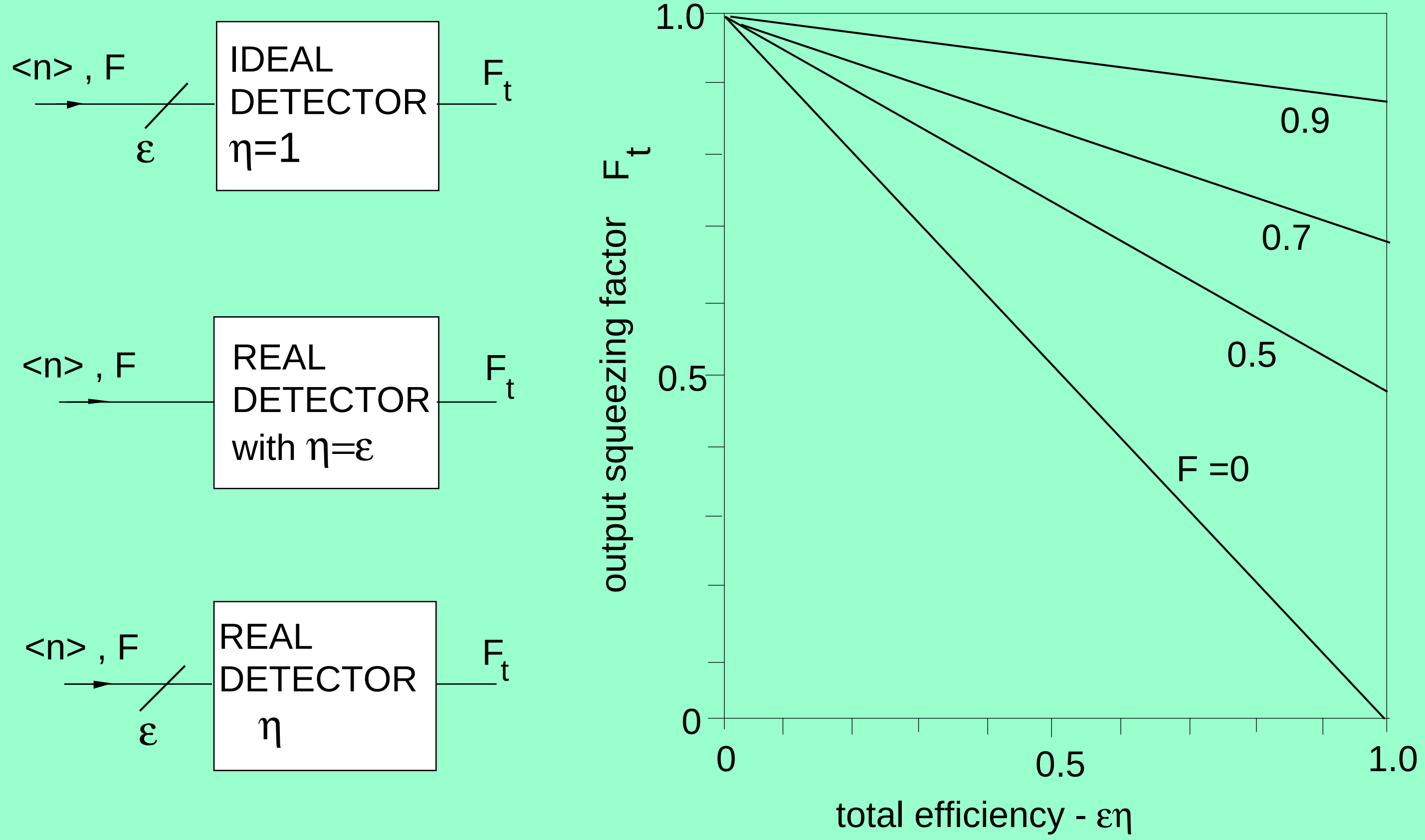
In homodyne detection and with a balanced detector, S/N ratio of the photocurrent is :

$$(S/N)^2_{hom,ss} = I^2/\langle \Delta I \rangle^2 = 4\mu^2 I\, I_0 / [2e(IF_0+I_0F+I_b)B+4kTB/R]$$

where F and F_0 are the squeezing factors of signal I and local oscillator I_0: squeezing factors interexchange in product. For $I_0 >> I+I_b+I_R$ the classical homodyne S/N is improved by the *signal* squeezing factor

$$(S/N)^2_{hom,ss} = (S/N)^2_{hom,cl}/F$$

Lability of SS radiation to attenuation



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SS in phase measurements

In phase measurements, attenuation can be made negligible and we fully exploit the squeezing factor improvement in the variance $\langle \Delta\Phi^2 \rangle$ of the phase Φ under measurement.

In a Mach-Zehnder read by a coherent radiation P entering at the input port, output at the balanced detector and phase are:

$$V_u = R\sigma P\cos\Psi \approx -RI\Phi, \qquad \langle \Delta\Phi^2 \rangle_{cl} = eB/2I$$

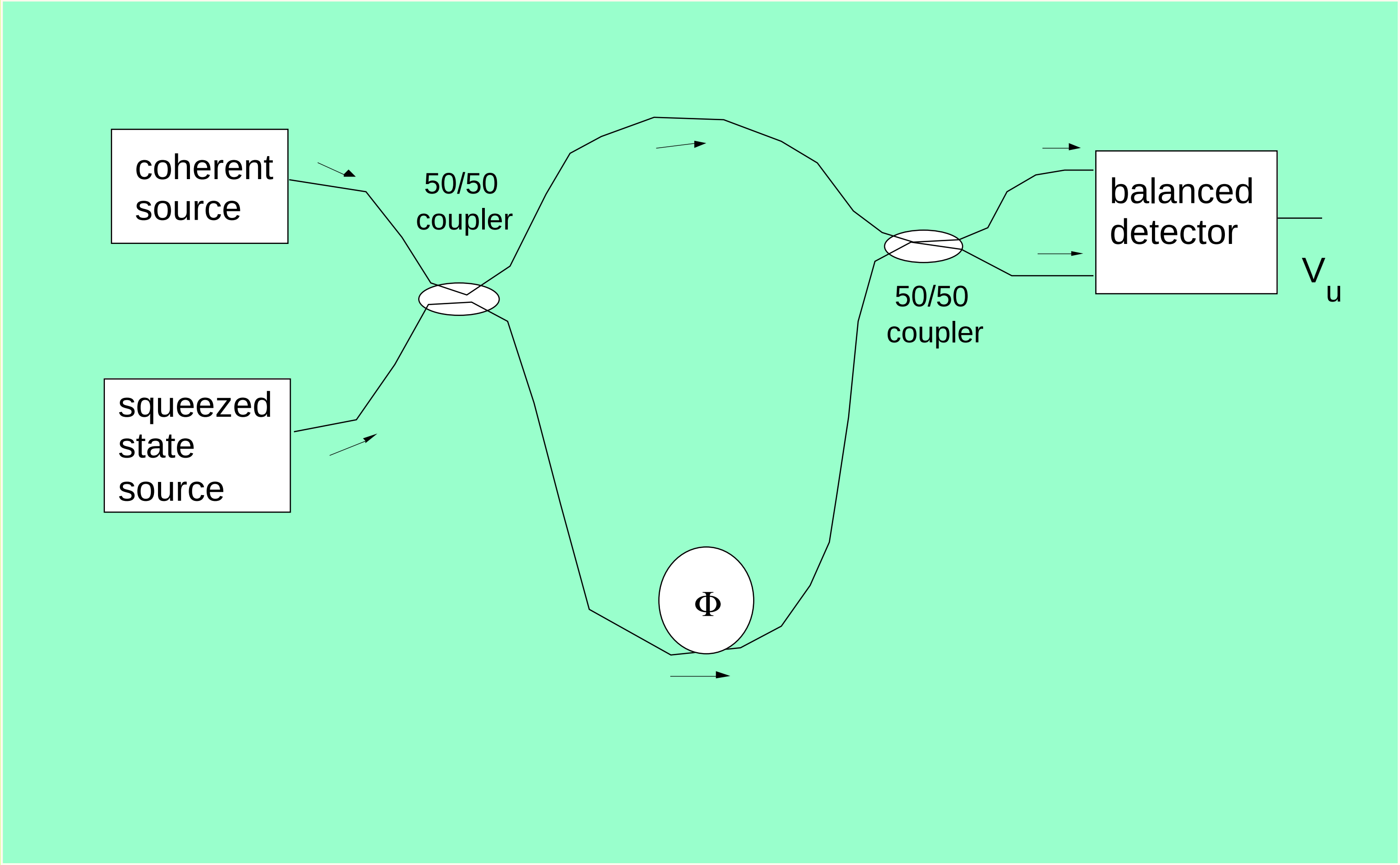
where $\Psi = nk(l_1 - l_2) = \pi/2 + \Phi$ is the optical pathlength difference, adjusted in quadrature so as to read Φ.

Adding a source of squeezed radiation at the normally unused input port, the phase variance is calculated as:

$$\langle \Delta\Phi^2 \rangle_{ss} = F_0 \langle \Delta\Phi^2 \rangle_{cl} = F_0\, eB/2I$$

($I = \sigma P$, F_0 = squeezing factor). Accuracy is improved by F_0.

SS in interferometry



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New model of photodetector noise

At optical frequencies, *first quantization* is no more sufficient. We need to account for HP embodying *second quantization* in a semiclassical model (SCM) . Postulates of SMC are:

◊ any electric field E of radiation has a Gaussian distributed fluctuation ΔE_{coh} with mean and variance given by:

$$\langle \Delta E_{coh} \rangle = 0, \quad \langle \Delta E_{coh}^2 \rangle = (2Z_0/A)^1/_2\, h\nu\, B$$

◊ in *any unused port*, with no applied signal, the vacuum field still exists with its fluctuations, and there we shall take account of $\langle \Delta E_{vac}^2 \rangle = \langle \Delta E_{coh} \rangle$ entering in the experiment.

◊ the vacuum fluctuation is *not* directly *observable* in itself, and thus in the basic photodetection relation we subtract $\langle \Delta E_{vac}^2 \rangle$:

$$I_{ph} = (\sigma A/2Z_0)\, \{\langle |E|^2 \rangle - \langle |\Delta E_{vac}|^2 \rangle \}$$

Application of photodetector noise model

Model gives correct results and explains:

• *Detection of a coherent (Poissonian) signal by an ideal detector:* classical result is found and is newly interpreted as the beating (or heterodyning) of the field fluctuation ΔE_{coh} with signal oscillation

•*Detection of a coherent signal with a real detector:* as above

• *Detection of a squeezed-state signal with $F\neq1$ by a real detector*: composition rule for the squeezing factor $F'=1-\eta(1-F)$ is found.

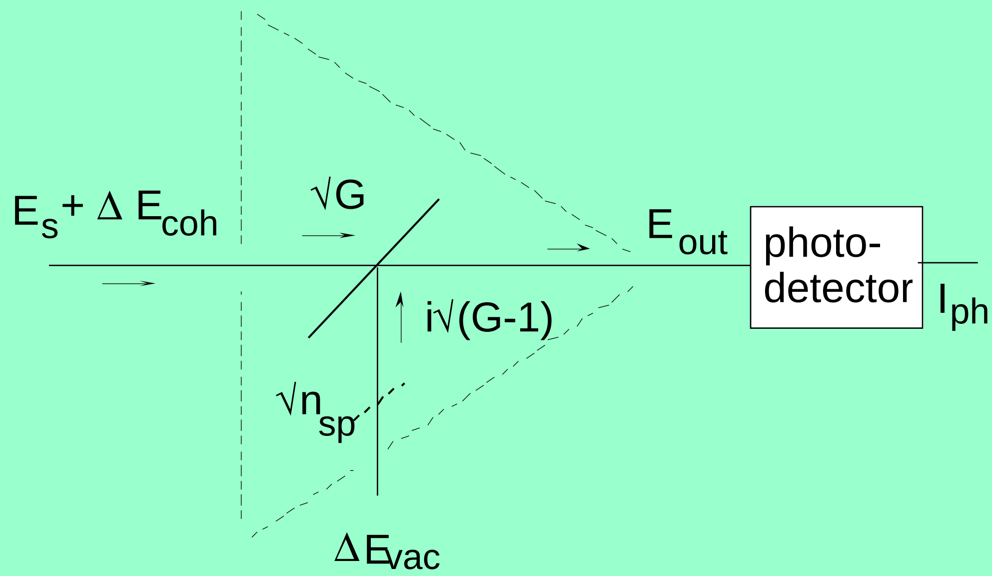
• *Coherent detection with signal and pump squeezed by F_s and F_0* : two features escaping first-quantization are explained (cross-multiplication of squeezing factors with signals, and that no shot noise accompanies the beating)

• *Detection of a wide-spectral bandwidth signal:* beating of the fluctuations of the various spectral components is found

• *Detection of a signal on a wide-spectral bandwidth background:* beating of the fluctuations ofspectral components and its heterodyning with signal is explained

from:"Photodetectors", by S.Donati, Prentice Hall 2000

OA model



OA is modeled by an amplifying beamsplitter opening a port on vacuum field fluctuations. Amplified vacuum field is just the ASE, and all the noise terms are obtained with no further assumptions. Adding a second beamsplitter n_{sp} accounts for the incomplete inversion of the medium.

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New Model: Example 1

• *Detection of a coherent (Poissonian) signal by an ideal detector*

A signal carrying a power P_s has an average field $E_s = \sqrt{[(2Z_0/A)P_s]}\, e^{i(\omega t+\phi)}$ and a fluctuation $\Delta E_s = \Delta E_{coh}$. By inserting the field at the PD as $E_{ph} = E_s + \Delta E_{coh}$, and developing the modulus, we get:

$$I_{ph} = (\sigma A/2Z_0) \cdot \{\langle E_s E_s^* + \Delta E_{coh} \Delta E_{coh}^* + 2 \mathrm{Re}\, E_s \Delta E_{coh}^* \rangle - \langle \Delta E_{vac} \Delta E_{vac}^* \rangle \}$$

in which $\sigma = e/h\nu$ for an ideal detector. The first term is the usual mean current $I_{ph} = \sigma P_s = \sigma A \langle E_s^2 \rangle / 2Z_0$, the second cancels out with the last, and the third has a zero mean and a spectral density:

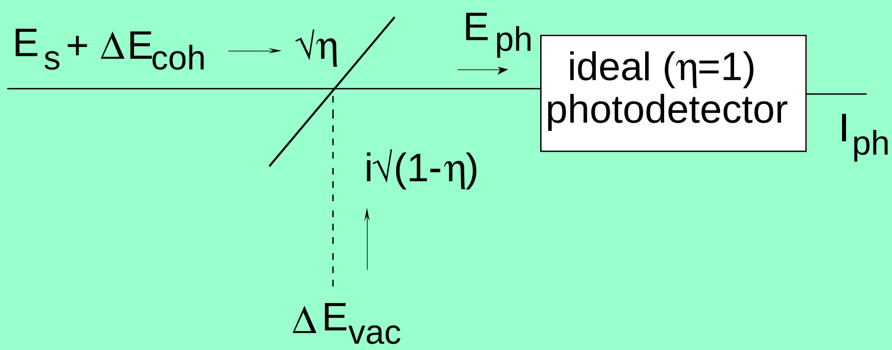
$$S = 4E_s^2 (2Z_0/A)\, {}^1/_2\, h\nu$$

whence the variance of the photogenerated current is:

$$\sigma^2_I = (\sigma A/2Z_0)^2 S = (\sigma A/2Z_0) 4E_s^2 \sigma\, {}^1/_2 h\nu B = 2I_{ph}\,(e/h\nu) h\nu B = 2\, e\, I_{ph}\, B$$

Thus, we have re-obtained the classical result and can newly interpret it as the beating (or heterodyning) of the field fluctuation ΔE_{coh} with signal oscillation.

New Model: Example 2



• *Detection of a coherent signal with a real detector*

A real detector with $\eta<1$ is equivalent to an ideal detector preceded by a BS with transmission η, thus we model it as shown and add, in the unused port, the vacuum field fluctuation ΔE_{vac}. Considering the BS, the field at PD is:

$$E_{ph} = \sqrt{\eta}\, E_s + \sqrt{\eta}\, \Delta E_{coh} + i\sqrt{(1-\eta)}\, \Delta E_{vac}$$

where field transmission is $\sqrt{\eta}$ and factor $i=\sqrt{-1}$ is due to the beamsplitter phaseshift. By inserting in above equation, we get the classical result:

$$I_{ph} = \eta(e/h\nu)\, P_s, \qquad \sigma^2_I = 2\, e\, I_{ph}\, B$$

New Model: Examples 3,4

• *Detection of a squeezed-state signal with $F \neq 1$ by a real detector*

In this case we have again last equation, but with ΔE_s substituted by $\sqrt{F}\Delta E_s$, and the variance is:

$$\sigma^2_I = 2\, e\, \eta\, I_{ph}\, B + 2\, e\, (1-\eta)\, F\, I_{ph}\, B$$

From this result, the composition rule for the squeezing factor $F' = 1-\eta(1-F)$ follows. We may also remark that this result comes from the beating of the signal with the vacuum fluctuation, of which we can find only a trace for $F \neq 1$ [otherwise we retum to classical result].

• *Coherent detection with signal and pump squeezed by F_s and F_0*

Repeating the calculations for the schemes of SS, quoted results are obtained. Thus, two features escaping first-quantization models are explained:

- that squeezing factor and signal are multiplied by each other and interchanged in the noise;
- that no shot noise need be attributed to the beating term signal-local oscillator $E_s E_0$ in coherent detection.

Example 5

• *Detection of a wide-spectral bandwidth signal*

In this case the signal is the superposition of many independent frequency oscillations and shot-noise variance no longer applies.

Let $g(f-f_0)$ be the spectral power distribution, which we assume with a unity peak value $g(0)=1$ at $f=f_0$, so that its integral on f, $\int_{-\infty,+\infty} g(f)\,df = B_{opt}$, represents the source linewidth. The spectral power in $f...f+\Delta f$ is $\Delta P=(P_s/B_{opt})g(f)\,\Delta f$ and the corresponding mean field is: $E_s(f) = \sqrt{[(2Z_0/A)\,\Delta P]}$.

As usual, we shall add to the mean field the contribution ΔE_{coh}. By repeating the calculations, mean and spectral density of photogenerated current are found as:

$$\langle I_{ph}\rangle = \sigma P_s, \qquad d\langle \Delta I_{ph}^2\rangle /df = 2\,eI_{ph}+2(I_{ph}^2/B_{opt})\,g(f)_*g(f)$$

where * indicates the convolution operation and g(f) is the emission spectral distribution translated in baseband. The second term is from the beating of the fluctuations in the spectral components $g(f-f_0)$ of the source. For a measurement on an electrical bandwidth $B \ll B_{opt}$, we get from the result, assuming $g(0)=1$:

$$\sigma_{Iph}^2 = [2\,eI_{ph}+2(I_{ph}^2/B_{opt})]\,B$$

New Model: Example 6

• *Detection of a signal on a wide-spectral bandwidth background*

Here, the signal is a narrow-band coherent state and is added to a wide spectrum background (such as dark current, ASE, etc.). Let I_S indicate the signal and I_{ASE} the background with a distribution $g_{ASE}(f)$ as in Example 5. From the calculations, one has:

$$\langle I_{ph} \rangle = \sigma(P_S+P_{ASE}) = I_S + I_{ASE}$$

$$d\langle \Delta I_{ph}^2 \rangle /df = 2eI_S+2eI_{ASE}+2(I_{ASE}^2/B_{opt})\, g(f)*g(f) + 4(I_S I_{ASE}/B_{opt})\, g(f)$$

The four terms can be interpreted as: signal shot noise, ASE shot noise, ASE-ASE beating and signal-ASE beating. Letting again $B<<B_{opt}$ we get:

$$\sigma_{Iph}^2 = [2eI_S+2eI_{ASE}+2(I_{ASE}^2/B_{opt}) + 4(I_S I_{ASE}/B_{opt})]\, B$$

If we now consider the ASE of an OA, $I_{ASE}=\sigma\, P_{ASE}=(e/h\nu)(G-1)h\nu B_{opt}$, and that the signal out is $I_S= GI_{s(in)}$, above eq. can be brought to coincide with already considered expressions.