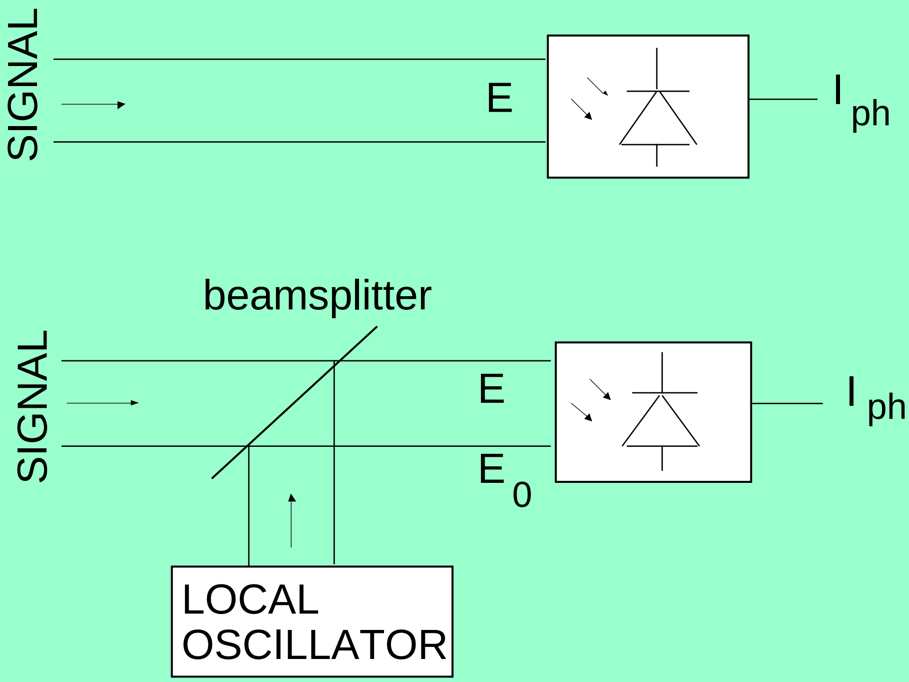
Direct and Coherent Detection



DIRECT

COHERENT

direct detection is linear in power $I_{ph} = \sigma P$

and quadratic in field amplitude: $I_{ph} = \sigma A \langle |E|^2 \rangle /2Z_0$

from:"Photodetectors", by S.Donati, Prentice Hall 2000

Coherent gain

In coherent detection, signal and local oscillator fields are:

$$E = \mathrm{E} \exp i(\omega t+\varphi), \quad E_0 = \mathrm{E}_0 \exp i(\omega_0 t+\varphi_0)$$

thus $\quad I_{ph}= (\sigma A/2Z_0) [\langle |E|^2\rangle + \langle |E_0|^2\rangle + 2 \mathrm{Re} \langle E_0 E^*\rangle] =$

$$= (\sigma A/2Z_0) \{\mathrm{E}^2 + \mathrm{E}_0^2 + 2 \mathrm{E} \mathrm{E}_0 \langle \cos [(\omega - \omega_0)t+\varphi-\varphi_0]\rangle \}$$

or, $\quad I_{ph} = I + I_0 + 2\mu \sqrt{(I I_0)}$

compared to I of direct detection, we find a

coherent gain $\quad G_{coh}= I_{ph}/I= 1+2\mu \sqrt{(I_0 /I)}$

μ is the coherence factor. When $\omega=\omega_0$ detection is called *homodyne*, while if $\omega\neq\omega_0$ we have *heterodyne* detection.

from:"Photodetectors", by S.Donati, Prentice Hall 2000

Coherence factor

$\mu = \langle \cos(\varphi - \varphi_0) \rangle$ ranges from $\mu = 0$ (uncorrelated phases of signal and local oscillator), to $\mu = 1$ (complete correlation).

Now consider homodyne detection ($\omega = \omega_0$) and write φ as the sum of a mean $\langle \varphi \rangle$ and a random part φ_r: $\varphi = \langle \varphi \rangle + \varphi_r$

Developing μ, $\mu = \langle \cos(\langle \varphi \rangle + \varphi_r - \varphi_0) \rangle$
$= \cos(\langle \varphi \rangle - \varphi_0) \langle \cos \varphi_r \rangle - \sin(\langle \varphi \rangle - \varphi_0) \langle \sin \varphi_r \rangle$.

As $\langle \varphi_r \rangle = 0$, also $\langle \sin \varphi_r \rangle = 0$ and if φ_r has a regular statistics. So:

$$\mu = \cos[\langle \varphi \rangle - \varphi_0] \langle \cos \varphi_r \rangle = \cos \Delta\varphi \, \mu_\Phi$$

Beating signal is multiplied by factor $\cos\Delta\varphi$, that is, homodyne detection is sensitive to the *in-phase* component with $\langle \varphi \rangle = \varphi_0$; the *in-quadrature* component with $\langle \varphi \rangle = \varphi_0 + \pi/2$ gives a zero output.

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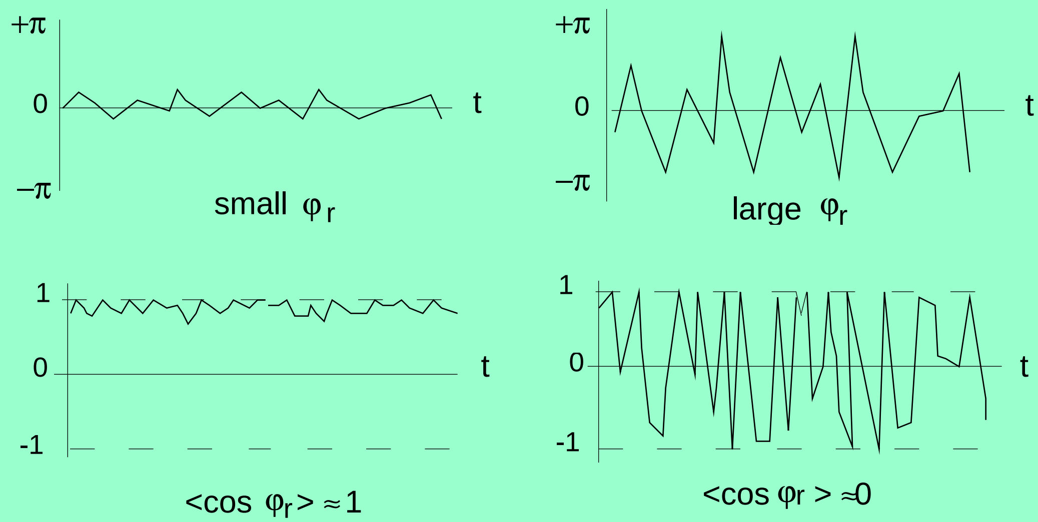
Phase fluctuations

The random part $\langle \cos\varphi_r \rangle = \mu_\Phi$ describes relative phase fluctuations. For φ_r small (<<1 rad), cosine is close to unity and its mean is ≈ 1; for large φ_r (over 2π), cosine spans from -1 to +1 and mean will be ≈ 0. For small φ_r <<1, developing cosine in series of φ_r:

$$\mu_\Phi = \langle \cos \varphi_r \rangle = \langle 1 - \varphi_r^2/2! + \varphi_r^4/4! + \ldots \rangle \approx 1 - \sigma_\Phi^2/2$$

we see that μ is connected to phase variance σ_Φ.



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Coherent S/N ratio

In direct detection, $(S/N)^2_{dir} = I^2 / [2e(I+I_d)B + 4kTB/R]$

and quantum limit is $(S/N)^2_{dir/q} = I/2eB$ for $I >> I_d + 4kT/R$.

In homodyne, signal is $(\sigma A/2Z_0)2\mu EE_0$, noise is sum of local oscillator and signal shot-noises, plus Johnson noise of load:

$$(S/N)^2_{hom} = \frac{[(\sigma A/2Z_0)2\mu EE_0]^2}{2e[(\sigma A/2Z_0)(E^2+E_0^2)+I_d]\,B+4kTB/R}$$

$$= \frac{4\mu^2 I\, I_0}{2e(I+I_0+I_d)B + 4kTB/R}$$

dividing by I_0 and letting $I_R = 2kT/eR$,

$$(S/N)^2_{hom} = \frac{4\mu^2\, I}{[2e(1+(I+I_d+I_R)/I_0]\, B}$$

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Quantum limited S/N

for $I_0 >> I_{0q} = I + I_d + I_R$, the quantum limit is always reached:

$$(S/N)^2_{hom/q} = 4\mu^2 I / 2eB$$

In coherent detection the Q-L condition is on local oscillator amplitude, *not* on signal amplitude as in direct detection. Making local oscillator $I_0 >> I + I_d + I_R$ large enough, Q-L is reached, even at weak signal levels.

Heterodyne detection follows the same arguments, but beating signal is now at the frequency $\omega - \omega_0$, so that

$$I_{ph} = 2\sqrt{(I\, I_0)} \cos[(\omega - \omega_0)t + \langle \varphi \rangle - \varphi_0] \langle \cos\varphi_r \rangle$$

$$(S/N)^2_{het} = 2\mu_\phi^2 I\, I_0 / [2e(I + I_0 + I_d)B + 4kTB/R]$$

i.e., it has a modest *penalty* - a factor of 2 (or 3dB) respect to homodyne but does not require the phase adjustment.

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Condition for coherent detection

Requirements:

- *phase matching* of signal and local oscillator (for homodyne), or beating will be reduced by a factor: $\cos(\langle \varphi \rangle - \varphi_0)$
- *phase coherence*, or signal will be reduced by: $\langle \cos\varphi_r \rangle = \mu_\phi$.
- superposition of E and E_0 on the PD with *spatial coherence* or beating will be reduced by a factor:

$$\mu_{sp} = \int_A E(x,y) \cdot E_0{*}(x,y)dxdy \,/ [\int_A |E(x,y)|^2 dxdy \int_A |E_0(x,y)|^2 dxdy\,]^{1/2}$$

- superposition of E and E_0 with *polarization matching* or signal is reduced by:

$$\mathbf{E}\cdot\mathbf{E_0} / |\mathbf{E}|\,|\mathbf{E_0}| = \mu_{pol} \qquad (\mathbf{E},\ \mathbf{E_0} = \text{Jones matrixes})$$

All previous expressions are generalized by using $\mu_\phi \rightarrow \mu_\phi\, \mu_{sp}\, \mu_{pol}$

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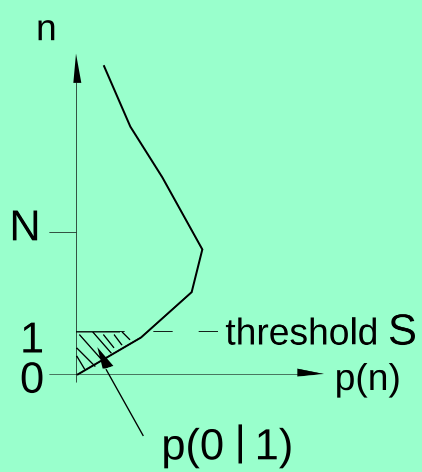
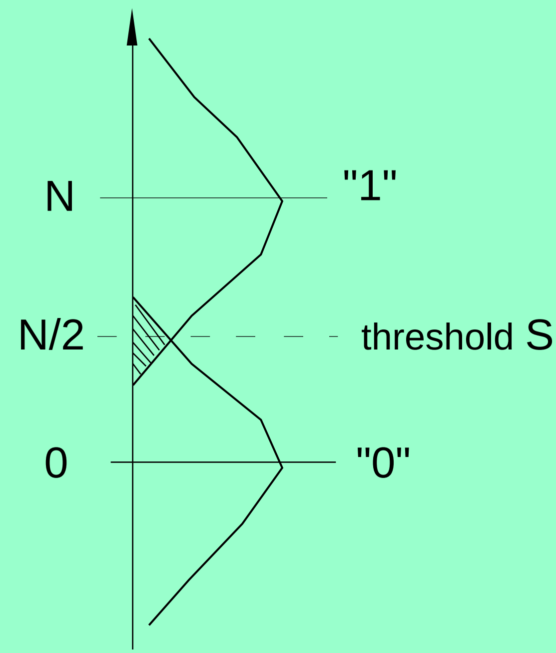
S/N, BER and photons/bit





ideal direct detection:

$BER = e^{-N}$

for $BER = 10^{-9}$: N=10 p/b

$(S/N)^2 = 10$

ideal coherent detection:

threshold at N/2

and $P(0|1) = P(1|0)$

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Photons per bit and modulation

- *homodyne detection* of amplitude modulated (ASK) signal:

$$BER = \mathrm{erfc}\; N/2\sigma_N \quad (N = \text{number of photons per bit})$$

$$N = 2(I \cdot I_0)^{1/2} T/e = 2(N_s N_0)^{1/2} \quad \sigma_N = (2eI_0/2T)^{1/2} T/e = N_0^{1/2}$$

then, $BER = \mathrm{erfc}\, \sqrt{N_s}$

and for $BER = 10^{-9}$ we get $N_s = 36$ p/b

- *homodyne* detection of a phase-modulated PSK signal:

$$BER = \mathrm{erfc}\; 2\sqrt{N_s}, \quad \text{and} \quad N_s = 9 \text{ p/b}$$

- *heterodyne* detection of a PSK-modulated signal:

$$BER = \mathrm{erfc}\sqrt{2N_s}, \quad \text{and } N_s = 18$$

homodyne detection of a 4Φ–PSK modulated signal

$$BER = \mathrm{erfc}\; 2\sqrt{2N_s}, \quad \text{and } N_s = 4.5$$

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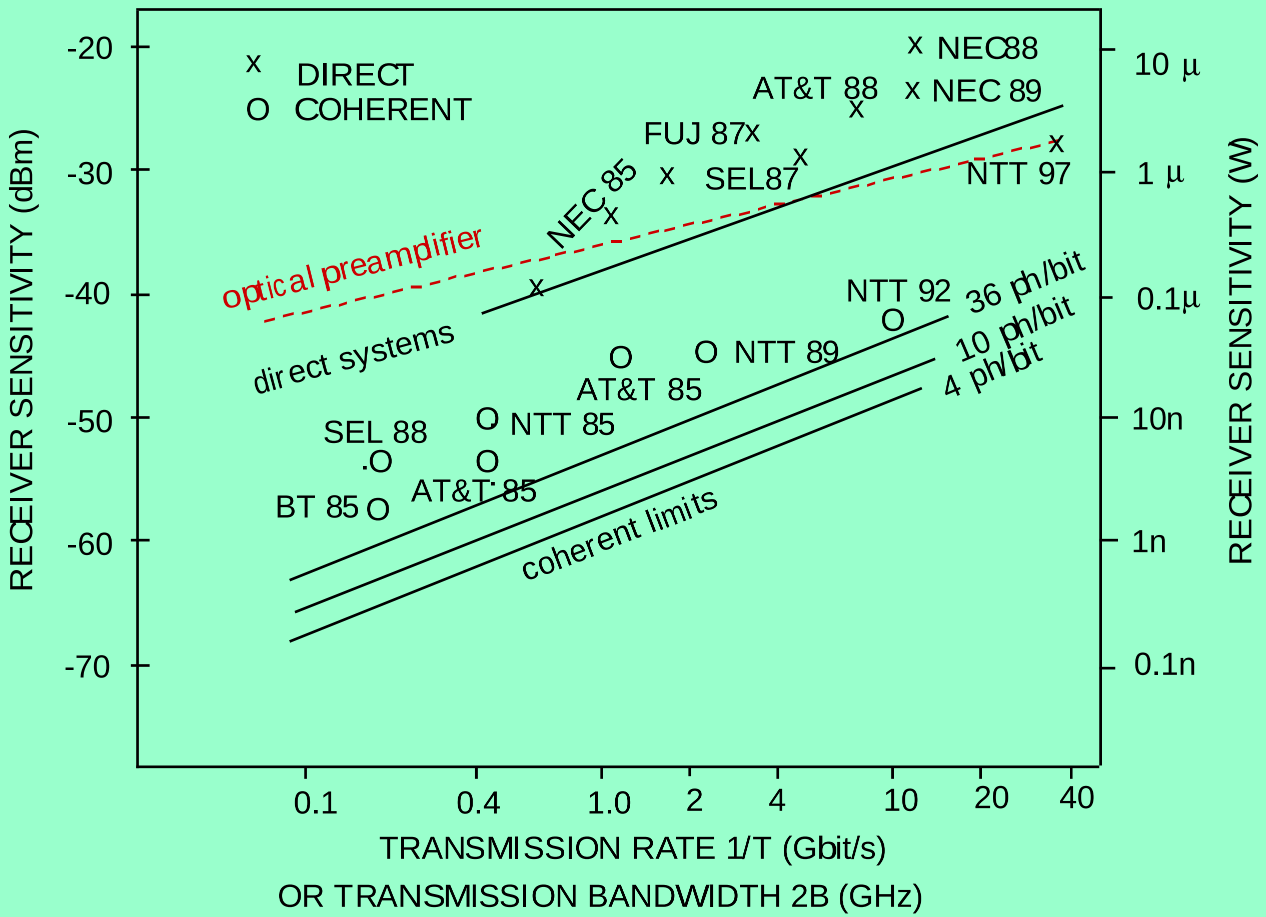
State-of-the-art receiver sensitivity



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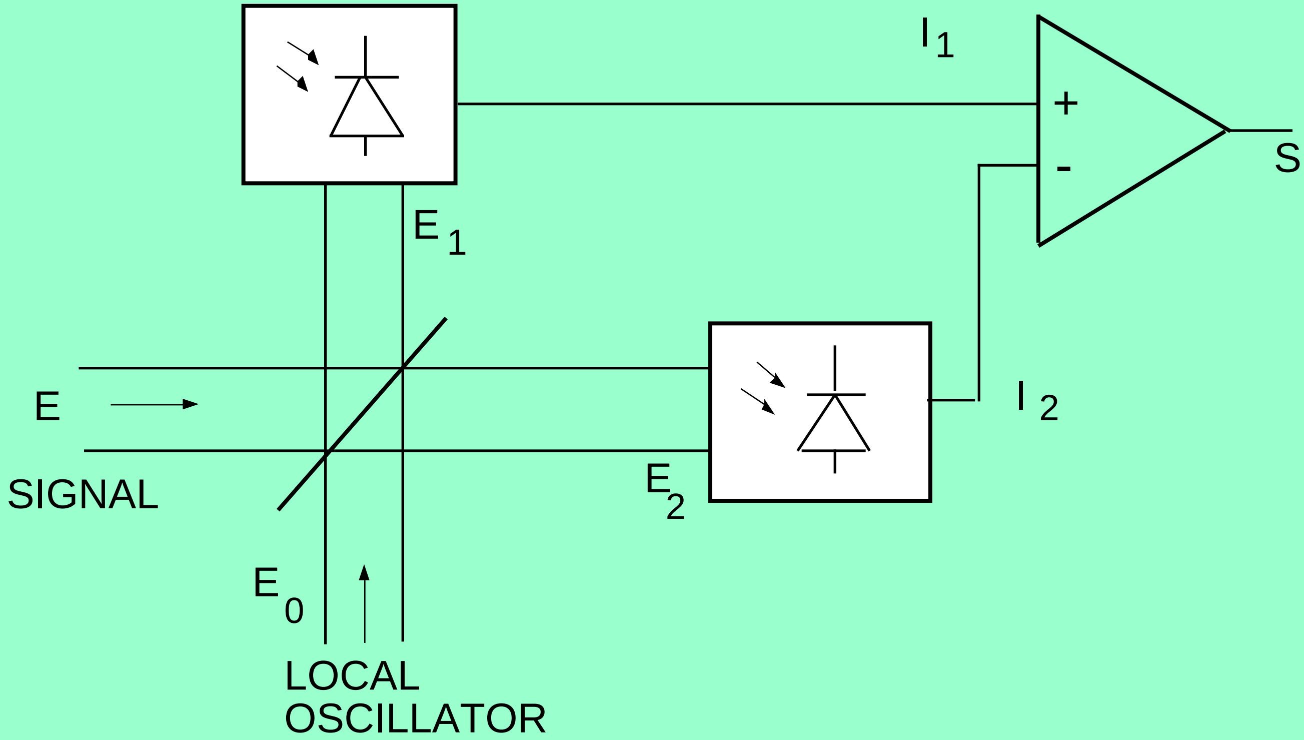
Balanced detector



$$I_1 = (1/2)(I_0+I)+\sqrt{(I_0 I)}\,\sin(\varphi-\varphi_0), \quad I_2 = (1/2)(I_0+I)-\sqrt{(I_0 I)}\,\sin(\varphi-\varphi_0)$$

$$S = I_1 - I_2 = 2\sqrt{(I_0 I)}\,\sin(\varphi-\varphi_0)$$

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Beamsplitter phaseshift

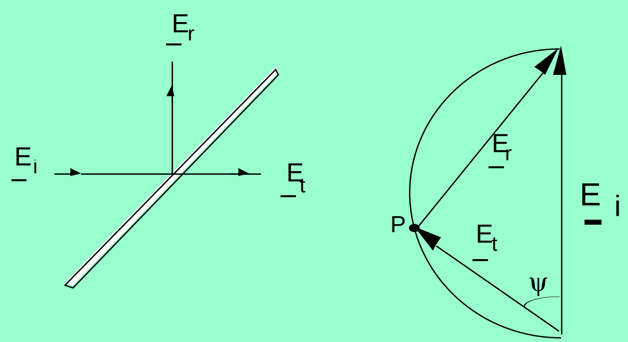
At a beamsplitter, the continuity condition of electric fields at the separation boundary requires that the incident E_i is always the sum of reflected E_r and transmitted E_t fields:

$$\underline{E}_i = \underline{E}_t + \underline{E}_r \qquad \text{(I)}$$

where the underlines indicate rotating vectors. Also, in a lossless beamsplitter power P is unchanged upon splitting and, as P is proportional to E^2, we have:

$$E_i^2 = E_t^2 + E_r^2 \qquad \text{(II)}$$

To have both equations satisfied, the three vectors must lie on a right-angle triangle, as shown in the figure below. Then, the angle - or phaseshift - between reflected $\underline{E}_r$ and transmitted $\underline{E}_t$ vectors is $\pi/2$ irrespective of the actual splitting ratio, while the angle ψ between incident and transmitted fields increases from 0 to $\pi/2$ as E_t decreases from E_i to 0 (or, R goes from 0 to 1). We can then write, for the lossless beamsplitter:



$$\underline{E}_t = \sqrt{(1-R)}\, E_i\, e^{i\psi}, \qquad \underline{E}_r = \sqrt{R}\, E_i\, e^{i(\psi-\pi/2)}$$

For a lossy beamsplitter, Eq.(I) still applies, while (II) holds with the ≥ sign; then point P in the figure shifts internal to the circle and the $\underline{E}_r$ $\underline{E}_t$ phaseshift becomes larger than $\pi/2$ (of an angle $p/2\sqrt{[R(1-R)]}$ where p is the loss).

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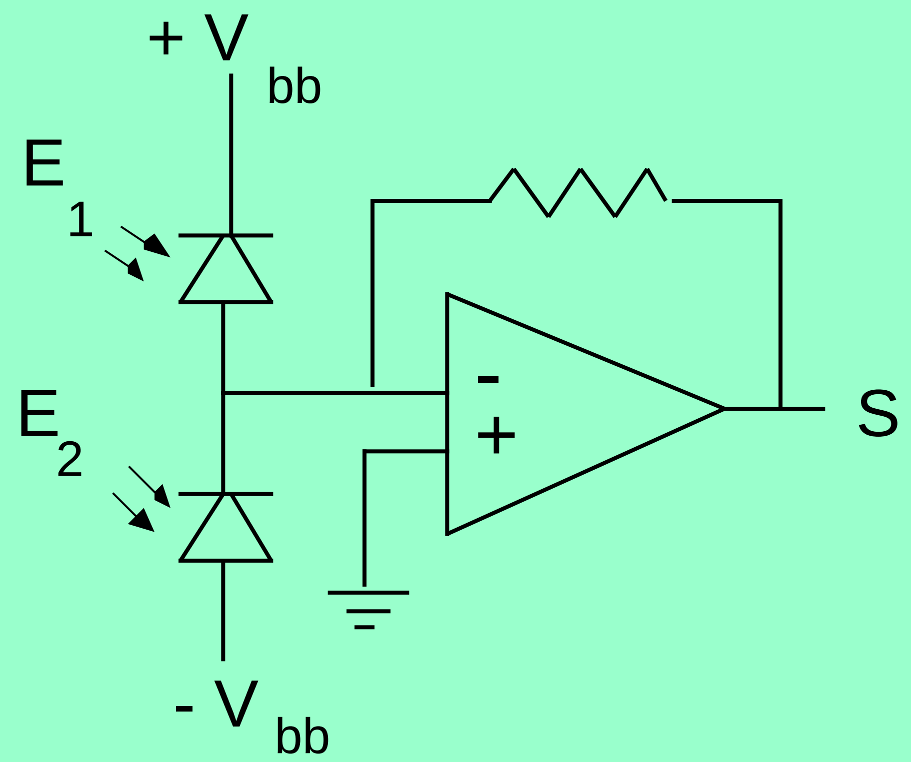
Balanced detectors with input subtraction



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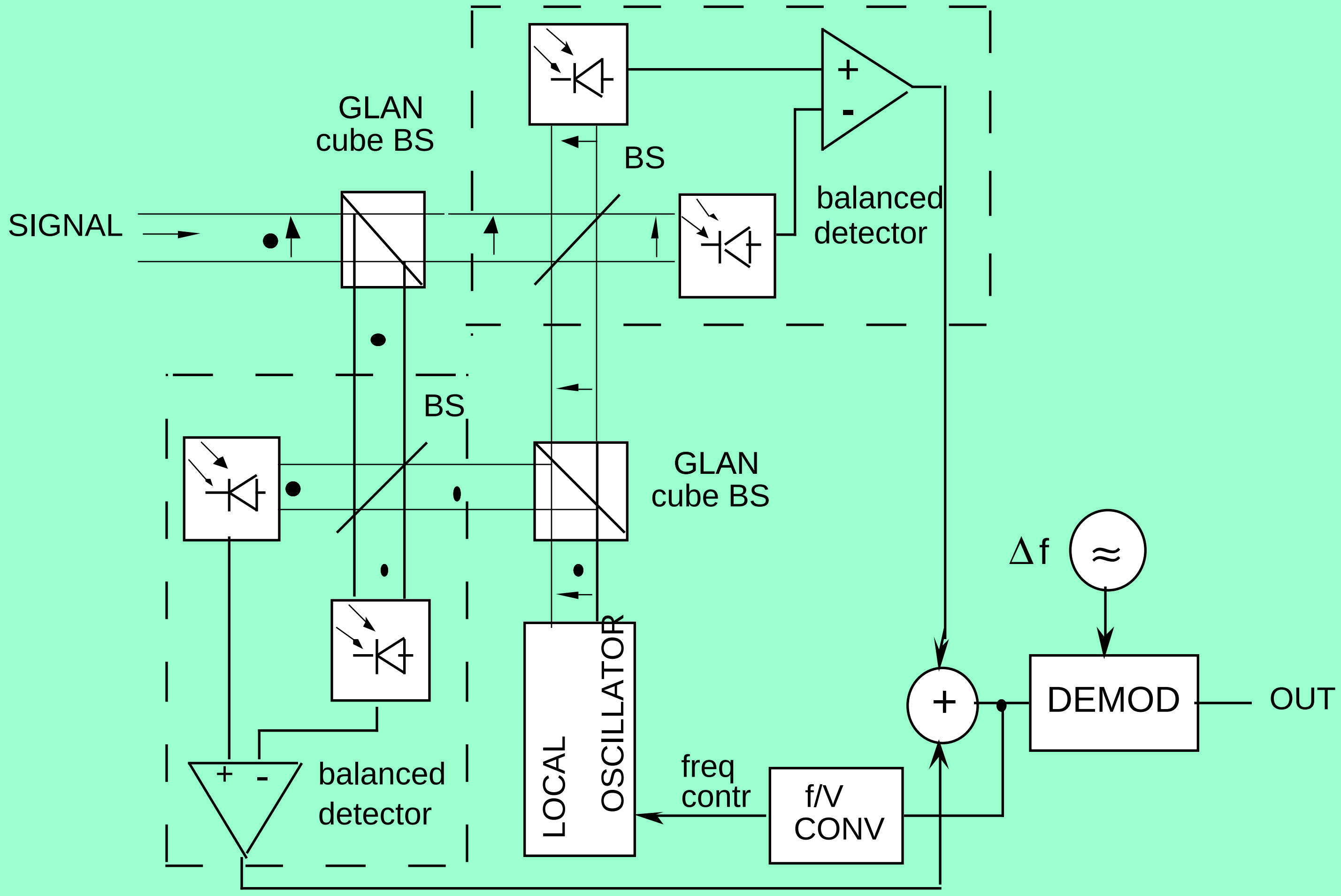
Coherent receiver with polarization diversity



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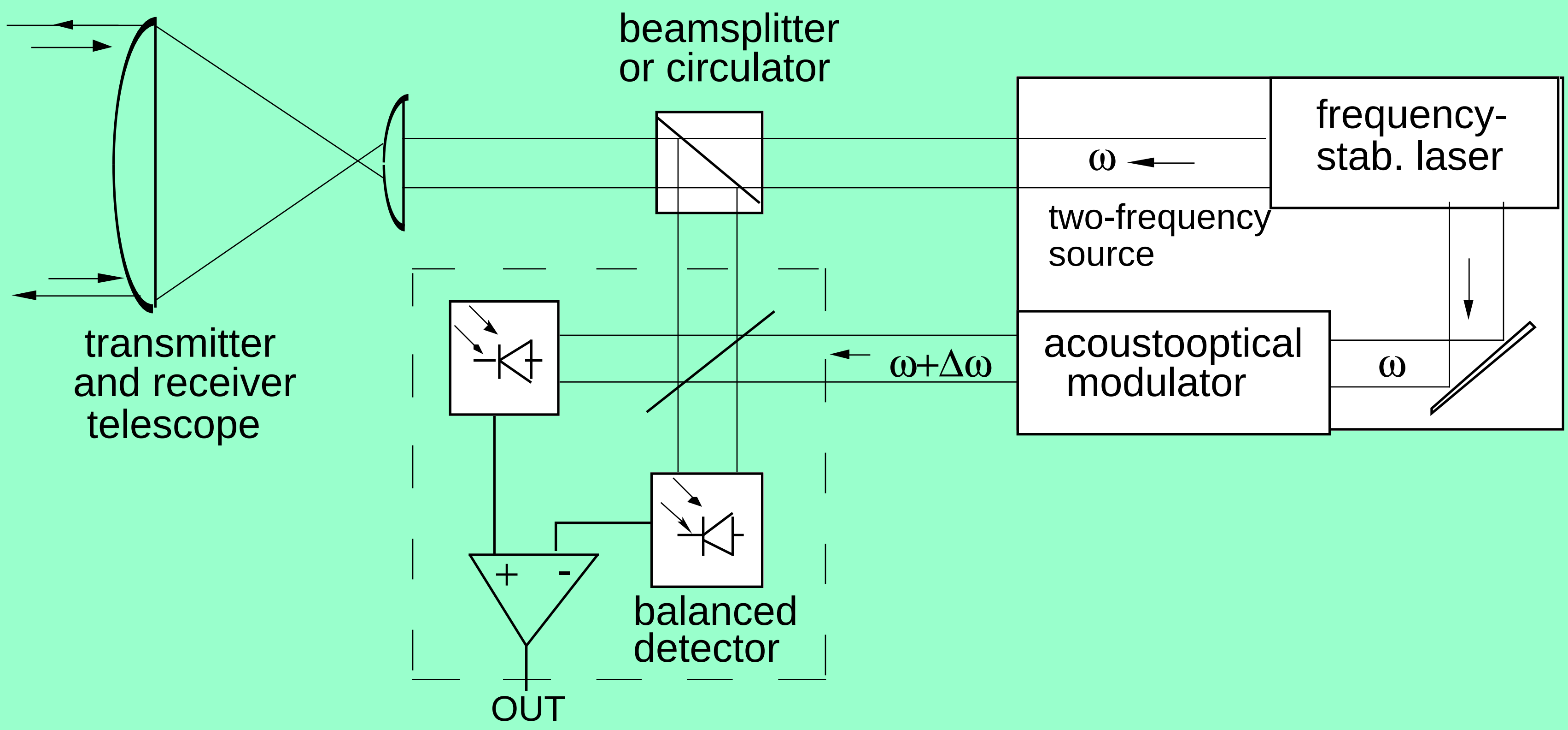
Two-frequency heterodyne receiver



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