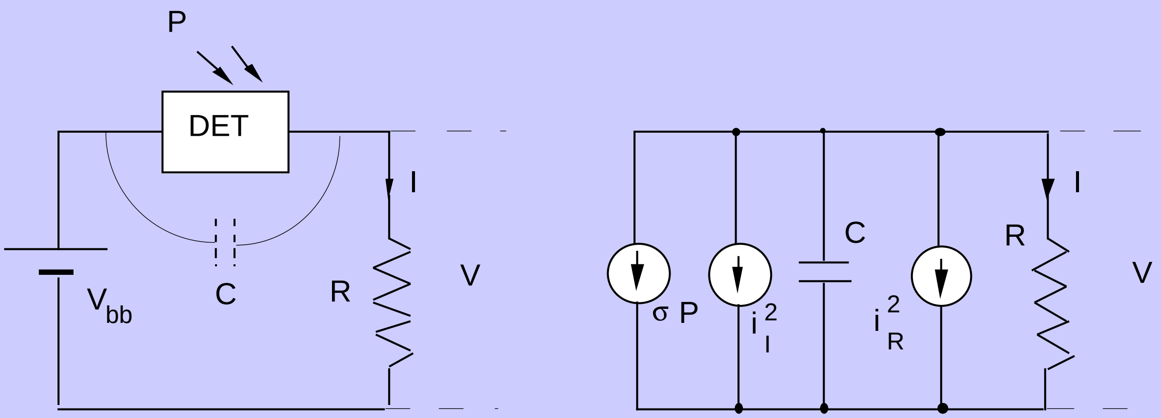
The Bandwidth-Sensitivity Tradeoff

All quantum detectors (phototubes, photodiodes, vidicon targets, etc.) are basically a current generator $I=\sigma P$ shunted by a capacitance C



output voltage across load: $V= \sigma P \cdot R$

3-dB high-frequency cutoff: $B = 1/2\pi\ RC$

noises superposed to signal : Johnson (or thermal) noise of R:

$$i^2_R = 4kTB/R$$

and shot noise (or granular, or quantum) of signal plus dark currents:

$$i^2_I = 2e\,(I_{ph}+I_d)\,B$$

from:"Photodetectors", by S.Donati, Prentice Hall 2000

The Bandwidth-Sensitivity Tradeoff (2)

Total noise is: $i^2_n = 2e\,(I_{ph}+I_d)\,B + 4kTB/R$

- for a high bandwidth we need R as small as possible,
- for low noise (or, the best S/N) we need R very large, such that $4kTB/R$ is negligible compared to $2e(I_{ph}+I_d)B$:

$$R_{min} > 4kTB/\,2e(I_{ph}+I_d)B = (2kT/e)/\,(I_{ph}+I_d)$$

$$= 50\text{ mV} / (I_{ph}+I_d) \quad \text{[at T=300 K]}$$

For example, R_{min}=10 GΩ for I_d=5 pA.

Penality for using a load $R<R_{min}$:

$$i^2_n = 2e(I_{ph}+I_d)B+4kTB/R = [2e(I_{ph}+I_d)B](1+R_{min}/R)$$

i.e., noise is $1+R_{min}/R=K$ larger the desired minimum, or, we need a current $I_{ph}+I_d$ increased by K

cures: circuit solutions or an internal gain

from:"Photodetectors", by S.Donati, Prentice Hall 2000

S/N ratio

In a quantum photodetector with an (eventual) internal gain G, the mean output current is $I = \sigma P G = I_{ph} G$ and the shot noise is:

$$i^2_I = 2e (I_{ph} + I_d) B G^2 F$$

where G^2 is the quadratic gain of noise, and F is the *excess noise factor* summarizing the extra noise introduced by amplification. By adding Johnson noise we get :

$$N = [2e (I_{ph}+I_d) B G^2 F + 4kTB/R]^{1/2}$$

and, being $S= I_{ph} G$, we get for the S/N ratio:

$$S/N = \frac{I_{ph}}{[\,2e(I_{ph}+I_b)\,B\,F + 4kTB/RG^2]^{1/2}}$$

Let us define a critical value I_{ph0}, we call it *threshold of quantum regime*

$$I_{ph0} = I_b + (2kT/e) / R F G^2$$

so that $S/N= I_{ph} / [2eB(I_{ph}+ I_{ph0})]^{1/2}$

1- When the signal is larger than the threshold, $I_{ph} > I_{ph0}$

from:"Photodetectors", by S.Donati, Prentice Hall 2000

S/N ratio (2)

then $S/N = [I_{ph}/2eB]^{1/2}$

this is the *quantum noise limit* or *regime* of detection,
physical boundary for any detection system with Poissonian radiation
The corresponding input power P_{ph0}, being $\sigma = I/P$, is:

$$P_{ph0} = I_{ph0}/\sigma = [I_b + (2kT/e)/RFG^2]/\sigma$$

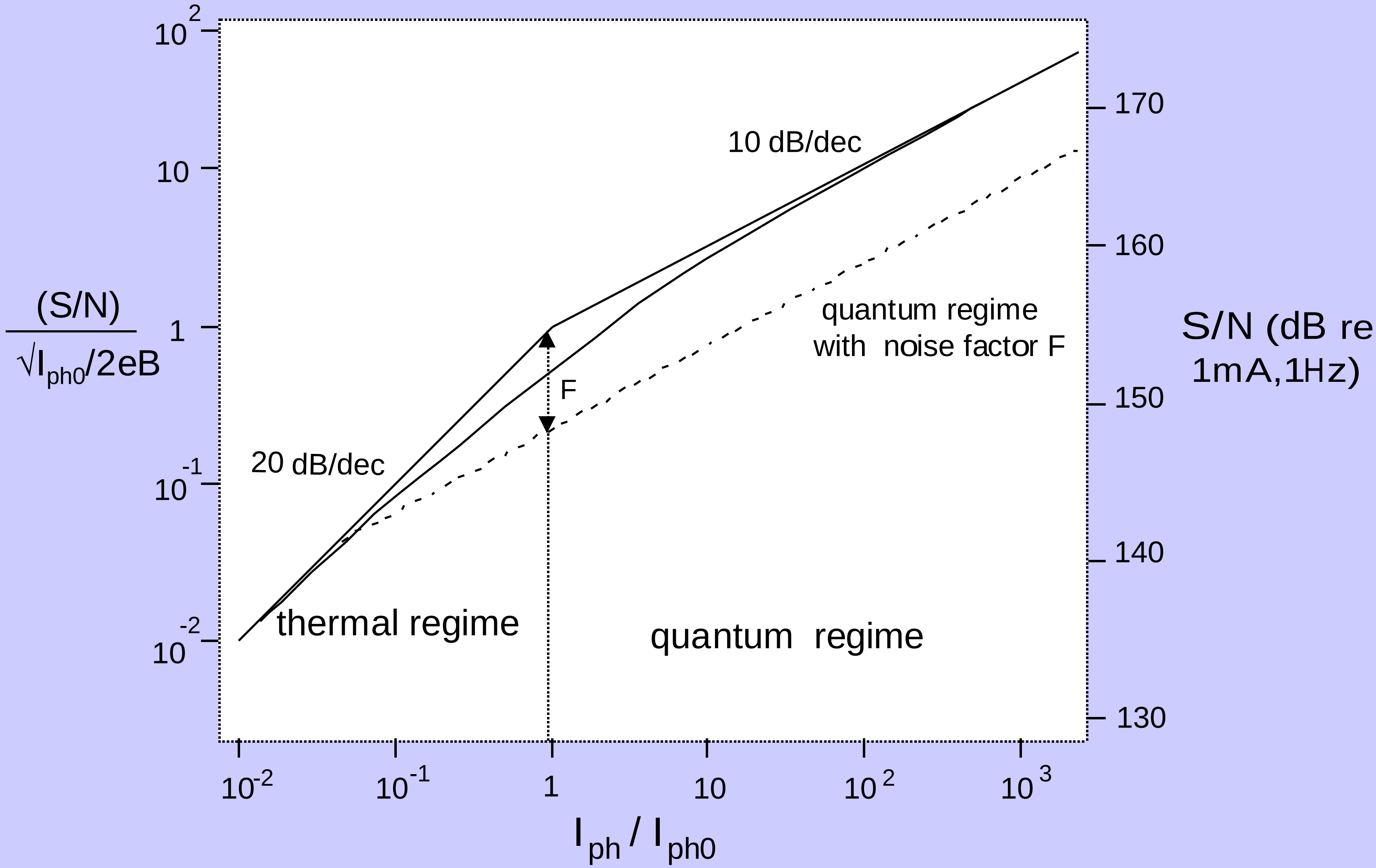
In a digital transmission with bit-period T and a Nyquist bandwidth B=1/2T,
the mean number of detected photons per bit is $N = (I_{ph}/e)T$, and

$$S/N = N^{1/2}$$

Note:
- the quantum regime is *not* associated with the photodetector *itself*:
- it is always reached at a high signal level, $I_{ph} > I_{ph0}$
- good sensitivity means a low value I_{ph0} (or P_{ph0}) of the quantum limit

2- when $I_{ph} < I_{ph0}$ we have $S/N = I_{ph} / [2e I_{ph0} B]^{1/2}$
this is the *thermal regime* of detection

Quantum detectors: S/N vs I_{ph}



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Noise and S/N terminology

- $I_{ph}=I_{ph0}=I_b+(2kT/e)/RFG^2$ is the signal giving the *break point* between thermal and quantum regimes or also, the
- *equivalent dark current* of the photodetector
- $R_{eq} =(2kT/e) / I_{ph0}\, G^2$ is the *equivalent-noise resistance*
- input noise *equivalent power* is:

$p_n = (1/\sigma)\,[2e(I_{ph}+ I_b)\, B\, F +4kTB/RG^2\,]^{1/2}$ [W]

- or, noise equivalent *power spectral density*:

$dp_n/df = (1/\sigma)\,[2e(I_{ph} +I_b)\, F+4\, kT/RG^2]^{1/2}$

- the *noise figure F* ratio of input and output S/N,

$F = (S/N)_i\,/(S/N)_u$, for a quantum photodetector is given by:

$F^2 = 1+ I_{ph0}/I_{ph}$

Figure of Merit of Detectors

To avoid confusion, the term sensitivity is *not* used.
Quantity of response is called *responsivity*:

$$R = Q_{ph} / P$$

where Q_{ph} is the output quantity produced when a power P is detected.

At equal R, the best detector has the smallest output noise q_n or, the least NEP (noise-equivalent input):

$$NEP = q_n / R$$

As a small NEP means a good performance, it is better to define its inverse as a merit figure, the *detectivity* D:

$$D = 1/NEP$$

from:"Photodetectors", by S.Donati, Prentice Hall 2000

Figure of Merit of Detectors (2)

Now, detectivity depends on incidental parameters, like area A and bandwidth of detection B. Dependence of noise is quadratic on AB, so we define (di-star) detectivity as:

$$D^* = \sqrt{AB} / NEP$$

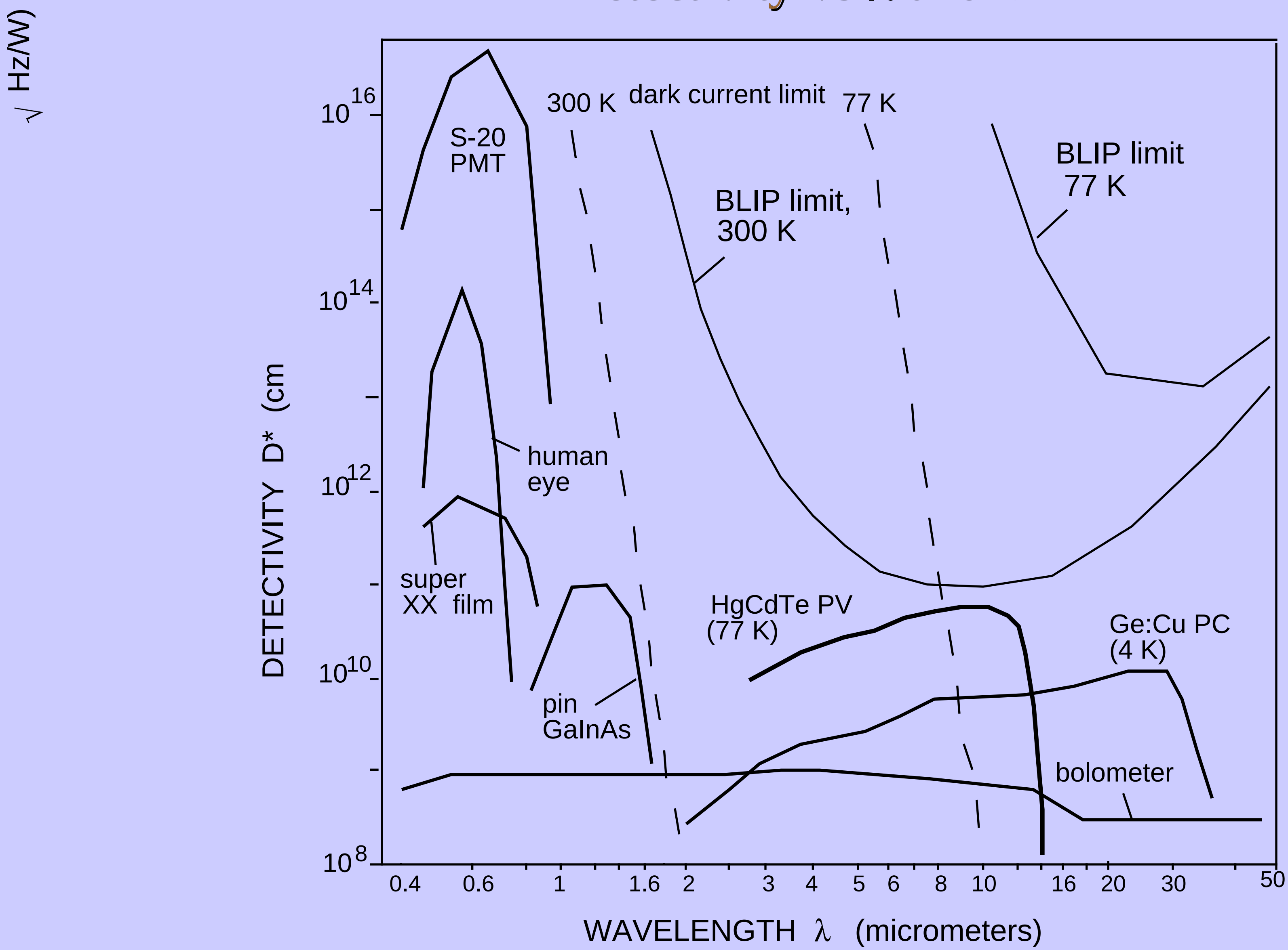
commonly employed measure unit for D* is cm $\sqrt{Hz}W^{-1}$. Typical values range from 10^7 to 10^{14}. NEP is simply $= \sqrt{AB} / D^*$. This D* allows to compare any kind of detector, as shown in next slide (eye, photodetectors, photo film, etc.) .

As a last correction to be considered, *not* really the *area* A, but the *acceptance* AΩ is an invariant. It is $\Omega = \pi NA^2$ (where NA= numerical aperture of detector lens). So D** is introduced:

$$D^{**} = NA \sqrt{AB} / NEP$$

from:"Photodetectors", by S.Donati, Prentice Hall 2000

Detectivity vs λ and BLIP



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BLIP limit of detectors

The minimum noise or NEP we can achieve is determined by shot noise of the background observed at temperature T. Blackbody radiance is:

$$r(\lambda) = 2h\nu^2/\lambda^3(\exp h\nu/kT - 1)$$

and this gives a detected current density:

$$J_{bg} = \sigma\, r(\lambda)\, \Delta\lambda\, \pi NA^2$$

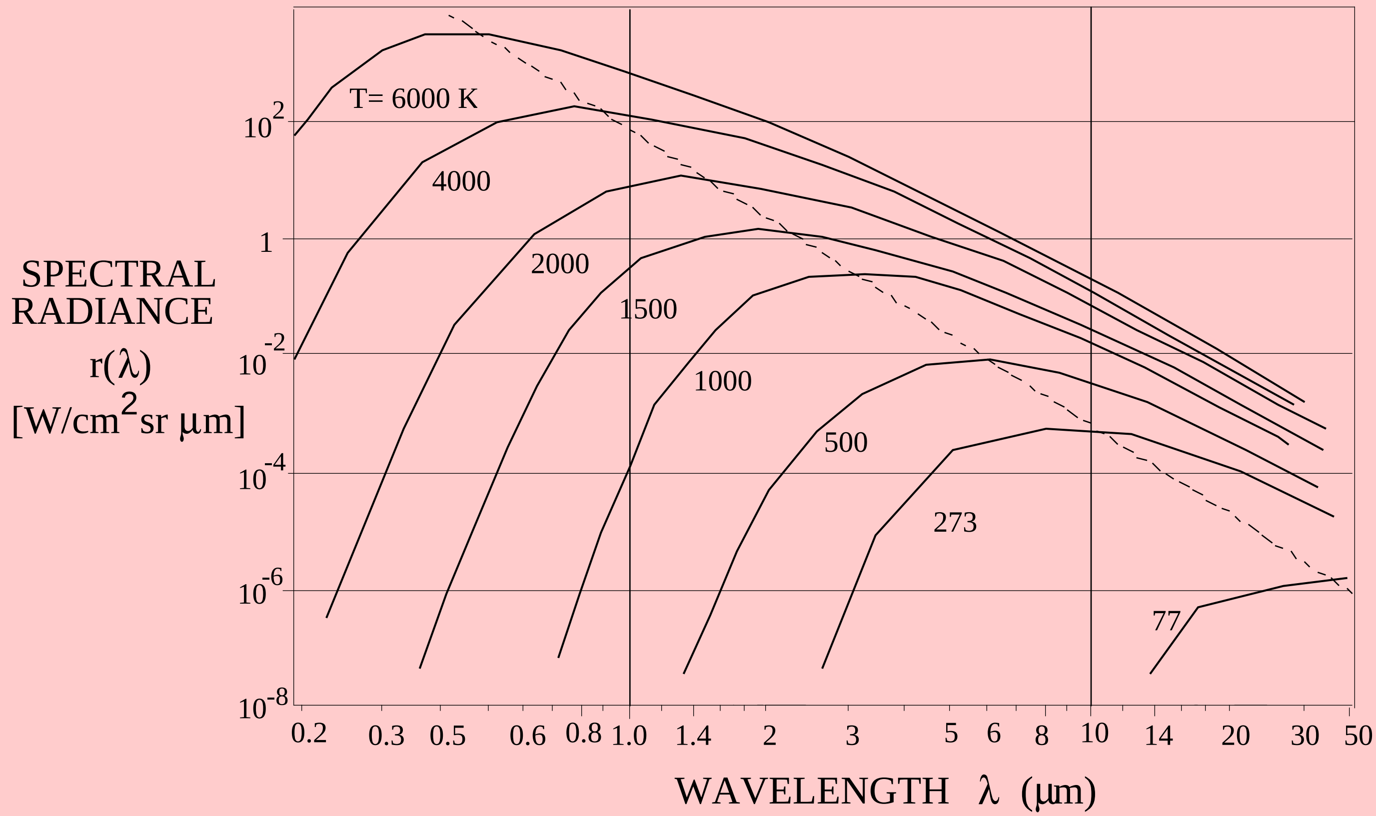
As we have: $NEP = i_{n(bg)} / \sigma = \sqrt{[2eB\, AJ_{n(bg)}]} / \sigma$, the D** obtained is the maximum detectivity D_{BLIP} that can be reached looking at the background, or, *background-limited- photodetector*:

$$D_{BLIP} = \sqrt{[\sigma / 2\pi e\, r(\lambda)\, \Delta\lambda]}$$

(see last slide)

from:"Photodetectors", by S.Donati, Prentice Hall 2000

Spectral Radiance of Blackbody



From Boltzmann equipartition principle: $dP = 1/2\ kT\ df$

$$p_m(\lambda) = [h\nu/(\exp h\nu/kT-1)]d\nu/d\lambda = h\nu^2/\lambda(\exp h\nu/kT-1)$$

$$r(\lambda) = h\nu^2/\lambda^3(\exp h\nu/kT-1), \qquad \lambda_m T = 2898\ \mu m \cdot K$$

from:"Photodetectors", by S.Donati, Prentice Hall 2000

BLIP and dark-current limit

- In MIR/FIR detectors, BLIP is the actual limitation.
- In VIS/NIR (say below 2μm) BLIP limit becomes loose and noise performance is better described in terms of dark-current limit.
  Using $J_d = C \exp -hc/\lambda_t kT$
  where C=const., and λ_t is the threshold wavelength to reveal the dependence of J_d,
  dark-limited-detectivity is found as:

$$D_{dark} = \sigma \, (2eC)^{-1/2} \exp hc/2\lambda_t kT$$

(see dotted lines in slide 9)

from:"Photodetectors", by S.Donati, Prentice Hall 2000

from:”Photodetectors”, by S.Donati, Prentice Hall 2000