ELECTROMAGNETIC MODELING
OF MM-WAVE AND OPTICAL
PERIODIC AND QUASI-PERIODIC STRUCTURES

PhD Thesis of
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For where your treasure is, there also will your heart be.

Luke, 12,34
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The adjective periodic recalls the idea of infinite: an event that recurs at regular interval is periodic in time; the weft of our clothes is periodic in space; a periodic decimal number is never ending (it cannot be represented on a standard calculator display!), etc. But only mathematics, and theoretical sciences in general, can deal with infinite and actual periodic entities. In the real world, rigorous periodicity is an approximation, because everything appears to be limited: the periodic event will eventually be interrupted and the fashion imposes how to cut and style our dresses. This happens also in electromagnetism: a periodic structure will never prove to be a perfect periodic structure. But, at least, the modeling can look at the ideal objects for a while, then the creativity of the designer is in charge of the best arrangement for the operating device. Moreover, a local break of the periodicity could provide very fascinating possibilities: these are the quasi-periodic structures.

From the very beginning, periodicity has been associated to very interesting applications. Isaac Newton was a pioneer also in this field: “The first work done in a one-dimensional lattice was that of Newton in his attempt to derive a formula for the velocity of sound. Newton assumed that sound was propagated in air in the same manner in which an elastic wave would be propagated along a lattice of point masses.” These are the starting words of the first chapter of a milestone in periodic structure literature: Wave Propagation in Periodic Structure, by Léon Brillouin [1]. In the meantime, the periodic electromagnetic community has intensively grown and at present its research covers a wide area of applications and devices, at different frequency ranges, both for antennas and guided components.

This thesis collects the most significant results of my PhD studies on electromagnetic modeling of periodic structures, at the Microwave Laboratory of the Department of Electronics of the University of Pavia. The report is divided in two major parts: the first one, with Chapters 2, 3 and 4, regards the analysis of frequency selective surfaces with a particular implementation of the method of moments technique, called MoM/BI-RME method. The second part, entitled breaking the periodicity, deals with two other kinds of applications, namely photonic crystal slabs (Chapter 5) and aperiodic phased arrays (Chapter 6), where the periodicity is locally interrupted in order to configure the final devices. The world of periodic and quasi-periodic modeling and applications is introduced in Chapter 1.

The electromagnetic group of the Microwave laboratory claims a leading role in both modeling and design of frequency selective surfaces, which basically are filters at microwave and mm-wave range. In fact, the MoM/BI-RME method and its basic block, the BI-RME method itself, have been entirely developed at the laboratory. Moreover, the group has provided successful designs of such periodic structures, mainly for deep-space communication antennas of the European Space Agency. My activity integrated with the previous one, especially in the modeling aspects, with the study of the convergence issue and the inclusion of updated analysis instruments.

The second kind of studies has been done in collaboration with external research groups. In particular, prof. Andreani, from the Department of Physics of the University of Pavia, proposed a
joined activity for the characterization of circuit components in photonic crystal technology. These structures are very promising for optical communications, but questions related to losses and coupling with external sources must be further investigated. Chapter 5 describes the combined use of two methods for accurate electromagnetic modeling of photonic crystals, as well as the idea of a new device layout that could reduce losses and it is based on microwave circuit design.

Finally, Chapter 6 reports the main results of a study on aperiodic arrays for space applications, carried out during a six-month traineeship at the European Space Operation Centre of the European Space Agency (ESA-ESOC), in Darmstadt (Germany). The main purpose of the study has concerned the investigation of the capabilities of aperiodic array layouts to be used in the new ground-based radar for ESA’s space surveillance program. In this starting phase of the project, the activity aimed to investigate the feasible and most important features the future array antenna should provide.
CHAPTER 1

Periodic Structures in Electromagnetism

Periodic structures are constituted by replicas of a unit cells, recurring at regular intervals. When a periodic structure is large enough, its inner part is totally surrounded by a perfect periodic environment. Only the few elements close to the edges can experience the real finiteness, therefore the assumption of infinite extent is predominantly valid. Under this approximation, every periodic structure is defined by its characteristic unit cell and its translational vectors, which delineate the positions of the replicas in space.

The above-mentioned description holds for electromagnetic devices also. The hypothesis of uniform and infinite periodicity leads to a significant reduction in the computational complexity, because the characterization of the whole structure can be reduced to the analysis of the unit cell. In fact, when the structure is lossless and its excitation is uniform, the computed electromagnetic field inside the single cell is repeated in all the other cells, with a proper phase shift, determined by the translational vectors.

Electromagnetic periodic structures find several applications in microwave, mm-wave and sub-mm-wave ranges, until the optical domain. For instance, they are operated as filters, antennas and polarizers, or they can modify the electric and magnetic properties of materials, in order to create discrete frequency bands for wave propagation. These structures are usually made of a combination of metal and dielectrics, depending on the frequencies of operation, which determine the expected material performance. Apart from the frequency range, a transversal and intuitive classification can be based on their geometric periodicity, that could be one-dimensional (such as in periodically loaded transmission lines and polarizing grids), two-dimensional (the widest class, with frequency selective surfaces, antenna and polarizing grids, standard metamaterials) or three-dimensional (mainly metamaterials, for few applications, due to the related complexity in analysis and fabrication), as Fig. 1.1 shows.

![Fig. 1.1](image)

Fig. 1.1 – Example of (a) one-dimensional, (b) two-dimensional, and (c) three-dimensional periodic structures [7].

In the subsequent sections, the structures that will be discussed throughout this work are introduced, and the basics of analysis theory and techniques are outlined. In particular, array
antennas and frequency selective surfaces, which represent classical electromagnetism by now, are presented, along with some insight into the open world of metematerials and photonic crystals. Subsequently, the fundamentals of both Floquet’s theorem and Brillouin-zones definition are described, to show details of numerical modeling of electromagnetic periodic structures [1].

1.1 **EXAMPLES AND APPLICATIONS**

Rigorously speaking, every periodic structure can be addressed as array. Nevertheless, in electromagnetism the word “array” is dedicated to array antennas, both linear and planar. The potential improvement in the antenna performances, due to the proper combination of more than one single antenna, has been widely investigated and documented [2], [3]. Most relevant features will be proposed in the first part.

Among the planar periodic structures, Frequency Selective Surfaces (FSS), described in the second section, recover a leading role. Since the earliest studies for military applications after the Second World War, they became the classical microwave and mm-wave filters [4], [5]. After the pioneering works, nowadays the research is mainly focused in multilayer layouts, extremely high-performance components for space applications, and miniaturization. This last concept states the aim of filling the gap between microwaves and optics, towards new monolithic technology for sub-mm-wave bands [6].

Periodically loaded waveguides and transmission lines are one-dimensional periodic structures. They are loaded with reactive elements at identical intervals: when these intervals correspond to multiples of half a wavelength, the loadings strongly attenuate the electromagnetic wave. Therefore, a discrete sequence of pass-bands and stop-bands is created [7]. This simple concept has been further developed in the last years, with the study of Electromagnetic BandGap (EBG) devices in the microwave range and Photonic Crystals (PhC) at optical frequencies [8], [9]. All these structures are usually collected under the name of metamaterials, even though the nomenclature is not perfectly defined yet [10]. Given that, the last section, under the common name of metamaterials, will highlight their peculiar properties.

1.1.1 **Array Antennas**

In many antenna applications, for example in selective or long distance communication, a very directive radiation pattern is required. This pattern can only be achieved by using antennas with large electrical size. Instead of increasing the dimension of a single element antenna, it is possible to form an assembly of radiating elements, with proper electrical and geometrical configuration: an *array*, namely. The simplest array is formed by placing elements along a line and is called *linear*; with the combination of two linear arrays, so that the elements are arranged on a two-dimensional grid, a *planar* array is derived (Fig. 1.2).

In an array antenna, it is then necessary that the fields radiated from all the elements interfere constructively in the nominal directions and cancel each other in the remaining space. Apart from the choice of the single elements, arrays offer different degrees of freedom for the design process, basically related to both position and spacing of the elements (parameters of the geometrical layout), and to the amplitude and phase of the excitation network.
Among all the interesting configurations, **phased array** antennas can produce a directive beam that can be scanned (or steered) electronically. They consist of multiple stationary antenna elements, which are fed coherently and use variable phase or time-delay control at each terminal to scan a beam to give angles in space [12]. In particular, the behavior of a phased array in a communication systems, often a radar system, is more complex than that of a passive, mechanically positioned antenna, because the performance characteristics vary with scan angle. Moreover, an analytical formulation of phased array radiation, including mutual coupling between elements, is very difficult. When considering aperiodic layout, with the incremental degree of freedom of the element position, also the properties of periodicity must be discarded, and the achievement of the required radiation pattern becomes a challenging task. Fig. 1.3 shows the transmitting and receiving antennas of GRAVES, a first example of European surveillance radar, in bistatic configuration. The system is based on four transmitting quasi-uniform arrays and on an aperiodic array of dipoles as receiver.

**Fig. 1.2** – Example of planar antenna array, in SIW (Substrate Integrated Waveguide) technology, with 13x32 slot elements [11].

**Fig. 1.3** – GRAVES radar system (France): (a) one of the four transmitting array panels; (b) receiving array antenna.
1.1.2 Frequency Selective Surfaces

Frequency selective surfaces are characterized in terms of transmitted and reflected electromagnetic waves due to an incident plane wave, which is impinging from an arbitrary direction \((\theta, \phi)\). As the name states, they represent distributed electromagnetic filters: at some frequencies the incident wave is totally transmitted, at other frequencies it is totally reflected in the specular direction. FSS are normally divided between capacitive and inductive structures, which exhibit complementary behavior. Capacitive FSS are defined by periodic replicas of conductive patches on a planar dielectric substrate and basically behave as stop-band filter; inductive FSS consist of apertures patterned on a thick metallic screen and act as pass-band filters. Fig. 1.4 reports an example of inductive FSS, which operates as dichroic mirror in a deep-space antenna of the European Space Agency.

![Fig. 1.4 – Example of inductive frequency selective surfaces, with cross-shaped apertures.](image)

The most important characteristic in the study and the design of FSS is the resonance frequency, which is determined by the dimension of the single elements. In particular, at low frequencies the dimension of the patch or of the aperture is much smaller than the wavelength. This means that in capacitive FSS the metallization is practically transparent and the incoming wave is totally transmitted, apart from some reflection due to the dielectric substrates. Vice versa, the metallic screen of inductive FSS results almost homogeneous, so that it reflects like a perfect mirror. When the frequency increases and reaches the first resonance, the length of the elements becomes comparable to half a wavelength. In the capacitive case, the patches resonate and scatter (i.e. reflect) all the energy. Due to the periodic repetition, the phase delay is constant and the reflection is coherent, by analogy with the antenna array principle. Conversely, apertures become transparent and the inductive FSS totally transmits the impinging wave. Finally, when the frequency further increases, the behavior exhibited before the resonance cannot be exactly reproduced, because of grating lobes. Grating lobes occur at angles or frequencies with higher-order constructive interference. As a result, they introduce wave propagation in different directions with respect to the nominal ones.

Different parameters contribute to the definition of the FSS frequency response. First of all, the single element, with its size and shape, then the periodicity and the direction of the incident wave,
along with the properties of the materials: dielectric permittivity, metal conductivity, thickness of the substrate and of the metallization. In most cases, canonical shapes are preferred, since they are easier to model and their fabrication presents a high degree of reliability. However, also innovative shapes have been studied, especially to improve the bandwidth, the cross-polarization rejection, the stability of the frequency response.

The presence of dielectrics in an FSS is referred as dielectric loading. In addition to structural support, they can stabilize the drift of the frequency response when steering the incident wave and they permit multiband FSS design. Dielectric substrate modifies the resonance frequency: when the electric permittivity increases, also the element size with respect to the wavelength increases, thus the resonance occurs at a lower frequency. Moreover, care must be taken in the choice of substrate thickness: it is important to ensure that the substrate thickness it is small enough to prevent the generation of surface waves (i.e. Wood’s anomaly [5]), especially at large incident angle.

Concerning the manufacturing, capacitive FSS are based on photolithographic technology. Therefore, with a substantially cheap process, accuracy, lightweight, low volume, and flexibility on the element shape are supplied. Inductive FSS require more expensive fabrication, since they are defined on thick metal blocks. Among the others, milling technique or drilling are employed in microwave and mm-wave range, whereas galvanizing growth and laser cutting are preferred at higher frequencies [13].

A classical commercial application of inductive FSS consists in the screen door of microwave ovens. In this case, the frequency selection clearly appears: the perforated metal screen totally reflect the microwave energy at 2.45 GHz, while it allows the light to pass through, so that we can see the food inside. Since in inductive FSS the presence of the thick metal screen offers good power handling capabilities in conjunction with low losses and enhanced band separation, they are very well suited for space applications. For instance, they are operated as dichroic mirrors in the beam-waveguide system of the feeding network of deep-space antennas (the first dichroic mirror by NASA and a state-of-the-art example by ESA are presented [14] and [15] respectively). Another typical use for FSS is as subreflector in dual reflector system on spacecrafts [16] and as band-pass radomes [4]. Capacitive FSS results better appropriate for these kinds of conformal applications, because they can exploit lightweight and easier manufacturing. At higher frequencies, FSS find application in quasi-optical systems: diplexers and dichroic mirrors in laser cavities and in Fabry-Perot interferometers.

### 1.1.3 Metamaterials and Photonic Crystals

In the introductory chapter of his basic book on photonic crystals [9], Johannopoulos states that “many of the true breakthroughs in our technology have resulted from a deeper understanding of the properties of materials”. He mentions three kinds of progress. First of all, the exploitation of mechanical properties to obtain artificial materials by means of metallurgy, ceramic, and plastic manufacturing. Then the spreading of material control, which in the previous century included electrical characteristics. For example, this led to the development of transistor and microelectronic circuits, based on the properties of semiconductors. Finally, in the last years, material science moved towards optical properties of material, with the aim of either inhibition or localization of light propagation. Photonic crystals (PhC) belong to this class; electromagnetic bandgap (EBG) structures are their counterpart at microwave frequencies, while metamaterials presents effective homogeneous arrangement, which exhibit unusual properties not readily found in nature. All these artificially engineered structures are based on periodic lattices and have two common objectives: the
control of the electromagnetic properties, and the consequent suppression of wave propagation in certain directions and at certain frequency bands.

In metamaterials, the average lattice constant of the periodic arrangement is much smaller than the nominal wavelength [10]. This means that when an electromagnetic wave is impinging on them, adjacent cells present negligible phase difference. Therefore, interference effects cannot take place, and the wave simply travels through the material in a straight line, only probing average (i.e. effective) constitutive parameters. The phenomenon is called long wavelength regime. As Veselago suggested first in 1960s [17], metamaterials can exhibit simultaneously negative real part of electric permittivity and magnetic permittivity. Therefore, the index of refraction results negative too and backward waves are generated, where phase and group velocities are anti-parallel. In this case, electric field, magnetic field, and the propagation vector form a left-handed system, instead of the right-handed system presented by natural materials. Several applications have been proposed at microwave and higher frequencies, such as subwavelength resonators [18], perfect lens [19], and optical cloaking [20]. Moreover, the possibility to further reduce the electrical size of the unit cells can drive metamaterials to micro and nanotechnologies [21].
Conversely, when considering the opposite trend, that is the increase of lattice size up to the order of the wavelength, the periodic structures became macroscopic. Instead of metamaterials, we deal with EBGs and PhC. In these structures, the basic idea is to reproduce the propagation of electrons in natural crystals, where the periodic arrangement of atoms or molecules determines gaps in the bands of energy, where the propagation of electrons is forbidden. In EBGs and PhC, electrons are replaced by electromagnetic waves: an analogous but artificial macroscopic lattice can produce the same behavior, and generate band-gaps (or photonic band-gaps in optics) of propagation, localized modes, and surface states. Most of the artificial lattices are characterized by a period equal to a multiple of half a guided wavelength: the Bragg regime [10]. In this case, for some specific angles of incidence, the waves scattered and reflected by adjacent periodic cells interfere constructively, in a similar way to Bragg diffraction in X-ray optics. Different configurations based on Bragg’s regime lattice have been proposed in the last years. Metallo-dielectric solutions are usually applied at microwave and millimeter ranges. At optical frequencies, the waves are quickly dissipated in metallic components, due to the properties of materials, therefore PhC are all-dielectric structures. Only three-dimensional lattices, periodic along the three axes, can provide a complete band-gap, where no propagating states are allowed in any direction. However, most of the application regards two-dimensional structures, which can localize modes in the plane, because they are easier to fabricate and can be integrated in monolithic circuits (i.e. Monolithic Microwave Integrated Circuits, MMIC, and integrated photonics). For instance, metamaterials and EBGs can improve the design of printed antennas. In fact, standard patches printed on dielectric substrates generally suffer from narrow beamwidth, low gain, and limited radiation efficiency. An EBG structure can be inserted around the antenna (Fig. 1.6), in order to suppress the propagation of surface waves, and then enhance both the efficiency and the radiation pattern [22]-[24]. Alternatively, an Artificial Magnetic Conductor (AMC) can be placed below the antenna, before the ground plane [25], [26]. AMC, or high impedance surface, are planar periodic structures which are able to reflect an incident plane wave without phase reversal. In this way, the thickness of the substrate can be greatly reduced and still direct and reflected wave can add constructively. Consequently, radiation efficiency increases and surface waves almost disappear. At higher frequencies, PhC are used for the realization of transmission lines and components in integrated optics circuits. These systems, also named PhC slabs, are constituted by two-dimensional photonic crystals embedded in planar waveguides [9], [27]. In the vertical direction, the light is guided by the step of refractive index between air and dielectric; the in-plane confinement is due to the presence of a defect in the planar periodic arrangement. They are usually defined in Silicon or Ga-As based substrates. Apart from optical waveguides, nanocavities [28], couplers [29], all-optical switches [30] and flip-flops (Fig. 1.7, [31]) are examples of PhC slab components. The employ of fully three-dimensional periodic devices is less common, because of the complexity in manufacturing volumetric lattice, with controlled defects, at optical wavelength.

1.2 OVERVIEW OF ANALYSIS THEORY AND TECHNIQUES

Numerical techniques in the analysis of periodic structures aim to the calculation of transmitted and reflected waves, when a uniform plane wave is impinging on them, and to the determination of the supported mode spectrum. Floquet’s theorem and Brillouin zones are two fundamental instruments. The theorem, presented in the first section, permits to determine the solution of wave equation in the case of periodic boundary conditions, while the study of Brillouin zones, associated to both the unit cell itself and the geometric arrangement, leads to the definition of the band diagram of the supported modes. In the second section, direct and reciprocal lattice, the drawing of the zones, and the concept of redundancy are discussed. During the studies and the design steps, these theories are
implemented with numerical tools which usually are able to fully characterize the whole structures by starting from the unit cell. The last section provides an overview of the most common techniques.

![Image](image1.png)

**Fig. 1.7** – Example of photonic crystal slab: all-optical flip-flop circuit, based on a triangular lattice; (a) layout of the device, where C1 and C2 are resonators, while P1, P2, P3, and P4 are input and output waveguides; (b) field distribution during flip-flop operation [31].

### 1.2.1 Floquet’s Theorem

At the end of the XIX century, the French mathematician Gaston Floquet published the solution of a differential equation with periodic coefficients [32]. This study perfectly applies to the analysis of one-dimensional periodic structures and is referred as Floquet’s Theorem. In 1920s, the Swiss physicist Felix Bloch, who was working in the field of quantum mechanics, generalized the theory for two- and three-dimensional cases [33]. Hence, in solid-state physics the name of the theorem usually changes into Bloch’s Theorem.

In a one-dimensional structure, periodic along the \( z \) direction (Fig. 1.8), the unit cell is defined by two parallel plates, perpendicular to \( z \), separated by a distance \( d \). Floquet’s theorem states that, under the hypothesis of lossless structure, the field at the parallel planes has the same amplitude, and differs only for a phase shift, due to the propagation factor \( e^{j\gamma d} \):

\[
\begin{align*}
\tilde{E}(x, y, 0) &= e^{-j\gamma d} \tilde{E}(x, y, d) \\
\tilde{H}(x, y, 0) &= e^{-j\gamma d} \tilde{H}(x, y, d)
\end{align*}
\]  

(1.1)

where \( \gamma \) is the complex propagation constant and is equal to \( j\beta \). Since the choice of the terminal planes of the unit cell is arbitrary, the relation holds for any point inside the unit cell and the corresponding point of another unit cell, at a distance \( nd \) (\( n \) is an integer value):

\[
\begin{align*}
\tilde{E}(x, y, z + nd) &= e^{-j\gamma nd} \tilde{E}(x, y, z) \\
\tilde{H}(x, y, z + nd) &= e^{-j\gamma nd} \tilde{H}(x, y, z)
\end{align*}
\]  

(1.2)
Consequently, both the electric and magnetic fields can be written in terms of two periodic functions, $\vec{E}_p$ and $\vec{H}_p$:

$$\vec{E}(x, y, z) = e^{-j\beta z} \vec{E}_p(x, y, z)$$

$$\vec{H}(x, y, z) = e^{-jnd} \vec{H}_p(x, y, z)$$

(1.3)

with

$$\vec{E}_p(x, y, z + nd) = \vec{E}_p(x, y, z)$$

$$\vec{H}_p(x, y, z + nd) = \vec{H}_p(x, y, z).$$

(1.4)

The two periodic functions can be expanded into Fourier series. After some mathematical elaboration (for an exhaustive theory, see for instance [34]), the electromagnetic field in a linear periodic structure results:

$$\vec{E}(x, y, z) = \sum_{n=-\infty}^{\infty} \vec{E}_{pn}(x, y)e^{-j\beta_n}$$

$$\vec{H}(x, y, z) = \sum_{n=-\infty}^{\infty} \vec{H}_{pn}(x, y)e^{-j\beta_n}$$

(1.5)

where $\vec{E}_{pn}(x, y)$ and $\vec{H}_{pn}(x, y)$ are vector functions that represent the coefficients of the Fourier series and depend only on the transverse coordinates, whereas the exponential term accounts for the propagation and the periodicity along $z$, with phase constant:

$$\beta_n = \beta + \frac{2n\pi}{d}.$$  

(1.6)

Expressions (1.5) show that in a linear periodic structure the field can be written as a summation of a fundamental mode, represented by $\beta$, and infinite terms $\beta_n$, called spatial harmonics. Since both the propagation constant and the phase velocity depend on $n$ and can result either negative or positive, some of the harmonics can present phase velocity and group velocity with opposite sign (backward propagating waves).
The formulation of the problem in two-dimensional lattices is slightly different. In fact, as in the case of metallic rectangular waveguide, Helmholtz equations are handled, with periodic boundary condition instead of electric wall condition. The equations and the resultant scalar potentials are defined on the unit cell. For example, in the case of a rectangular grid on the x-y plane (Fig. 1.9a), the following problem must be solved:

\[
\begin{align*}
\nabla^2 \psi(x, y, z) + \kappa^2 \psi(x, y, z) &= 0 \\
\psi(x + a, y + b, z) &= e^{-j(\psi_x, + \psi_y)} \psi(x, y, z)
\end{align*}
\]  

(1.7)

where \(a\) and \(b\) represent the amplitude of the translational vectors along \(x\) and \(y\) directions; \(\psi_x\) and \(\psi_y\) are the inter-element phase shift, again along \(x\) and \(y\) directions, which depend on the angle of incidence \((\theta, \phi)\) and on the frequency; finally, \(\xi\) and \(\kappa\) indicate the unknowns of the problem, that are the scalar potentials and the eigenvalues respectively. The potentials present separate behavior in the transverse \(x\)-\(y\) plane and in direction \(z\), expressed by:

\[
\psi(x, y, z) = \xi(x, y) e^{-j\kappa z}
\]

(1.8)

hence, the equation can be recast in:

\[
\nabla^2 \xi(x, y) + (\kappa^2 - \beta^2) \xi(x, y) = 0
\]

(1.9)

with the relative periodic boundary conditions. The problem in the transverse Laplace operator is solved in analogy to the classical waveguide procedure, by the separation of variables. The resultant eigenvectors and eigenvalues are:

\[
\begin{align*}
\xi_{mn}(x, y) &= -\frac{j}{\sqrt{ab}} e^{-j(\kappa_{xmn} x + \kappa_{ymn} y)} \\
\kappa_{mn} &= \sqrt{\kappa_{xmn}^2 + \kappa_{ymn}^2}
\end{align*}
\]

(1.10)
\[
\begin{align*}
\kappa_{x_{mn}} &= k \sin \theta \cos \varphi + \frac{2m\pi}{a} \\
\kappa_{y_{mn}} &= k \sin \theta \sin \varphi + \frac{2n\pi}{b}
\end{align*}
\]  
(1.11)

and \( k = \omega \sqrt{\varepsilon_0 \mu_0} \).

The Helmholtz potentials permit to derive the orthonormal set of modal vectors, whom superposition represent the field inside the unit cell. Again, like in metallic waveguides, the set is constituted by TM (magnetic modal field transverse to \( z \)) and TE (electric modal field transverse to \( z \)) modes, given by:

\[
\begin{align*}
\text{TM} & \quad \begin{cases} 
\tilde{E}_{mn}^* = -\nabla \times \frac{\psi_{mn}}{\kappa_{mn}} \\
\tilde{H}_{mn}^* = -\hat{u}_z \times \frac{\psi_{mn}}{\kappa_{mn}}
\end{cases}, \\
\text{TE} & \quad \begin{cases} 
\tilde{E}_{mn} = -\nabla \times \frac{\psi_{mn}^*}{\kappa_{mn}} \\
\tilde{H}_{mn} = -\nabla \frac{\psi_{mn}^*}{\kappa_{mn}} \hat{u}_z
\end{cases}.
\end{align*}
\]  
(1.12)

These modes, termed Floquet modes in microwaves and Bloch states in solid-state physics, satisfy the boundary conditions and propagate along \( z \) with the following propagation constant:

\[
\gamma_{mn} = \begin{cases} 
\sqrt{\kappa_{mn}^2 - k^2} & \rightarrow k < \kappa_{mn} \\
j \sqrt{k^2 - \kappa_{mn}^2} & \rightarrow k > \kappa_{mn}
\end{cases}
\]  
(1.14)

When the plane wave is impinging from the normal direction, i.e. \( \theta = 0^\circ \), the orthonormal set is not complete and two more modes are needed. They are the solutions of the Laplace equation with periodic boundaries and are called TEM modes, because both their magnetic and electric fields are transverse to the direction of propagation:

\[
\begin{align*}
\text{TEM}_{01} &= \begin{cases} 
\tilde{E}_{01} = \frac{\hat{u}_x}{\sqrt{ab}} \\
\tilde{H}_{01} = \frac{\hat{u}_y}{\sqrt{ab}}
\end{cases}, \\
\text{TEM}_{10} &= \begin{cases} 
\tilde{E}_{10} = \frac{\hat{u}_y}{\sqrt{ab}} \\
\tilde{H}_{10} = -\frac{\hat{u}_x}{\sqrt{ab}}
\end{cases}.
\end{align*}
\]  
(1.15)

Finally, if the translational vectors \( \tilde{a} \) and \( \tilde{b} \) are not perpendicular, the \( \gamma \)-term of the cutoff wavenumber (1.11) and the normalization factor of the potentials in (1.10), (1.15) and (1.16) must be modified to account for the skew angle \( \alpha \):
\[
\kappa_{\text{max}} = k \sin \theta \sin \varphi + \frac{2n\pi}{b} - \frac{2m\pi}{a \tan \alpha},
\]

(1.17)

\[
\frac{1}{\sqrt{ab}} \rightarrow \frac{1}{\sqrt{ab} \sin \alpha}
\]

(1.18)

The electromagnetic field in an ideally infinite periodic structure will be a superposition of the computed Floquet modes.

### 1.2.2 Brillouin Zones

The study of the lattice properties provides all the information needed in the characterization of electromagnetic wave propagation in periodic structures. In particular, it permits to identify the redundancy due to the periodicity, and therefore to focus the analysis as much as possible.

For example, in the one-dimensional case, the solution of the wave equation yields a series of replicas of the fundamental mode, as indicated by the formula \( \beta_n = \beta + 2n\pi/d \). In fact, since the propagation phase constant \( \beta \) is periodic, it is totally described by the behavior inside one period. This is clear when studying the \( \omega-\beta \) diagram (Fig. 1.10), which is the band diagram of the structure: outside the interval \(-\pi < \beta d < \pi\), only exact replicas of the mode bands appear. The period interval \(-\pi < \beta d < \pi\) is usually named fundamental interval or first Brillouin zone [1]. The diagram presents an alternation of pass- and stop-bands, where the propagation along the structure is allowed or inhibited, respectively.

![Fig. 1.10 – Example of \( \omega-\beta \) (or dispersion) diagram of a one-dimensional periodic structure. In the stop-bands (grey-shadowed), there are no values of the propagation phase constant that corresponds to frequency values.](image)

As shown in Fig. 1.10, an additional redundancy can be exploited: the band diagram can be restricted to values of the phase constant inside the irreducible Brillouin zone. This further step considers the symmetries of the structure: let us consider the two-dimensional case to describe the details. For instance, a generic triangular lattice is reported in Fig. 1.11a. As already mentioned, the geometry of a periodic structure can be defined by means of two independent translational vectors, say \( \vec{a}_1 \) and \( \vec{a}_2 \), and the angle they form, \( \alpha \). Given that, every point of the geometry is then infinitely repeated at a particular distance, along the two vectors. Hence, a lattice of points (usually called Bravais lattice) is identified:

\[
\vec{r} = l_1 \vec{a}_1 + l_2 \vec{a}_2
\]

(1.19)

where \( l_1 \) and \( l_2 \) are integer number. Vectors \( \vec{a}_1 \) and \( \vec{a}_2 \) also permit to delineate the unit – or primitive – cell of the periodic structure (in Fig. 1.11a the hexagonal example). It should be noted that the
definition of both the translational vectors and the unit cell is not unique. The most common choice considers the Weigner-Seitz unit cell, defined as the region of space closer to a particular point rather than to any other point in the Bravais lattice [7]. The relative \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) vectors characterize the so-called direct lattice.

Each direct lattice is associated to a reciprocal lattice (Fig. 1.11b), which is defined in the Fourier space, i.e. \( k_x-k_y \) plane in two-dimensional cases. The reciprocal lattice is determined by the two reciprocal vectors \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) which are related to \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) by the following condition:

\[
\mathbf{a}_i \cdot \mathbf{b}_j = 2\pi \delta_{ij}
\]

(1.20)

where \( \delta_{ij} \) is the Kronecker symbol, and \( i,j=1,2 \). This relationship states that \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) are perpendicular to \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) respectively, while their length is such that the scalar product between the corresponding vectors in the two spaces is equal to \( 2\pi \). In the reciprocal lattice, the unit cell identified with the Weigner-Seitz definition is denominated first Brillouin zone. Once \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) are known, first and higher Brillouin zones can be derived on the reciprocal lattice following a graphical method (see [7] and [9] for detailed procedures). For example, a triangular lattice presents a hexagonal unit cell in both spatial and Fourier domain (Fig. 1.11).

As observed in the \( \omega-\beta \) diagram of Fig. 1.10 in one-dimensional case, the considered part of the structure can be further reduced. In particular, by exploiting the symmetries in the reciprocal lattice, the smallest and most relevant region of the \( k_x-k_y \) plane which contains no redundancy is identified. It is termed irreducible Brillouin zone and in two-dimensional-periodicity case it is delineate by three points. Referring to the hexagonal lattice in Fig. 1.11b, the points are:

\[
\begin{align*}
\Gamma : & \quad k_x = k_y = 0 \\
M : & \quad k_x = 0; k_y = \frac{2\pi}{\sqrt{3}a_1} \\
K : & \quad k_x = \frac{2\pi}{3a_1}; k_y = \frac{2\pi}{\sqrt{3}a_1}
\end{align*}
\]

(1.21)
In this case, the irreducible Brillouin zone corresponds to one-eighteenth of the first Brillouin zone. The propagation constants that lay inside the irreducible zone hold all the information related to pass- and stop-bands of the periodic structure. In practice, since the band edges usually occur at the boundary of the irreducible Brillouin zone, it is convenient to study only the propagation constants of the three segments $\Gamma M$, $MK$, and $K\Gamma$. This last concept will be outlined in Chapter 5, where band diagrams of planar photonic crystals based on triangular (or hexagonal) lattices are computed and described.

1.2.3 Numerical Techniques

Analysis of periodic structure is typically performed under the hypothesis of perfect periodicity. Therefore, it is reduced to the investigation of a single unit cell, thanks to Floquet’s theorem, or to the study of the irreducible Brillouin zone, if the delineation of the band diagram is addressed. This approximation permits a significant reduction in the computing time, whatever the employed numerical method. Subsequently, and only when the finite extension cannot be neglected, it is advisable to relax the hypothesis of periodicity and account for actual edge effects.

Several techniques have been proposed for the analysis and the characterization of periodic structures. They can be classified in two groups: dedicated and general purpose methods. Dedicated methods are expressly developed to work with a selected class of problems and devices, for example planar photonic crystals, or one-dimensional periodic transmission lines. Vice versa, general-purpose tools are founded on basic equations and rules (Maxwell’s and wave equations, for example), and permit to model most of electromagnetic components. The Integral Equation Method (IEM [35], sometimes called Boundary Integral method) in conjunction with the Method of Moments (MoM [36]), the Finite Element Method (FEM [37]), and the Finite-Difference Time-Domain method (FDTD [35]) belong to the second group. Of course all these methods can be modified and joined together to improve their characteristics, especially in the solution of limited classes of problems. Combinations of methods are named hybrid approaches.

Among all the proposed techniques for the analysis of periodic structure, the integral equation method proved to be very efficient. It is based on either an electric or magnetic integral equation, to be solved by the MoM, which transforms the analytical problem into a matrix one. In the formulation of the IEM, the unknown is defined at the interfaces of the domain of interest. This means that, for example, in a generic three-dimensional problem the unknown will be defined on a surface, the one which represents the discontinuity between different media: instead of volumetric functions, surface functions and operators are involved. Then, with the MoM the continuous unknown are expanded in a summation of basis functions, scalar or vector, analytic or not, computed at discrete points. They are multiplied by unknown scalar coefficients, which represent the actual unknowns of the matrix problem. Finally, the MoM enforces the starting analytical integral equation on the discrete domain: this is accomplished by means of proper test functions. For instance, when using a Galerkin’s implementation, the test functions coincide with the basis functions [36]. The solution of the matrix system yields the scalar coefficients and subsequently all the related quantities are derived.

In the application of the IEM, the choice of a suitable set of basis and test functions is a key point in the implementation of the algorithm. In fact, the dimension of the matrix problem (i.e. the order of the problem) is equal to the number of basis functions, and the computing time for the solution of a matrix system is proportional to the square of its order. It is important to remind that numerical MoM systems are characterized by dense matrices.
In the modeling of periodic structures, which are usually analyzed under the hypothesis of uniform plane wave impinging on the unit cell, the equation is written in terms of a Green’s integral (from George Green, English mathematician of the XIX century). The Green’s function is the kernel of the integral equation and relates the electromagnetic field in the domain of interest, i.e. the effect, to the unknowns of the problem, i.e. the equivalent source, generated by the external sources. In the most general case, the Green’s function is a dyadic function. In fact, it accounts for the relationship between every vector component of the effect and every vector component of the source. When an IEM is implemented, the Green’s function has to be determined in advance and this could be an extremely demanding step, concerning both mathematical and electromagnetic aspects. Moreover, since it contains all the information about the boundaries of the problem, the Green’s function is specific for particular classes of electromagnetic problems. An overview of its properties and expressions can be found in [38]. For a numerical resolution of the integral problem, also the Green’s function must be written in a discrete form: finite series. Hence, the efficiency of the IEM depends on the convergence rate of these series that impacts mainly in the filling time of the system matrix. Several works dealing with the acceleration of Green’s function convergence have been published. They are based for example on Ewald transformation and Poisson summation formula ([39] and [40] in the case of periodic structures), but also windowing of Green’s function spectrum ([41] for antenna problems).

The integral equation method has been applied to almost all possible problems where periodic structures are involved. Thanks to the fact that Green’s functions include the information of the boundaries as well as of the surrounding medium, IEM is actually well suited for the analysis of antennas. In fact, it permits to exactly simulate the radiation condition. In particular, the use of IEM in conjunction with MoM has been widely proposed for the characterization of large arrays (up to hundreds and thousands of elements), by means of special basis functions, named synthetic or characteristics, and a multiscale or multilevel MoM [42]-[45]. In fact, due to the dimensions of the problem, conventional formulations are severely limited by the size of the matrix system. Synthetic and characteristic functions overcome the restriction, because they are defined by starting from the solution of small-size problems; then, they are incorporated in larger features to describe the whole array. For example, a tool of the software ADF [46], used in the frame of the work on aperiodic phased arrays presented in Chapter 6, is based on a MoM implementation with synthetic basis functions.

In array analysis, the periodicity approximation leads to reasonably accurate results in predicting the behavior of elements which are far out from the edges. Nevertheless, for wide-beam angle scanning applications, the effects of truncation can be relevant for many array elements. The IEM/MoM implementation can be slightly modified to include the finite dimension of the antenna [47]-[49]. However, it should be noted that array analysis by the MoM cannot be restricted to the investigation of a single unit cell, even in the case of periodicity approximation. This is due to the nature of the problem, where radiation and mutual coupling among all the element must be considered for an accurate design.

Concerning FSS, IEM has been widely implemented in the analysis of both capacitive and inductive structures [5], [50]. The integral equation is enforced in the single unit cell, as a result of Floquet’s theorem, and the unknowns are defined on either the metallization of the patches or the aperture of the holes. After solving the matrix system, reflection and transmission coefficient are derived. A detailed description of the MoM modeling of frequency selective surfaces will be outlined in Chapter 2.

IEM has also been applied to the characterization of EBG and PhC [51]-[53]. The analysis of electromagnetic band-gap structures should provide two kinds of information: the dispersion
diagram of the supported modes and the phase of the reflection coefficient of the structure under plane-wave illumination. The second kind of calculation can be carried out by following a procedure similar to the FSS one. On the contrary, in the determination of the dispersion diagram, no incident field is considered and the MoM matrix results homogeneous. In addition, the matrix depends on both the propagation constant and the frequency, and it is not possible to separate the two dependencies. Therefore, frequency iteration is needed in order to find non-trivial solutions: the so computed pairs of propagation constant and frequency value determine the points in the dispersion diagram. Again, the analysis is reduced to the study of a single unit cell, thanks to Fourier's theorem.

When EBG and FSS are based on structure with complicate geometry, the use of IEM becomes unpractical, due to the problem of finding a proper expression of the Green's function and a suitable set of basis functions. Sometimes a volume integral equation is required (see, for instance, [54]), which can still be solved by the MoM. However, this leads to large, fully dense matrix problems, typically inefficient from the computational point of view. Consequently, more flexible modeling can be implemented by using the FEM or the FDTD methods.

In the finite element method, the electromagnetic problem is formulated in terms of a functional, which must be minimized in order to find the electric and magnetic field inside the structure. The problem is solved by meshing the whole domain of interest with tetrahedra, and expressing the field in terms of a set of edge-based expansion functions. FEM implementation is very general and can be applied to almost every kind of electromagnetic structure. It requires the discretization of the whole domain. Thus, in scattering problems, where most of the periodic structures are involved, the investigation volume must be truncated by imposing a suitable set of boundary conditions. When the investigation of the periodic structure starts from the study of the single unit cell, the domain truncation is performed by exploiting the periodicity first (one-, two- or three-dimensional), then by imposing radiation condition to the open boundaries. For instance, a Perfectly Matched Layer (PML) can be placed in close proximity to the periodic structure and can be used to absorb the scattered field, simulating open boundaries (a use of PML in the analysis of FSS can be found in [55]). The use of PML in conjunction with FEM originates a numerical problem with fully sparse matrices. Most commercial software are based on FEM, thanks to its flexibility. In the following chapters, some presented results are computed with HFSS, a FEM electromagnetic simulator.

FDTD is another general method. As the name states, it computes the fields in the time domain, by starting from differential equation expressed with finite differences. The analysis is constituted by subsequent iterations in time until the steady state is reached; then the frequency response can be derived through a Fourier transform. Similar to the FEM, also in this case the field must be evaluated in the whole domain of interest, hence a truncation in requested. Again, the periodicity is exploited: since the analysis is performed in the time domain, the periodicity is represented by a time-delay condition (a periodic boundary condition implementation for the analysis of FSS is reported in [56]). Concerning the open boundaries, the radiation condition can be performed by ABC surfaces (Absorbing Boundary Conditions), which are the FDTD counterpart of PML in the FEM approach. FDTD is characterized by sparse system matrix.

With respect to the IEM, both FEM and FDTD method permit to analyze a much wider class of devices. In particular, this is true when the periodicity is broken, and it is no longer possible to make use of the Floquet's theorem: FEM and FDTD are able to simulate also non-perfect periodic structures. However, they require total volume mesh, so they pay the flexibility with poor efficiency in computation time and memory allocation, even though both originate numerical problems with fully sparse matrices. Moreover, due to the need of domain truncation, FEM and FDTD are not used in practice for the analysis of antenna arrays. An alternative solution consists in the development of dedicated software that are less versatile but more powerful in handling specific
classes of problems. For example, the Guided Mode Expansion method (GME), used to analyze PhC in Chapter 6, is a numerical technique expressly implemented for the investigation of PhC slabs that can present defect in the crystal structure [57].
PART ONE

Frequency Selective Surfaces
CHAPTER 2

FSS Analysis by the Method of Moments

Frequency selective surfaces are planar periodic structures, consisting of either an array of metallic patches on a dielectric substrates, or a metallic screen periodically perforated with apertures.

The frequency response of FSS depends on the resonance of the periodic elements, since their dimension is comparable with the wavelength. The electromagnetic problem relates the fields scattered from the FSS to the surface currents induced on the surface by the incident field. In this way, the FSS is characterized in terms of transmission and reflection coefficients, referred to the incident wave [4]-[5], [58].

These structures can be analyzed under the hypothesis of an infinite array illuminated by a uniform plane wave: the Floquet’s theorem can be applied, the analysis of the whole FSS reduces to the investigation of its unit cell, and the computational effort is significantly reduced, whatever the used numerical technique is. As previously mentioned, the most popular formulations are based on an integral equation, either in a spatial- or spectral-domain. The equation is then solved by reducing it to a system of a finite number of equations with the same number of unknowns, in the context of the method of moments, for example. This step involves two different discretizations; therefore, the final solution must suffer of a relative convergence phenomenon, which will be discussed in the last paragraph of the chapter. Nevertheless, in general the integral methods are very efficient and they have the advantage of straightforwardly handling arbitrary incident angles.

In this chapter, a specific implementation of the integral equation method is described. It is named MoM/BI-RME method and is based on a MoM solution with entire-domain basis functions, under the infinite array approximation. The technique has been developed at the University of Pavia and was implemented in some computer codes in the last years. It proved to be very accurate for the analysis of FSS with arbitrarily shaped elements, both capacitive and inductive, from the microwave to the THz region [59]-[61]. In the following, the relevant theory is detailed and all the aspects discussed in the subsequent chapters are introduced.

2.1 ANALYSIS OF THIN CAPACITIVE FSS

The analysis of capacitive FSS (Fig. 2.1) is performed under the hypothesis of a multilayered infinite array of metal patches embedded in a stratified dielectric medium, and is based on the formulation of an electric field integral equation. Patches on different layers may have different shapes, but must have the same periodicity. The incident field at the frequency of interest is a uniform plane wave travelling along an arbitrary direction (θ,φ), shown in Fig. 2.1. Under these hypotheses, the analysis of the whole structure reduces to the investigation of the unit cell with periodic boundary conditions (Fig. 2.2).
CHAPTER 2: FSS Analysis by the Method of Moments

It is easy to demonstrate that an arbitrarily polarized incident wave can always be expressed as the combination of the two fundamental Floquet modes: TE\textsubscript{00}-TM\textsubscript{00} in case of oblique incidence, \(\theta \neq 0^\circ\), and TEM\textsubscript{01}-TEM\textsubscript{10} in case of normal incidence, \(\theta = 0^\circ\) (see Section 1.2.1). Then, since the FSS performance is characterized in terms of transmitted and reflected fields, matrices \(T\) and \(R\) can be defined as:

\[
\begin{bmatrix}
\vec{E}_{\text{trans}}^I \\
\vec{E}_{\text{trans}}^II
\end{bmatrix} =
\begin{bmatrix}
T_{I-I} & T_{I-II} \\
T_{II-I} & T_{II-II}
\end{bmatrix}
\begin{bmatrix}
\vec{E}_{\text{inc}}^I \\
\vec{E}_{\text{inc}}^II
\end{bmatrix}
\tag{2.1}
\]

\[
\begin{bmatrix}
\vec{E}_{\text{refl}}^I \\
\vec{E}_{\text{refl}}^II
\end{bmatrix} =
\begin{bmatrix}
R_{I-I} & R_{I-II} \\
R_{II-I} & R_{II-II}
\end{bmatrix}
\begin{bmatrix}
\vec{E}_{\text{inc}}^I \\
\vec{E}_{\text{inc}}^II
\end{bmatrix}
\tag{2.2}
\]

where \(\vec{E}_{\text{trans}}, \vec{E}_{\text{refl}},\) and \(\vec{E}_{\text{inc}}\) are the transmitted, reflected, and incident electric field, respectively; the superscripts I and II indicate TM\textsubscript{00} and TE\textsubscript{00} modes in the case of oblique incidence, and TEM\textsubscript{01}, TEM\textsubscript{10} in the case of normal incidence. The elements of the matrices are computed after solving the MoM numerical system that derives from the integral equation formulation.

In particular, let us consider a unit cell comprising a total of \(P\) metallic patches arbitrarily located at the interfaces of the stratified dielectric medium including \(N\) layers (Fig. 2.2). The analysis is based on the solution of the following 2P integral equations:

![Fig. 2.1 – Model of capacitive FSS, consisting of a periodic array of metal patches in a stratified medium.](image)

![Fig. 2.2 – Elementary cell of the capacitive FSS shown in Fig. 2.1.](image)
CHAPTER 2: FSS Analysis by the Method of Moments

\[ \vec{E}_n^{\mu e}(x,y) + \sum_{p=1}^{P} \int_{S_p} \vec{G}(x,y,z_n;x',y',z_p) \cdot \vec{J}_p^{\mu}(x',y') \, dS' = Z_S \vec{J}_n^{\mu}(x,y) \]  

(2.3)

with \( n = 1, \ldots, P \), and \( \mu = I, II \). The integral equation is obtained by enforcing the boundary condition on the surfaces of all the patches. In (2.3), \( \vec{E}_n^{\mu e} \) is the transverse component of the so-called “excitation field” on the \( n \)th patch (i.e., the electric field at the location of the \( n \)th patch, in the absence of all patches [62]), \( Z_S \) is the “sheet impedance” of the metallizations, \( S_p \) indicates the surface of the \( p \)th patch, \( z_p \) is the longitudinal coordinate of the \( p \)th patch, \( \vec{J}_p^{\mu} \) is the (unknown) current density on \( S_p \). Finally, \( \vec{G} \) is the dyadic Green’s function which relates the transverse electric delta current density in \((x',y',z_p)\) to the transverse electric field in \((x,y,z_n)\), given by [38]:

\[ \vec{G}(x,y,z_n;x',y',z_p) = \sum_{m=1}^{M} V_m(z_n,z_p) \vec{E}_m(x,y) \vec{E}_m^*(x',y') \]  

(2.4)

where \( \vec{E}_m \) denotes the transverse electric modal vector of the \( m \)th Floquet mode (TE, TM or TEM), the asterisk denotes the complex conjugate, and the functions \( V_m \) are determined by considering the equivalent modal transmission lines for the layered medium in the periodic cell. Note that, for sake of simplicity, a single subscript \( m \) is used in the following to indicate all the ordered Floquet modes. Moreover, the expression of Floquet modes is analytical (Section 1.2.1).

Equations (2.3) are solved by applying the MoM in the Galerkin’s form (i.e. test functions equal to the basis functions). The unknown current density \( \vec{J}_p^{\mu} \) is represented through a set of \( N_p \) vector basis functions \( \vec{e}_{p,j} \) defined on the \( p \)th patch, namely

\[ \vec{J}_p^{\mu}(x',y') = \sum_{j=1}^{N_p} x_{p,j}^{\mu} \vec{e}_{p,j}(x',y') \]  

(2.5)

where \( x_{p,j}^{\mu} \) are the unknown coefficients. By substituting (2.5) in (2.3), the integral equation is transformed into the matrix equation \( \mathbf{A} \mathbf{X} = \mathbf{B} \), where \( \mathbf{A} \) is the system matrix, \( \mathbf{X} \) is the column vector of the unknown amplitudes \( x_{p,j}^{\mu} \), and vector \( \mathbf{B} \) is related to the incident field:

\[
\begin{bmatrix}
A^{1,1} & \cdots & A^{1,p} & \cdots & A^{1,P} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
A^{n,1} & \cdots & A^{n,p} & \cdots & A^{n,P} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
A^{P,1} & \cdots & A^{P,p} & \cdots & A^{P,P}
\end{bmatrix}
\begin{bmatrix}
X^{1,1} \\
\vdots \\
X^{1,p} \\
\vdots \\
X^{1,P}
\end{bmatrix} =
\begin{bmatrix}
B^{1,1} & B^{1,II} \\
\vdots & \ddots & \vdots \\
B^{P,1} & B^{P,II}
\end{bmatrix}
\]

(2.6)

\[ A_{ij}^{\mu,\nu} = \sum_{m=1}^{M} V_m(z_n,z_p) \int_{S_n} \vec{e}_{n,j}(x,y) \cdot \vec{E}_m(x,y) dS \int_{S_p} \vec{e}_{p,i}(x',y') \cdot \vec{E}_m^*(x',y') dS' + Z_S \delta_{ij} \]  

(2.7)

\[ B_{i}^{\mu} = \int_{S_n} \vec{e}_{n,j}(x,y) \cdot \vec{E}_p^{\mu e}(x,y) dS \]  

(2.8)

\[ X_{j}^{\mu} = x_{p,j}^{\mu} \]  

(2.9)

The solution of the matrix equation yields the unknown coefficients \( x_{p,j}^{\mu} \) which permit to calculate the reflection and transmission coefficients of the expressions (2.1) and (2.2), following the line of [63]:
where \( \mu, \nu = I, II, \) and \( T_d^{\mu-\nu} \) and \( R_d^{\mu-\nu} \) are the transmission and reflection coefficients of the Floquet mode across the multilayered dielectric medium in the absence of the metal patches. Expressions (2.10) and (2.11) can be extended to compute reflection and transmission contributions of the fundamental Floquet mode to higher-order modes.

### 2.1.1 Entire-Domain Basis Functions: the MoM/BI-RME Method

The MoM analysis of capacitive FSS can be implemented by using either sub-domain or entire-domain basis functions. In the case of sub-domain basis functions, linear functions defined on rectangular (roof-tops, Fig. 2.3) or triangular sub-domains (Rao-Wilton-Glisson functions, [64]) are widely adopted [36]. The dimension of the sub-domains determines the number of basis functions, and smaller dimensions permit a more accurate representation of the unknown electric current density \( \mathbf{J}_p \). In the case of entire-domain basis functions, the vector functions span the entire domain of the patch and are tangential to its boundary (like the current density itself). A proper set of entire-domain basis functions is represented by the modal vectors of a waveguide with cross-section coincident with the shape of the patch, ordered according to their cutoff frequency. Increasing the accuracy means to include modes with higher cutoff frequency, which are basis functions with better spatial resolution. Entire-domain basis functions can be determined analytically for canonical shapes [50] or numerically for arbitrary shapes, e.g. by using the Boundary Integral-Resonant Mode Expansion (BI-RME) [59] or the FEM method [65]. All the results presented in the next chapter, in the case of entire-domain basis functions are obtained with the MoM/BI-RME method.

More specifically, the basis functions are the electric modal vectors of a waveguide with a cross-section \( S_p \) bounded by magnetic walls. These modal vectors refers to TM (\( \tilde{e}_{p,j}^{\nu} \)), TE (\( \tilde{e}_{p,j}^{\nu} \)), and, in the case of a multiply connected surface, TEM (\( \tilde{e}_{p,j}^{0} \)) modes. They can be defined in terms of scalar potentials.
\[ \tilde{e}_{p,j}^{\nu} = -\bar{u}_z \times \frac{\nabla \psi_{p,j}^{\nu}}{k_{p,j}} \]  
(2.12)

\[ \tilde{e}_{p,j}^{\nu} = -\frac{\nabla \psi_{p,j}^{\nu}}{k_{p,j}} \]  
(2.13)

\[ \tilde{e}_{p,j}^{0} = -\bar{u}_z \times \nabla \psi_{p,j}^{0} \]  
(2.14)

where the pairs \{\psi_{p,j}^{\nu}, k_{p,j}^{\nu}\} and \{\psi_{p,j}^{\nu}, k_{p,j}^{\nu}\} are the eigensolutions of the homogeneous Helmholtz equation in the domain \( S_p \) with Dirichlet and Neumann boundary condition, respectively. In the case of \( N \)-times connected surfaces, the basis functions of type (2.14) are \( N-1 \), and they are defined by solving the Laplace equation for \( \psi_{p,j}^{0} \), with the boundary condition \( \psi_{p,j}^{0} = 1 \) on an internal contour and \( \psi_{p,j}^{0} = 0 \) elsewhere.

When considering arbitrarily shaped patches, the entire-domain basis functions must be determined numerically. The efficiency of the numerical method used for their calculation is of paramount importance, since it practically determines the overall efficiency of the algorithm. In particular, the BI-RME method permits the calculation of some tens of eigensolutions of the Helmholtz equation and, therefore, of basis functions (2.12) and (2.13) - in a very short computing time (few seconds on a standard PC). The BI-RME method is a modified Boundary Integral Method (BIM), which permits to transform the Helmholtz equation into a linear eigenvalue problem [66]-[68]. In the BIM approach, the unknowns are current sheets defined over the boundary and acting in free-space. On the contrary, in the BI-RME method the same current sheets are embedded in an exterior rectangular waveguide of cross-section \( \Omega \) (Fig. 2.4).

**Fig. 2.4** – The geometry considered for the application of the BI-RME method: the cross-section \( S \) of the arbitrarily shaped waveguide is enclosed in the external rectangular waveguide omega \( \Omega \).

The solution of this problem yields, in a single shot, all the eigenvalues \( k_{p,j}^{\nu} \) and \( k_{p,j}^{\nu} \) up to a prescribed value \( k_{max} \), and the corresponding eigenfunctions \( \partial \psi_{p,j}^{\nu} / \partial n_p \) and \( \psi_{p,j}^{\nu} \) over the boundary \( \partial S_p \) (\( \partial n \) is the outward normal derivative on \( \partial S_p \)). From these boundary values, potentials \( \psi_{p,j}^{\nu} \) and \( \psi_{p,j}^{\nu} \) can be calculated on the whole domain \( S_p \), and therefore the basis functions can be obtained through (2.12) and (2.13). In the case of multiply-connected domains, the solution of the Laplace equation by the standard BIM described in [69] provides \( \partial \psi_{p,j}^{0} / \partial n_p \) on \( \partial S_p \) and, finally,
the basis functions (2.14). As an example, Fig. 2.5 shows some entire-domain basis function in the case of a multiply-connected arbitrary domain: the vector modes are calculated by the BI-RME method.

![Fig. 2.5](image)

**Fig. 2.5** – Some of the first entire-domain vector basis functions used to represent the electric current density on a multiply-connected patch with a rounded cross shape.

After the determination of the basis functions, the calculation of the MoM matrices (2.7) and (2.8) involves the coupling coefficients

\[
\int_{S_p} \bar{e}_{p,j}(\bar{r}) \cdot \bar{E}_m^*(\bar{r}) \, dS
\]

between the \( j \)th basis function on the \( p \)th patch and the \( m \)th Floquet modal vector. Their evaluation can be directly performed by a surface integration. However, such integration is a time consuming task, especially in case of basis functions determined by a boundary integral method, since it requires an additional numerical effort for computing the basis functions in many points within the
integration domain $S_p$. Conversely, by applying the Green’s identity and the properties of the modal vectors used as basis functions (see [60], Appendix), coupling integrals (2.5) can be transformed from surface to line integrals. In particular, we have

$$\int_{S_p} \mathbf{\varepsilon}_{p,j}^* \cdot \mathbf{E}_m^* \, dS = 0 \quad (2.16)$$

$$\int_{S_p} \mathbf{\varepsilon}_{p,j}^* \cdot \mathbf{E}_m^* \, dS = \frac{\kappa_{p,j}^*}{k_m (\kappa_{p,j}^2 - k_m^2)} \int_{\partial S_p} \psi_{p,j}^* \frac{\partial \chi_m^*}{\partial n_p} \, d\ell \quad (2.17)$$

$$\int_{S_p} \mathbf{e}_{p,j}^0 \cdot \mathbf{E}_m^* \, dS = 0 \quad (2.18)$$

$$\int_{S_p} \mathbf{\varepsilon}_{p,j}^* \cdot \mathbf{E}_m^* \, dS = \frac{k_m}{\kappa_{p,j}^*} \int_{\partial S_p} \frac{\partial \psi_{p,j}^*}{\partial t_p} \chi_m^* \, d\ell \quad (2.19)$$

$$\int_{S_p} \mathbf{e}_{p,j}^0 \cdot \mathbf{E}_m^* \, dS = \frac{1}{k_m} \int_{\partial S_p} \psi_{p,j}^* \frac{\partial \chi_m^0}{\partial n_p} \, d\ell \quad (2.20)$$

$$\int_{S_p} \mathbf{e}_{p,j}^0 \cdot \mathbf{E}_m^0 \, dS = 0 \quad (2.21)$$

$$\int_{S_p} \mathbf{\varepsilon}_{p,j}^0 \cdot \mathbf{E}_m^0 \, dS = \frac{1}{\kappa_{p,j}^*} \int_{\partial S_p} \psi_{p,j}^0 \frac{\partial \chi_m^0}{\partial n_p} \, d\ell \quad (2.22)$$

$$\int_{S_p} \mathbf{e}_{p,j}^0 \cdot \mathbf{E}_m^0 \, dS = 0 \quad (2.23)$$

where $\partial/\partial t_p$ represents the derivative along the boundary. The evident advantage of this transformation comes from the possibility of calculating the coupling integrals by a one-dimensional numerical integration. Furthermore, the contour integrals involve $\partial \psi_{p,j}^* / \partial n_p$, $\psi_{p,j}^0$ and $\partial \psi_{p,j}^0 / \partial n_p$, which are obtained as basic outputs of the BI-RME analysis. For this reason, the use of the BI-RME method leads to a dramatic computational advantage.

Finally, it is worthy observing that, in cases where $\kappa_{p,j}^2 = k_m^2$, expression (2.17) and (2.19) are not applicable. The BI-RME solution to this problem consists in deriving alternative line-integral expressions, as it is indicated and discussed in [69].

### 2.2 Analysis of Inductive FSS

An example of inductive FSS, consisting of a thick metal screen perforated periodically by apertures with an arbitrary shape and illuminated by a uniform plane wave incident from the direction $(\theta, \phi)$ is shown in . Again, due to the double periodicity of the structure, the analysis reduces to the investigation of a single unit cell, which is a rectangular waveguide with periodic boundary
CHAPTER 2: FSS Analysis by the Method of Moments

conditions Fig. 2.7. Then, the analysis of inductive FSS is based on the formulation of an integral problem on the unit cell, which is then solved by the Method of Moments (MoM).

![Inductive frequency selective surfaces, consisting of a thick metal screen perforated periodically with holes.](image)

Fig. 2.6 – Inductive frequency selective surfaces, consisting of a thick metal screen perforated periodically with holes.

![Elementary cell of the inductive FSS shown in Fig. 2.6.](image)

Fig. 2.7 – Elementary cell of the inductive FSS shown in Fig. 2.6.

As discussed in [5], aperture \( S_1 \) and \( S_2 \) on both sides of the screen (Fig. 2.8) are closed by perfect conductors and equivalent magnetic current densities \( \tilde{M}_1 \) and \( \tilde{M}_2 \) are defined over them. In order to guarantee the continuity of the tangential component of the electric field, the magnetic currents on both sides of \( S_1 \) and \( S_2 \) must be equal in amplitude and opposite in phase. Moreover, to ensure the continuity of the tangential component of the magnetic field, the following equations must be satisfied:

\[
\begin{align*}
\left(2\tilde{H}_{\text{inc}}^{L_1} + \tilde{H}_{M_1}^{L_1}\right) \times \tilde{u}_z &= (\tilde{H}_{M_1}^{R_1} + \tilde{H}_{M_2}^{R_1}) \times \tilde{u}_z \\
(\tilde{H}_{M_1}^{L_2} + \tilde{H}_{M_2}^{L_2}) \times \tilde{u}_z &= \tilde{H}_{M_z}^{R_2} \times \tilde{u}_z \quad \text{(2.25)}
\end{align*}
\]

on \( S_1 \) and \( S_2 \) respectively, where \( \tilde{H}_{\text{inc}} \) is the magnetic field of the incident plane wave, the subscripts refer to the source who generates the field (namely, the incident field, \( \tilde{M}_1 \) and \( \tilde{M}_2 \)), and the superscripts indicate the left and right sides of the pertinent aperture. The magnetic fields \( \tilde{H}_{M_1}^{L_1} \) and \( \tilde{H}_{M_2}^{R_2} \) (in region I and III, respectively) are expressed as a summation of Floquet modal vectors \( \mathbf{H}_{mn} \), whereas in region II the fields \( \tilde{H}_{M_1}^{R_1}, \tilde{H}_{M_2}^{R_1}, \tilde{H}_{M_1}^{L_2} \) and \( \tilde{H}_{M_2}^{L_2} \) are given as a summation of the normalized modal vectors \( \tilde{h}_r \) of the waveguide with a cross-section \( S \) (Fig. 2.9).
Fig. 2.8 – Side view of the equivalent circuit of the unit cell. Region II is a waveguide section of cross-section $S$, connected to Region I and III through the apertures $S_1$ and $S_2$, respectively.

Fig. 2.9 – Periodicity of a FSS: $\bar{T}_1 = a\hat{u}_x, \bar{T}_2 = b / \tan(\alpha)\hat{u}_x + b\hat{u}_y$.

The integral problem deriving from (2.25) is solved by using the method of moments. The magnetic currents are expressed as combinations of the modal vectors of the first $Q$ waveguide modes:

$$\bar{M}_1(x, y) = \sum_{q=1}^{Q} x_q \bar{h}_q(x, y)$$

$$\bar{M}_2(x, y) = \sum_{q=1}^{Q} y_q \bar{h}_q(x, y)$$

where $x_q$ and $y_q$ are unknown coefficients. With the MoM in the Galerkin’s form (i.e., choosing $\bar{h}_r$ as test functions), the following matrix problem is obtained:

$$\begin{bmatrix} A + B & C \\ C & A + B \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} D \\ 0 \end{bmatrix}$$

where $X$ and $Y$ are the column vectors of the coefficients $x_q$ and $y_q$, 0 is a null column vector of $Q$ elements, and
where $r$ and $q$ range from 1 to $Q$; the summation in (2.29) extends to all types of Floquet modes (TM, TE and, possibly, TEM); $V_{mn}$ denotes the modal admittance of the Floquet modes; $y_r$ and $\gamma_r$ are the modal admittance and the propagation constant of the waveguide modes, respectively; $t$ is the thickness of the metal screen (Fig. 2.8).

The solution of the matrix problem (2.28) yields the unknown coefficients $x_q$ and $y_q$. Once these coefficients have been calculated, the transmission and the reflection coefficients can be determined, along the lines of [61]. In conclusion, the application of the MoM algorithm to the analysis of FSS basically requires two steps: the former step is the evaluation of the modes of the waveguide (which again must be performed numerically in the case of arbitrarily shaped waveguides). The latter step is the calculation of the integrals in (2.29) and (2.32), which are the coupling integrals between the waveguide modes and the Floquet modes. The technique for calculating the modes of the waveguide with a cross-section $S$ corresponds to the one used in the capacitive structure.

### 2.2.1 Entire-Domain Basis Functions: the BI-RME Method

Similarly to the case of capacitive FSS, the accuracy of the solution depends on the number $Q$ of basis functions used at each terminal cross-section of the hole and on the number $M$ of Floquet modes adopted in the representation of the Green’s function. Since the waveguide modes (coinciding with the entire-domain basis functions) are needed to describe the propagation inside the hole perforating the metal screen, no sub-domain basis functions are typically adopted in the case of thick inductive FSS. In fact, even though in principle it is possible to consider sub-domain basis functions on both ends and waveguide modes in the perforating hole, this implementation is not as efficient as the entire-domain approach. In particular, entire-domain basis functions are computed by the BI-RME method, as in the capacitive case. They are electric modal vectors of a metallic waveguide with the same section of the periodic apertures, and they can be written in terms of scalar Helmholtz potentials $\Phi$ and $\psi$ [60].

The second step of the MoM procedure address the calculation of the matrix elements, hence of the coupling integrals that appear in (2.29) and (2.38) in the form

$$\int_S \vec{h}_q(\vec{r}) \cdot \vec{H}_{mn}(\vec{r}) dS$$

for all the six combinations of TE or TM waveguide modes ($\vec{h}_q$ and $\vec{H}_{mn}$ respectively) with TE, TM, or TEM Floquet modes ($\vec{H}_{mn}$, $\vec{H}_{mn}$, and $\vec{H}_{mn}$ respectively). In fact, also in the inductive case, the
incident field $\vec{H}_{\text{inc}}$ in (2.32) can be expressed as a combination of the two proper Floquet modal vectors.

Again, integral (2.33) can be transformed from a surface into a contour integral. The different expressions are

$$\int_S \vec{h}_q \cdot \vec{H}_{mn}^* dS = \frac{k_{mn}}{k_q^2} \int_{\partial S} \frac{\partial \psi_q^*}{\partial n} \chi_{mn}^* d\ell$$

(2.34)

$$\int_S \vec{h}_q \cdot \vec{H}_{mn}^* dS = 0$$

(2.35)

$$\int_S \vec{h}_q \cdot \vec{H}_{mn}^0 dS = 0$$

(2.36)

$$\int_S \vec{h}_q \cdot \vec{H}_{mn}^* dS = -\frac{1}{k_q} \int_{\partial S} \phi_q \frac{\partial \chi_{mn}^*}{\partial t} d\ell$$

(2.37)

$$\int_S \vec{h}_q \cdot \vec{H}_{mn}^* dS = \frac{k_{mn}^*}{k_q (k_{mn}^* - k_{mn}^2)} \int_{\partial S} \phi_q \frac{\partial \chi_{mn}^*}{\partial n} d\ell$$

(2.38)

$$\int_S \vec{h}_q \cdot \vec{H}_{mn}^0 dS = -\frac{1}{k_q} \int_{\partial S} \phi_q \frac{\partial \chi_{mn}^0}{\partial t} d\ell$$

(2.39)

where $\chi$ is the scalar potential of the Floquet modes, and $\partial/\partial t$ and $\partial/\partial n$ are the derivative with respect to the tangent and normal vector respectively (Fig. 2.9). As in the capacitive case, all the integrals turn into one-dimensional operations, and the quantities there involved are standard outputs of the BI-RME methods. Finally, the singular condition $k_q^2 = k_{mn}^2$ needs a special integral evaluation, described in [70].
CHAPTER 3

Study of the Convergence Properties of the MoM in the Modeling of FSS

In the past and present years, many papers have been published on the development of an efficient numerical solution to various types of electromagnetic problems, but only some of them have seriously considered the convergence issue with respect to the truncation of the infinite system of equations. Often, a numerical solution is regarded “satisfactory” because “it remains practically unchanged even though more and more modes are used in the computations,” or because “energy is conserved” [72]. The purpose of the study presented in the chapter is to closely examine these somewhat vague criteria and establish some general guidelines for efficient numerical computations in the analysis of FSS with the integral method.

The MoM resolution of the FSS problem may result highly dependent upon the manner in which the kernel and the unknown function are approximated in the process of constructing the matrix equation (i.e. the number of retained Floquet modes and basis functions, respectively). In particular, the MoM may exhibit a relative convergence phenomenon, because it may happen that unless the relative numbers of the degrees of freedom are chosen properly, the numerical solution do not converge to the correct value as the number of modes in the kernel and in the unknown increase indefinitely.

The term relative convergence was first introduced by Mittra [73] in 1963 in solving a waveguide bifurcation problem through a particular mode-matching procedure different from the moment method. In that problem, the issue was due to the choice of the particular mode-matching procedure, and it could be removed by using the more stable moment method. After the work of Mittra, a few papers that deal with the relative convergence have been published. In [72] and [74] a convergence rule is analytically derived in the case of iris-type discontinuity in metallic waveguides. When the electromagnetic problem is complex, hence an analytical relationship does not exist, and when the analysis of a whole class of structure is addressed, other criteria must be adopted to investigate the rate of convergence. For instance, the condition number of the MoM matrix is studied in [75], where the requirement to obtain accurate results is related to the properties of “well-conditioned” linear systems (i.e. stability, when modifying either input data or matrix elements). Finally, [77]-[80] discuss the relative convergence that appears in the MoM resolution of FSS problems, by considering the condition number, the spectral formulation, and simple geometries.

This chapter presents a comprehensive investigation of the convergence properties of the spatial-domain MoM convergence properties in the modeling of FSS with different element shapes, lattice geometry, angle of incidence, and characteristics of the dielectric substrate or metal layer. Both capacitive and inductive FSS are considered, and both the use of entire-domain and of sub-domain basis functions is discussed, by following the MoM/BI-RME implementation. First of all, different convergence criteria are defined, based on the stability of the resonance frequency, on the condition number, and on the variation of the transmission/reflection coefficients. Then convergence rules to
properly select the number of Floquet modes and basis functions to be retained are derived, for different FSS configurations. In fact, the availability of general and reliable convergence rules is of paramount importance when implementing the MoM in a computer code, with the purpose of either providing an automatic convergence procedure, or helping the user with clear guidelines. At last, the properties of entire- and sub-domain basis function, of capacitive and inductive FSS are compared and related to the physical aspects of the electromagnetic problem.

3.1 CONVERGENCE CRITERIA

The analysis of a capacitive FSS with entire-domain basis functions is considered first, simultaneously three different convergence criteria are defined and balanced. A capacitive FSS with free-standing cross-shaped patches (inset of Fig. 3.1) is analyzed under the hypothesis of illumination by a linearly polarized plane wave incident from the normal direction. The analyses are performed by using a numerical code based on the MoM/BI-RME method [59].

3.1.1 Resonance Frequency

The first convergence criterion is based on the error in the evaluation of the resonance frequency $f_0$, defined as the frequency where the transmission coefficient presents its minimum in the band of interest. Since capacitive FSS are basically stop-band filters, and in most cases they are operated near the resonance frequency, this convergence criterion is a reasonable choice. In the case that more resonances occur in the band of interest, the convergence is checked for all resonance frequencies, and in the following $f_0$ represents the highest resonance frequency.

![Fig. 3.1 – Study of the convergence for the analysis by the method of moments of the capacitive FSS with cross-shaped patches, when varying the number of entire-domain basis functions and of Floquet modes: percent relative error in the calculation of the resonance frequency (contour lines with 0.25% step).](image)
In this study, the convergence is said to be achieved if the error is smaller than 0.5%. Analyses have been performed by varying the number of basis functions from \( N = 2 \) (which corresponds to a maximum cutoff frequency of the waveguide modes used as basis functions \( f_N = f_0 \), with \( f_0 = 27.9 \) GHz) to \( N = 170 \) (\( f_N = 20 f_0 \)) and the number of Floquet modes from \( M = 2 \) (which corresponds to a maximum cutoff frequency of the Floquet modes \( f_M = f_0 \)) to \( M = 4900 \) (\( f_M = 30 f_0 \)). Fig. 3.1 shows the percent error in the determination of the resonance frequency versus the number of entire-domain basis functions and of Floquet modes.

The relative error is referred to the resonance frequency calculated with 170 basis functions and 4900 Floquet modes. Fig. 3.1 clearly shows that a minimum number \( M \) of Floquet modes is required for a given number \( N \) of basis functions, and when \( N \) increases, the minimum \( M \) must be increased accordingly. Since the number of basis functions and of Floquet modes are related to the respective maximum cutoff frequencies (see top and right axes in Fig. 3.1), a spectral rule can be set to guarantee the convergence. From this criterion it results that a relative error smaller than 0.5% is achieved when

\[
\frac{f_N}{f_0} > 6 \quad \frac{f_M}{f_0} > f_N \tag{3.1}
\]

that correspond to the use of 15 entire-domain basis functions and 194 Floquet modes. If these conditions are not met, increasing \( N \) without increasing \( M \) accordingly could have a detrimental effect on the accuracy of the result. To better appreciate this phenomenon, Fig. 3.2 shows the transmission coefficient of the FSS versus frequency for some values of \( N \) and for a given number of Floquet modes (\( M = 274 \)). Simulations from the commercial software HFSS, based on the finite element method, are used as a reference, to validate the results. It is noted that the calculated resonance frequency approaches the correct result when \( N < 30 \), but wrong results are obtained once \( N \) further increases.

![Graph showing transmission coefficient vs. frequency](image)

**Fig. 3.2** – Study of the convergence for the analysis by the method of moments of the capacitive FSS in Fig. 3.1: frequency response for selected values of the number \( N \) of basis functions, for a given number of Floquet modes (\( M = 274 \)).
3.1.2 Scattering Parameters

Another possible way to determine the conditions that guarantee the convergence requires the evaluation of the relative error in the reflection/transmission coefficient (or in general in the scattering parameters). This is a common convergence criterion, widely adopted in many commercial computer-aided design tools, which anyway exhibits a serious drawback: the choice of the frequency where the error is calculated can strongly affect the value of the error itself. For instance, in any filter the error in the scattering parameters is significantly larger near the resonance than elsewhere. Consequently, setting the convergence to an error of 2% rather than 5% is an arbitrary choice, which can be typically satisfactory at some frequencies but not at others. The drawback could be partially mitigated by considering several frequency points in the band of interest.

![Fig. 3.3 – Study of the convergence for the analysis by the method of moments of the capacitive FSS in Fig. 3.1, when varying the number of entire-domain basis functions and of Floquet modes: percent relative error in the calculation of the transmission coefficient at 27 GHz (contour lines with 2% step).](image)

In our case, the relative error in the calculation of the transmission coefficient is considered, and this error is referred to the value of the transmission coefficient calculated with 170 basis functions and 4900 Floquet modes. Analyses have been performed at a single frequency point (27 GHz), varying the number of basis functions from \(N=2\) to \(N=170\) and the number of Floquet modes from \(M=2\) to \(M=4900\). The results, shown in Fig. 3.3, confirm that this convergence criterion is more critical than the previous one, based on the resonance frequency. On the other hand, it also shows that a more strict convergence criterion could be required to achieve very high accuracy in a specific quantity, for instance in the value of the cross-polarization level.
### 3.1.3 Condition Number

The third criterion used to investigate the convergence is based on the condition number of the MoM matrix $\mathbf{A}$ (2.6). The condition number is a measure of that problem's amenability to digital computation, that is, how numerically well-conditioned the problem is. If the condition number is large, a small error in the data (vector $\mathbf{B}$) may cause a large error in the results (vector $\mathbf{X}$). As already mentioned, a criterion based on the condition number was already used to study the convergence properties of the MoM in simple cases [75], [77]. The condition number is defined as

$$\kappa(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$$  \hspace{1cm} (3.2)

where $\| \cdot \|$ represents the 2-norm of matrix $\mathbf{A}$ [81]. Since the condition number is related to the stability of the matrix problem solution, it is important to keep its value reasonably small. Fig. 3.1 shows the normalized condition number $\kappa(\mathbf{A}) / N^2$ calculated at 27 GHz, versus the number of basis functions and Floquet modes in the ranges considered above. In fact, it results that the value of the condition number grows with the number of basis functions: this is a normal behavior, due to the increased matrix dimension. Conversely, the normalized condition number is more significant, because it tends to rapidly increase when the number of basis functions is too small (below the limit defined by the convergence rule). Moreover, when the number of Floquet modes is too small (below the value imposed by the convergence rule $f_M > f_N$), the condition number is very large, whereas it tends to become small and constant when increasing the number of Floquet modes. Consequently, the condition number can be used to determine if the number of Floquet modes is large enough, for a given number of basis functions. It is worth noting that the condition number is calculated at a given frequency, but we verified in several different cases that its behavior does not significantly change with frequency. Therefore, the choice of the calculation frequency is less critical than in the case of the reflection/transmission parameters.

![Fig. 3.4](image)

**Fig. 3.4** – Study of the convergence for the analysis by the method of moments of the capacitive FSS in Fig. 3.1, when varying the number of entire-domain basis functions and of Floquet modes: normalized condition number of the MoM matrix at 27 GHz (contour lines with 1 step).

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As a final comment on the convergence rule (3.1), we can observe that the relation $f_M=f_N$ guarantees the same spatial resolution for the unknown, i.e. the electric current density $J$, expressed as a linear combination of basis functions (2.5), and the scattered field, expressed as a linear combination of Floquet modes (2.4) in the kernel of the integral equation (2.3). It is interesting to observe that $f_M>f_N$ (excess of Floquet modes) is not critical, whereas the opposite ($f_M<f_N$, excess of basis functions) severely deteriorates the result. This is clearly shown by the deterioration of the condition number of the MoM matrix, observed in Fig. 3.4, which happens in the region where $f_M<f_N$.

3.2 Capacitive FSS: Comparison between Entire-Domain and Sub-Domain Basis Functions

The same capacitive FSS with free-standing cross-shaped patches considered to introduce the convergence criteria (inset of Fig. 3.1) is analyzed by the MoM with sub-domain basis functions. In particular, roof-top basis functions are adopted, defined on rectangular domains according to a uniform mesh. The analyses are performed by using a numerical code based on the MoM/BI-RME method, with the change of basis functions. In this investigation, only the convergence criteria based on the resonance frequency and on the normalized condition number are adopted, whereas the criterion based on the transmission/reflection coefficient is discarded for the reasons previously discussed.

The first result is related to the convergence criterion based on the resonance frequency. Analyses have been performed by varying the number of basis functions (ranging from $N=10$ to $N=220$) and the number of Floquet modes (ranging from $M=2$ to $M=4900$). Fig. 3.5 shows the percent error in the determination of the resonance frequency versus the number of sub-domain basis functions and of Floquet modes. The relative error is referred to the resonance frequency calculated with 220 basis functions and 4900 Floquet modes. The same phenomenon already observed in the case of entire-domain basis functions is noted: when increasing the number of basis functions, more Floquet modes are needed to reach the convergence. Whereas entire-domain basis functions are related to a cutoff frequency, sub-domain basis functions are not. Nevertheless, it is possible to introduce a relation between the dimension of the sub-domains (the mesh size) and the wavelength at the operation frequency. This permits to define a spectral criterion for the choice of the number of sub-domain basis functions. From Fig. 3.5, it results that the convergence is reached when

$$d < \frac{\lambda_0}{15} = \frac{c}{15f_0}, \quad f_M > \frac{c}{2d}$$

where $d$ is the side of the sub-domain, $\lambda_0$ is the wavelength in vacuum at the resonance frequency $f_0$, and $c$ is the speed of light in vacuum. The first relation given in (3.3) represents a typical convergence rule adopted in many numerical codes based on the MoM, where the ratio $d/\lambda$ usually ranges from 1/10 to 1/20, depending on the required accuracy. The second relation in (3.3) basically means that all Floquet modes with a half cutoff wavelength larger than the mesh size need to be included. In the case of sub-domain roof-top basis functions, the convergence can be achieved by using 40 basis functions and 650 Floquet modes. Obviously, the number of sub-domain basis functions required to achieve the convergence is higher than the number of entire-domain basis functions. Note that also the number of Floquet modes needed to reach the convergence is larger for sub-domain basis functions than for entire-domain basis functions. This is due to the larger
spectrum of sub-domain basis functions, which depends on the combination of two effects: first of all, sub-domain basis functions are defined on a smaller spatial domain; second, entire-domain basis functions are $C_\infty$-class functions within the domain, whereas sub-domain basis functions are only $C_0$-class functions, and therefore their spectrum is larger.

\[ C_{nm} = \int_{S_n} \vec{e}_n \cdot \vec{E}_m^* dS \]  \hspace{1cm} (3.4)

which practically coincides with the Fourier transform of the basis functions [5]. In (3.4), $S_n$ represents the domain of the basis function $\vec{e}_n$. The simulation with 15 entire-domain basis functions is compared to the one with 40 sub-domain basis functions: these are the numbers that permit to achieve the convergence of the MoM according to the criterion of the resonance frequency. In the first case, the last entire-domain basis function is considered, because it exhibits the highest cutoff frequency and, therefore, the highest spectral components. In the case of sub-domain basis functions, a generic function is considered, because all functions have the same spectral components since the grid is uniform and the dimension of all sub-domains is practically identical. Fig. 3.6a shows the amplitude of the coupling integrals between the 15th entire-domain basis function and the first 4000 Floquet modes on the wavenumber $k_x-k_y$ plane. Similarly, Fig. 3.6b shows the amplitude of the coupling integrals between a sub-domain basis function and the first 4000 Floquet modes. It is noted that, in the case of the entire-domain function, the coupling integrals with significant amplitude are more concentrated in the center of the $k_x-k_y$ plane, which corresponds to Floquet modes with smaller cutoff frequency. This result explains why a smaller number of Floquet modes is needed to reach the convergence when using entire-domain basis functions.
Finally, the convergence criterion based on the condition number was also applied to the case of sub-domain basis functions: analyses have been performed by varying the number of basis functions (from $N=10$ to $N=220$) and of Floquet modes (from $M=2$ to $M=4900$). Fig. 3.7 shows the condition number versus the number of sub-domain basis functions and of Floquet modes. It is observed that the condition number is higher than in the case of entire-domain basis functions, and this effect can be attributed to two reasons: first of all, a larger number of basis functions is used and this leads to larger MoM matrices, which naturally exhibit higher condition number. The second
effect is due to the slower convergence of the kernel of the integral equation, due to the worse regularity of the functions and therefore to higher spectral components.

Fig. 3.7 – Study of the convergence for the analysis by the method of moments of the capacitive FSS in Fig. 3.1, when varying the number of sub-domain basis functions and of Floquet modes: normalized condition number of the MoM matrix at 27 GHz (contour lines with 5 step).

3.3  CAPACITIVE FSS: SYSTEMATIC STUDY OF THE CONVERGENCE PROPERTIES

The convergence criteria defined for the MoM analysis of capacitive FSS, as well as rules (3.1) and (3.3), represent a helpful instrument for computer-aided design tools only if they are validated for a wide range of element shapes and in different operation conditions. Since the frequency response of the FSS depends on the element shape and size, their periodicity, the incidence angle of the impinging wave, and the thickness and permittivity of the dielectric substrate, the effect of all these parameters has been systematically investigated, in order to demonstrate the general validity of the convergence rules.

3.3.1  Geometry of the Periodic Lattice

First of all, the effect of the unit cell dimension is considered. The capacitive FSS with cross-shaped patches, already discussed in the previous sections, has been analyzed with different dimensions of the unit cell. Rules (3.1) and (3.3) permit to reach the convergence in all cases: it means that the needed number of basis functions does not change, whereas the number of Floquet modes is modified according to the cell dimension, but the same maximum cutoff frequency is kept. It is noted that, if the elements are extremely close, capacitive coupling effects significantly change the
current density distribution on the patch: in this case, a larger number of basis functions could be needed.

A similar study was performed by keeping the same dimension of the unit cell, but modifying the skew angle in the range 45°-90°, in the case of the capacitive FSS with cross-shaped patches. Also in this case rules (3.1) and (3.3) permit to reach the convergence in all cases.

3.3.2 Element Shape

Another important test is the verification of the validity of convergence rules (3.1) and (3.3) for different patch shapes. Capacitive FSS with six different element shapes have been considered (Fig. 3.8), and in all cases the patch dimensions have been optimized in order to obtain the resonance frequency \( f_0 \) around 27.5-28 GHz. Analyses have been performed under the hypothesis of free-standing metal patches, normal incidence and linear vertical polarization. In the case of entire-domain basis functions, the resonance frequency has been calculated by setting the maximum cutoff frequency of the entire-domain basis functions to \( f_N = 7 f_0 \) and the maximum cutoff frequency of the Floquet modes to \( f_M = 9 f_0 \). The relative error has been computed with respect to the resonance frequency calculated with a large number of basis functions and of Floquet modes, obtained by setting \( f_N = 20 f_0 \) and \( f_M = 30 f_0 \), respectively.

![Capacitive FSS: different element shapes considered in the performance comparison.](image)

Table I reports the number of entire-domain basis functions adopted in the analysis, along with the relative error found for the six considered shapes, which is smaller than 0.5% in all cases. It was also verified that, in general, very thin structures may result more critical, and a higher maximum cutoff frequency for the basis functions is recommended. In the case of sub-domain basis functions, rule (3.3) has been applied to some shapes shown in Fig. 3.8; rounded shapes have been avoided, because in those elements the mesh is primarily related to the accuracy in the representation of the geometry. The analyses have been performed with a mesh size smaller than \( d_{\text{MAX}} = \lambda / 20 \), and a maximum cutoff frequency of the Floquet modes \( f_M = c / (2d) \). Table II reports the number of sub-domain basis functions adopted in the analysis, along with the relative error found for the three considered shapes, and also in this case the relative error is always smaller than 0.5%.
CHAPTER 3: Study of the Convergence Properties of the MoM in the Modeling of FSS

TABLE I

Performance comparison of different element shapes (dimensions shown in Fig. 3.8): number $N$ of entire-domain basis functions and relative error in the calculation of the resonance frequency.

<table>
<thead>
<tr>
<th>element shape</th>
<th>$N$</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>square</td>
<td>54</td>
<td>0.32%</td>
</tr>
<tr>
<td>dipole</td>
<td>14</td>
<td>0.11%</td>
</tr>
<tr>
<td>circle</td>
<td>73</td>
<td>0.22%</td>
</tr>
<tr>
<td>cross</td>
<td>23</td>
<td>0.18%</td>
</tr>
<tr>
<td>ring</td>
<td>33</td>
<td>0.35%</td>
</tr>
<tr>
<td>Jerusalem cross</td>
<td>20</td>
<td>0.18%</td>
</tr>
</tbody>
</table>

TABLE II

Performance comparison of different element shapes (dimensions shown in Fig. 3.8): number $N$ of sub-domain basis functions and relative error in the calculation of the resonance frequency.

<table>
<thead>
<tr>
<th>element shape</th>
<th>$N$</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>square</td>
<td>112</td>
<td>0.07%</td>
</tr>
<tr>
<td>dipole</td>
<td>19</td>
<td>0.11%</td>
</tr>
<tr>
<td>cross</td>
<td>40</td>
<td>0.07%</td>
</tr>
</tbody>
</table>

3.2.4 Effect of the Dielectric Substrate

The last verification performed on capacitive FSS is related to the effect of a dielectric substrate. The main consequence of the substrate is a reduction of the resonance frequency $f_0$ of the FSS of a factor $\sqrt{\varepsilon_{\text{eff}}}$, where $\varepsilon_{\text{eff}}$ is the effective dielectric permittivity, which can be calculated by the approximate formula $\varepsilon_{\text{eff}} = (\varepsilon_r + 1)/2$. The resulting value is independent on substrate thickness and frequency, but it represents the highest possible value of effective dielectric permittivity for capacitive FSS with dielectric on one side only [4], page 394. Consequently, rule (3.1) must be modified to account for this effect, thus resulting

$$f_N > 6f_0\sqrt{\varepsilon_{\text{eff}}} \quad f_M > f_N$$

(3.5)

Results similar to those shown in Fig. 3.1, Fig. 3.3, and Fig. 3.4 have been obtained in this case, and they are not reported for sake of shortness.
3.4 **Inductive FSS: Systematic Study of the Convergence Properties**

The convergence criteria defined in previous sections to investigate capacitive FSS are applied here to the study of inductive FSS. A thick metal screen perforated periodically with cross-shaped holes (inset of Fig. 3.9) is analyzed under the hypothesis of illumination by a linearly polarized plane wave incident from the normal direction. The thickness of the metal screen is $t = 2\,\text{mm}$ and the holes are arranged according to a square grid. The analyses of the inductive FSS are performed by using the numerical code based on the MoM/BI-RME method [60].

The first analysis is related to the convergence criterion based on the resonance frequency $f_0$, defined as the frequency where the FSS presents the minimum of its reflection coefficient in the band of interest. Since inductive FSS are basically band-pass filters, they exhibit a complementary behavior with respect to capacitive FSS, and this is the reason for considering the reflection coefficient instead of the transmission one. Similarly to the previous case, the convergence is said to be achieved if the relative error in the evaluation of the resonance frequency is smaller than 0.5%.

![Fig. 3.9](image_url) – Study of the convergence of the analysis by the method of moments of the inductive FSS with cross-shaped holes, when varying the number of entire-domain basis functions and of Floquet modes: percent relative error in the calculation of the resonance frequency (contour lines with 0.1% step).

Analyses have been performed by varying the number of basis functions from $N=2$ (which corresponds to a maximum cutoff frequency of the waveguide modes used as basis functions $f_N=f_0$, with $f_0=27.9\,\text{GHz}$) to $N=66$ ($f_N=9f_0$) and the number of Floquet modes from $M=2$ (which corresponds to a maximum cutoff frequency of the Floquet modes $f_M=f_0$) to $M=1700$ ($f_M=18f_0$). Fig. 3.9 shows the percent error in the determination of the resonance frequency versus the number of entire-domain basis functions and of Floquet modes. The relative error is referred to the resonance frequency calculated with 66 basis functions and 1700 Floquet modes. Since the number of basis functions and of Floquet modes are related to the respective maximum cutoff frequencies...
(see top and right axes in Fig. 3.9), a spectral criterion can be set, which guarantees to reach the convergence. From this criterion it results that a relative error smaller than 0.5% is achieved when

\[ f_N > 3f_0 \quad f_M > 4f_0 \]  

(3.6)

thus permitting to achieve the convergence with \( N = 5 \) entire-domain basis functions and 90 Floquet modes. This result is substantially different from the one obtained in the capacitive case (3.1), where the maximum cutoff frequency of the Floquet modes was related to the maximum cutoff frequency of the waveguide modes used as basis functions. In the case of inductive FSS, as shown in (3.6), the two quantities \( f_N \) and \( f_M \) can be set independently. It is clear from this result that the convergence issue is less critical in the case of inductive FSS than in the case of capacitive FSS. To better appreciate this difference, Fig. 3.10 shows the reflection coefficient of the FSS versus frequency for some values of \( N \) and for a given number of Floquet modes (\( M = 274 \)). Simulations from the commercial software HFSS are used as a reference, to validate the results. It is noted that the calculated resonance frequency is very stable for all considered values of \( N \), and the relative error is small in all cases.

![Image of Fig. 3.10](image-url)

**Fig. 3.10** – Study of the convergence of the analysis by the method of moments of the inductive FSS in Fig. 3.9: frequency response for selected values of the number \( N \) of basis functions, for a given number of Floquet modes (\( M = 274 \)).

The second analysis refers to the error in the evaluation of the reflection coefficient. Fig. 3.11 shows the relative error in the calculation of the reflection coefficient at 27 GHz, versus the number of basis functions and of Floquet modes. The relative error is referred to the value of the reflection coefficient calculated with 66 basis functions and 1700 Floquet modes. By comparing Fig. 3.11 with Fig. 3.3 (which shows the corresponding quantity in the case of capacitive FSS), it is clear that the convergence properties of the MoM are much better in the case of inductive FSS.
Fig. 3.11 – Study of the convergence of the analysis by the method of moments of the inductive FSS in Fig. 3.9, when varying the number of entire-domain basis functions and of Floquet modes: percent relative error in the calculation of the reflection coefficient at 27 GHz (contour lines with 2% step).

Fig. 3.12 – Study of the convergence of the analysis by the method of moments of the inductive FSS in Fig. 3.9, when varying the number of entire-domain basis functions and of Floquet modes: normalized condition number of the MoM matrix at 27 GHz.

The condition number of the MoM matrix has been also considered in this case. The normalized condition number $\kappa(A)/N^2$ calculated at 27 GHz is shown in Fig. 3.12, versus the number of basis functions and of Floquet modes. The value of the normalized condition number rapidly grows if the number of basis functions is too small and is practically insensitive to the number of Floquet modes.
Also in the case of inductive FSS, in order to guarantee the applicability of the convergence rule (3.6) as an helpful instrument for computer-aided design tools, its validity has been verified in a large number of different cases and operation conditions, including a number of different hole shapes, normal and oblique incidence of the impinging plane wave, rectangular and skewed grid. The analyses have been performed with $f_N=4f_0$ and $f_M=6f_0$, and in all cases the relative error in the calculation of the resonance frequency was smaller than 0.5%.

### 3.5 INDUCTIVE FSS: EFFECT OF METAL THICKNESS

An interesting analysis on inductive FSS is related to the effect of the thickness of the metal screen on the convergence properties. The inductive FSS with cross-shaped holes, shown in the inset of Fig. 3.9, is analyzed in this case with 15 basis functions ($f_N=4f_0$) and 200 Floquet modes ($f_M=6f_0$), for different values of the thickness of the metal screen. Fig. 3.13 shows the relative error in the calculation of the resonance frequency versus the metal thickness. For each value of the metal thickness, the error is referred to the resonance frequency calculated with 66 basis functions ($f_N=9f_0$) and 1700 Floquet modes ($f_M=18f_0$). It results that the relative error is small in general for all thickness values, but it increases when the metal thickness is very small: in particular, the largest error is obtained with the minimum metal thickness ($t=10\,\mu m$), when the FSS approaches the condition of infinitely thin metal screen. If the metal thickness is infinitely small, the frequency response of the inductive FSS can be obtained from the one of the corresponding capacitive FSS by applying the Babinet principle. Obviously, this analysis exhibits the same convergence properties of the capacitive case: therefore a larger number of basis functions and Floquet modes is required to achieve the same accuracy, as it results from the comparison of the convergence rules (3.1) and (3.6). This is also confirmed by the relative error calculated for the inductive FSS with zero thickness metal screens, which has been analyzed by using the corresponding capacitive FSS and applying the Babinet principle. The result for zero thickness is shown in Fig. 3.13 (round marker): the relative error in the case of $t=0$ is slightly larger than the error in the case $t=10\,\mu m$, and this confirms that the error increases when the metal thickness becomes small.

![Fig. 3.13 – Effect of the thickness of the metal screen on the relative error in the calculation of the resonance frequency of the inductive frequency selective surface with cross-shapes holes.](image)
In conclusion, it results that the convergence issue appears less critical in the case of thick inductive FSS than in the case of capacitive FSS. There is a possible physical explanation, which is related to the metal thickness and to the singularity of the field in the two cases. In fact, free-standing capacitive FSS present a sharp edge, and the electric field tangential to the patch diverges as $O(\rho^{-1/2})$ [82], where $\rho$ is the distance from the edge (Fig. 3.14). Conversely, in the case of thick inductive FSS, there is a right-angle conductive wedge, and the magnetic field tangential to the patch diverges as $O(\rho^{-1/3})$. For this reason, the singularity of the field is weaker in the case of inductive FSS, and therefore the convergence in this case appears less critical than in the case of capacitive FSS. Moreover, if the thickness of the metal screen is very small (like in the extreme case of Fig. 3.13), the situation is similar to a sharp edge, and therefore this case is similar to the capacitive case, thus explaining the larger error found in the analysis.

![Fig. 3.14 – Effect of metal thickness: (a) thin capacitive FSS; (b) inductive FSS defined on a thick metal layer.](image)

### 3.6 Final Remarks and Conclusions

A significant test case for the convergence rule (3.6) is the analysis of the dichroic mirror presented in [15], designed, manufactured and measured for the European Space Agency. This inductive FSS comprises cross-shaped holes in a thick metal screen, arranged in a tightly packed way according to a skewed lattice. The incidence angle is $\theta=30^\circ$ and the frequency response exhibits two close resonances for both TE and TM polarization. It results that the convergence rule (9) holds for the first TE and TM resonance (Fig. 3.15a and Fig. 3.16a, respectively) as well as for the second TE and TM resonance (Fig. 3.15b and Fig. 3.16b, respectively). It is noted that rule (3.6) has been applied with $f_0=8.5$ GHz, which approximately corresponds to the second resonance frequency for both TE and TM modes.

In principle, a correct value of the resonance frequency can still produce wrong reflection/transmission values and/or have different sensitivity to convergence. To verify the accuracy in the calculation of the wideband frequency response, the simulated and measured reflection coefficients of the TE and TM modes are shown in Fig. 3.17. The simulations have been performed with $N=34$ and $M=400$. Due to the extremely high fabrication accuracy of this FSS (which is currently operating in the deep-space antenna of the European Space Agency in Perth, Australia [15]), a perfect agreement between simulations and measurements can be achieved.
Fig. 3.15 – Study of the convergence of the analysis by the method of moments of the inductive FSS with cross-shaped holes [15], when varying the number of entire-domain basis functions and of Floquet modes; percent relative error in the calculation of the resonance frequency of: (a) the first TE resonance; (b) the second TE resonance (contour lines with 0.1% step).
Fig. 3.16 – Study of the convergence of the analysis by the method of moments of the inductive FSS with cross-shaped holes [15], when varying the number of entire-domain basis functions and of Floquet modes; percent relative error in the calculation of the resonance frequency of: (a) the first TM resonance; (b) the second TM resonance (contour lines with 0.1% step).
To conclude, this chapter has presented an investigation of the convergence properties of the MoM in the modeling of FSS. Different convergence criteria have been adopted and compared: they are based on the resonance frequency, on the reflection/transmission coefficients, and on the condition number of the MoM matrix. These criteria have been applied to the study of the convergence for capacitive FSS (using either entire-domain or sub-domain basis functions) and for inductive FSS (using entire-domain basis functions).

The most significant result of this study has been the definition of general convergence rules, based on a spectral principle, which permit to properly select the number of entire-domain basis functions and of Floquet modes. In the case of entire-domain basis functions, the convergence rule provides a relation between the numbers of basis functions and the operation frequency, through the concept of cutoff frequency of the modes. In the case of sub-domain basis functions, the convergence rule provides a relation between the mesh size and the operation frequency. In both cases, the number of Floquet modes is also related to the operation frequency, through the concept of cutoff frequency of the modes. Although these convergence rules cannot be derived analytically in the case of FSS with arbitrary elements, they provide a useful guideline both for setting the initial value of the convergence parameters and for defining a strategy to enhance the numerical accuracy of the results. These rules have been verified in a large number of operation conditions, including different incident angles, lattice periodicities, element shapes, and characteristics of the dielectric substrate or metal thickness. In all cases, they guarantee a good accuracy of the result, thus showing their general validity and applicability. Therefore, these rules can be implemented in the computer codes, with the aim of either providing an automatic convergence procedure, or helping the user with clear guidelines.

Finally, it has been shown that, in the case of sub-domain basis functions, a larger number of basis functions as well as of Floquet modes is usually needed to achieve the convergence. Moreover, it has been highlighted that the convergence appears less critical in the case of thick inductive FSS: a possible physical explanation has been given, based on the weaker field singularity in the case thick inductive FSS.
CHAPTER 4

Development of the MoM/BI-RME Method for the Analysis of Thick Capacitive FSS

The analysis of capacitive FSS is typically based on the hypothesis of infinitely thin metal patches. In fact, the complete analysis of an actual finite thickness conductor with finite bulk conductivity in electromagnetic analysis software can be numerically intensive. Also the MoM/BI-RME approach has been developed for the analysis of thin metal patches with an equivalent surface impedance [59]. The approximation implies that the current density is completely located on one side of the patch. Nevertheless, in some practical cases, this approximation is not accurate enough. In particular, the equivalent surface impedance is correct only for good conductors much thicker than the skin depth and for current flowing on one single side of the conductor [83]. On the contrary, in the mm- and submm-wave frequency band, patches become electrically thick, and the current is no longer located completely on one face of the conductor: this means that the ohmic losses are not carefully estimated. Moreover, the thin-patch approximation leads to an incorrect shift in the computed frequency response of the FSS.

A more accurate solution requires the exact modeling of thick metal patches. Obviously, general-purpose numerical codes, based on the finite-difference or the finite element method, can be applied to solve this problem, but they are usually inefficient and lead to long computing time. A few numerical techniques have been specifically proposed for the analysis of thick conductors, for different kind of electromagnetic applications. For instance, dielectric slab periodically loaded with thick metal strips have been studied with an integral equation methods, a mode matching technique and a combined FEM/MoM solution, in order to study the influence of the conductor thickness [84]-[86]. The problem of accurate modeling of ohmic loss has been investigated also in the interconnections for multi-GHz-bandwidth digital signals in the distribution network used for state-of-the-art integrated circuits. In this case, at high frequency, when a fine volume discretization should be necessary, the behavior of the conductor has been on the contrary related to a global surface impedance associated with a surface integral equation method [87], [88]. The global impedance is computed by solving for the fields inside the conductors. In addition, as already mentioned, also in high-frequency capacitive FSS the metal thickness cannot be neglected. Above all, three different approaches have been proposed in the last years: a surface integral equation method with roof-top basis functions [89], a hybrid modeling based on both FEM and IE formulations [90], and a volume integral equation resolution with volume currents [91]. All these three techniques are highly flexible, but may suffer of poor efficiency depending on the application, especially in the case of volume unknowns.

In this chapter, a novel technique for the modeling of thick capacitive FSS (Fig. 4.1) with the MoM/BI-RME method and its entire-domain basis functions is presented. In particular, the description of the unknown electric current density is modified in order to account for the actual thickness of the metal patches. Preliminary studies of the frequency response and of the current distribution have been carried out, with the aim of defining an efficient electromagnetic model for
the unit cell of the FSS. These results, discussed in the first paragraph, are integrated in the analysis theory, which is detailed in the second section. Then some examples are reported and some final remarks on the computational performances conclude the description of the “extended” MoM/BI-RME method.

Fig. 4.1 – Unit cell of a thick capacitive FSS.

### 4.1 CURRENT DISTRIBUTION AND ELECTROMAGNETIC MODELS

Capacitive frequency selective surfaces are usually fabricated by chemical processing of copper-coated films and dielectric layers, using standard circuit board manufacturing. Several dielectric substrates are available, with different composition, permittivity, flexibility, thermal stability, etc. Each of these laminates has a specific metal cladding, whose description is typically reported in terms of surface roughness, machinability and thickness. As a consequence, the electrical design and performance of the FSS are directly affected by the electrical characteristics of the complete metallo-dielectric structure [4], [5].

Since the presence of substrates strongly affect the transmission and reflection performances of capacitive FSS, all the numerical techniques must account for dielectric loading effects. For example, the MoM/BI-RME method directly includes permittivity constants and loss tangents in the Green’s function, in the equivalent transmission line model for the calculation of all the $V_m$ coefficients (2.4). Conversely, in most cases the analysis of capacitive FSS is based on the hypothesis of infinitely thin patches: since the actual thickness of the metal is neglected, the model may result not accurate enough, especially at high frequencies. Fig. 4.2 shows the frequency response of the capacitive FSS with free-standing cross-shaped patches arranged in a square grid, already analyzed for the convergence study (inset of Fig. 4.2), when varying the thickness $t$ of the metallization, in the case of normal incidence. From technical datasheets of commercial dielectric substrates, the more interesting thickness result to range between 15 and 100 $\mu$m. For the considered FSS, which presents a resonance frequency around 28 GHz, the related electrical thickness at resonance ranges between about $\lambda/700$ and $\lambda/100$. The analyses have been performed with the FEM commercial software HFSS, under the hypothesis of perfect electric conductors.

It is possible to observe from Fig. 4.2 that if the thickness increases up to 200 $\mu$m, the FSS response presents a frequency shift of about 500 MHz. In fact, as the thickness becomes larger with respect to the wavelength, the approximation of a thin single layer cannot account for the real current distribution on the three-dimensional patch. Moreover, since the model considers all the current concentrated on the equivalent upper plane, the ohmic losses due to the surface resistance are overestimated.
4.1.1 Electromagnetic Models for MoM/BI-RME method

The native MoM/BI-RME method, described in Chapter 2, deals with the approximation of infinitely thin metallization (Fig. 4.3). The extension of the MoM/BI-RME method to the modeling of thick capacitive FSS can be preliminary implemented by simply considering two infinitely thin patches, located at the proper distance, corresponding to the upper and lower faces of the thick patch (Fig. 4.4a). This solution is very efficient when implemented in the MoM/BI-RME method, because it can already handle multilayered capacitive structures. Furthermore, the basis functions need to be computed only once (for one patch), and the additional time required for the solution of the MoM matrix problem is small. Nevertheless, this approach permits to approximate only partially the effect of the metal thickness, since the current flowing on the side wall of the patch is completely neglected.
A more accurate study requires the modeling of the top, bottom and side currents (Fig. 4.4b). In such a way, the MoM/BI-RME approach should be modified in order to account for the quantities flowing in the vertical direction. If new components are added to the current representation (2.5), new components of the dyadic Green’s function are also needed, since it is necessary to relate all kinds of sources to the generated scattered field (2.4). In principle, this requested revision to the theory is straightforward, but the subsequent numerical implementation may present increment in both complexity and computation time.

### 4.1.2 Current Distribution on a Thick Patch

Preliminary studies have been performed with the software HFSS, to characterize the current distribution on the actual thick patches of a capacitive FSS. Again, the free-standing cross-shaped patch has been analyzed in the case of normal incidence. The metal thickness has been set equal to 100 μm. In Fig. 4.5 and Fig. 4.6 the three separate components of the electric current density on the surfaces of the patch are displayed, with top and bottom views. It clearly appears that the $z$-component of the current density on the side wall generally remains negligible with respect to its transverse component. This behavior suggests the opportunity to represent the electric current density by retaining only the transverse vector components of the current, in conjunction with a scalar variation on the side wall. Moreover, since the thickness of the patch is small, a linear function of the current amplitude along the vertical direction can be considered.

![Fig. 4.5 – Electric current density on the surfaces of the thick patch (top view): (a) $x$ component; (b) $y$ component; (c) $z$ component. The figure shows a quarter of the whole patch, because the software considers electromagnetic symmetries.](image)
In the MoM/BI-RME approach, the new current modeling is achieved by extending the basis functions defined on the top and bottom faces to the side of the patch. Hence, the vector basis functions still consist of electric modal vectors of a waveguide with the shape of the patch and magnetic-wall boundary condition (2.12), (2.13) and (2.14). Then, the basis functions defined on $S_{\text{bottom}}$ are extended to the domain $S_1 = S_{\text{bottom}} + S_{\text{side}}$, whereas the ones defined on $S_{\text{top}}$ are extended to $S_2 = S_{\text{top}} + S_{\text{side}}$ (Fig. 4.4b).

The extended MoM/BI-RME scheme presents an interesting advantage: there is no need to calculate new basis functions for representing the side current and no need to introduce additional terms of the dyadic Green’s function, since no $z$ components of the current are added.

### 4.2 Outline of the Theory

The analysis of thick capacitive FSS by the MoM/BI-RME method is based on the formulation of an electric field integral equation for the unit cell of the periodic array, in analogy with (2.3). It is obtained by enforcing the Leontovich boundary condition on the surface $S_{\text{bottom}} + S_{\text{top}} + S_{\text{side}}$ of the thick patch (Fig. 4.4b)

$$\vec{E}^{\text{inc}}(x, y, z) + \vec{E}^{\text{scat}}(x, y, z) = Z_s \vec{J}(x, y, z)$$

where $\vec{E}^{\text{inc}}$ represents the tangential component of the incident field (combination of fundamental Floquet modes, see Chapter 2), $Z_s$ is the sheet impedance of the metallization, and $\vec{J}$ is the unknown electric current density on the surface of the patch. Finally, $\vec{E}^{\text{scat}}$ is the scattered field

$$\vec{E}^{\text{scat}}(x, y, z) = \sum_{p=1}^{2} \int \vec{G}(x, y, z; x', y', z') \cdot \vec{J}_p(x', y', z') dS'$$

where $\vec{J}_p$ is the unknown current density on $S_p$ ($p=1,2$), and $\vec{G}$ is the dyadic Green’s function [38]

$$\vec{G}(x, y, z; x', y', z') = \sum_{m=1}^{M} V_m(z; z') \vec{E}_m(x, y) \vec{E}_m^*(x', y').$$

The formulation strictly follows the standard MoM/BI-RME one, with the addition of the $z$ variable. In the subsequent sections, all the involved expressions are detailed.
4.2.1 Green’s Function

The general expression of the dyadic Green’s function consists of nine components

\[ \tilde{G}(x, y, z; x', y', z') = G_{xx}xx' + G_{xy}xy' + G_{xz}xz' + G_{yx}yx' + G_{yy}yy' + G_{zy}zy' + G_{zx}zx' + G_{zy}zy' + G_{zz}zz' \] (4.4)

and it relates the three vector components of the electric delta current density in \((x', y', z')\) to the three vector components of the scattered electric field in \((x, y, z)\). Since the study of the current distribution indicates that the \(z\) components can be neglected, only four terms of the dyadic functions must be retained: \(G_{xx}, G_{xy}, G_{yx}, G_{yy}\). In this way, the Green’s function can be written as (4.3). Moreover, since the thickness \(t\) of the patch will be \(t << \lambda\), the assumption that the impedance does not vary significantly along \(S_{\text{side}}\) can be made, so the Green’s function further simplifies as:

\[ \tilde{G}(x, y, z; x', y', z') \approx \sum_m V_m(z_n; z_p)E_m(x, y)E_m^*(x', y') \] (4.5)

where, again, \(E_m\) denotes the transverse electric modal vector of the \(m\)-th Floquet mode, and the coefficients \(V_m\) are the functions determined with the equivalent transmission lines model that represents the unit cell.

4.2.2 Basis Functions

In the solution of the integral equation by the method of moments, the novel expression of the electric current density \(\tilde{J}_p\), in terms of \(N\) entire domain basis functions, becomes:

\[ \tilde{J}_p(x', y', z') = \sum_{j=1}^{N} a_{p,j} \tilde{e}_{p,j}(x', y')f_p(z') \] (4.6)

where \(a_{p,j}\) are the unknown coefficients, and \(\tilde{e}_{p,j}\) are the electric modal vectors of a waveguide with \(S_p\) as cross-section, bounded by magnetic walls (2.12), (2.13) and (2.14); finally, \(f_p(z')\) is the scalar function that accounts for a linear variation along \(z\):

\[ f_1(z') = \frac{z'-z_2}{z_1-z_2} \quad \text{and} \quad f_2(z') = \frac{z'-z_1}{z_2-z_1} \] (4.7)

with

\[ f_a(z') = \begin{cases} 1 & \text{if } z' = z_p \\ 0 & \text{if } z' = z_p \end{cases} \] (4.8)

By means of this definition, the new basis functions are defined on the larger domains \(S_1 = S_{\text{bottom}} + S_{\text{side}}\) and \(S_2 = S_{\text{top}} + S_{\text{side}}\) (Fig. 4.4b). In particular, \(f_1\) (equal to 1 on \(S_{\text{bottom}}\) and linearly vanishing on \(S_{\text{top}}\)) extends the amplitude of the vector modes from the contour of \(S_{\text{bottom}}\) to the side wall; vice versa \(f_2\) (1 on \(S_{\text{top}}\) and linearly vanishing on \(S_{\text{bottom}}\)) extends the amplitude of the vector modes from the contour of \(S_{\text{top}}\) to the side wall.

Since the study of the distribution of the electric current density \(\tilde{J}_p\) on the lateral surface of the patch showed that the \(z\) component is negligible, it is possible to keep the same set of vector basis functions already introduced in the MoM/BI-RME approach for thin capacitive FSS (Chapter 2 and...
Moreover, this hypothesis implies that also the Green’s function does not need the definition of new terms which deal with the longitudinal components of the source and the scattered field.

### 4.2.3 Matrix Problem

The final matrix problem is obtained after substituting (4.6) into (4.2) and (4.1). Then, by applying the Galerkin’s method, the matrix problem results

\[
\begin{bmatrix}
A_{1,1}^{1} & A_{1,2}^{1} \\
A_{2,1}^{2} & A_{2,2}^{2}
\end{bmatrix}
\begin{bmatrix}
X^{1} \\
X^{2}
\end{bmatrix}
= 
\begin{bmatrix}
B^{1} \\
B^{2}
\end{bmatrix}
\tag{4.9}
\]

with

\[
A_{i,j}^{n,p} = \sum_{m=1}^{M} V_{m}(z_{n}, z_{p}) \left[ \vec{e}_{n,j}(x, y) f_{n}(z) \right] \cdot \vec{E}_{m}(x, y) dS_{n} \times \]

\[
\left[ \vec{e}_{p,j}(x', y') f_{p}(z') \right] \cdot \vec{E}_{m}(x', y') dS_{p} + Z_{0} \delta_{np} \left[ \vec{e}_{n,j}(x, y) f_{n}(z) \right] \cdot \left[ \vec{e}_{p,j}(x, y) f_{p}(z) \right] dS_{n}
\tag{4.10}
\]

\[
X_{j}^{p} = a_{p,j}
\tag{4.11}
\]

\[
B_{i}^{n} = \left[ \vec{e}_{n,i}(x, y) f_{n}(z) \right] \cdot \vec{E}_{inc}(x, y) dS_{n}
\tag{4.12}
\]

where \(A\) is a \(2N \times 2N\) matrix, \(X\) is the \(2N \times 2\) matrix of the unknown amplitudes, and \(B\) is related to the incident field.

Once the problem (4.9) has been solved and the \(a_{p,j}\) coefficients are known, the reflection and transmission coefficients can be derived with the expressions (2.10) and (2.11), after modifying the domain of the surface integrals according to the definitions \(S_{1} = S_{\text{bottom}} + S_{\text{side}}\) and \(S_{2} = S_{\text{top}} + S_{\text{side}}\).

### 4.2.4 Coupling Integrals

The calculation of the matrices (4.10) and (4.12) involves the coupling integrals between basis functions and Floquet modes and between two basis functions, respectively:

\[
\left[ \vec{e}_{n,j}(x, y) f_{n}(z) \right] \cdot \vec{E}_{m}(x, y) dS_{n}
\tag{4.13}
\]

\[
\left[ \vec{e}_{n,j}(x, y) f_{n}(z) \right] \cdot \left[ \vec{e}_{p,j}(x, y) f_{p}(z) \right] dS_{n}
\tag{4.14}
\]

with \(S_{n} = S_{n,xy} + S_{\text{side}}\) (where \(S_{n,xy}\) denote the surfaces of the patch parallel to the \(xy\) plane, i.e. either \(S_{\text{top}}\) or \(S_{\text{bottom}}\), Fig. 4.4b). The first coupling integral (4.13) is involved in the calculation of the scattered field through the Green’s function. The second one (4.14) accounts for the losses related to the surface impedance; please note that this coefficient will differ from zero only when \(p = n\), otherwise the vector functions will span two different planes.

In both cases, it is possible to split the integrals considering \(S_{n,xy}\) first and then the scalar integration along \(z\). For example, the coefficient (4.13) becomes:
\[ \int \left[ \tilde{e}_{n,j}(x, y) f_n(z) \right] \cdot \mathbf{E}_m(x, y) dS_n = \int \left[ \tilde{e}_{n,j}(x, y) \cdot \mathbf{E}_m(x, y) \right] dS_{n,xy} + \frac{1}{2} \int \left[ \tilde{e}_{n,j}(x, y) \cdot \mathbf{E}_m(x, y) \right] d\ell \quad (4.15) \]

where \( \partial S \) is the boundary of \( S_{n,xy} \). By applying the Green's identity and the properties of the modal vectors used as basis functions to the surface integrals (see Chapter 2 and [59]), also the first term turns into a contour integral, reducing the computational effort. The final expression of the first terms, for all the combinations of basis functions and Floquet modes (superscripts \(^,\) and 0 indicate TM, TE, and TEM modes, respectively), are:

\[ \int \left[ \tilde{e}_{n,j}^i f_n(z) \right] \cdot \mathbf{E}_m^* dS = 0 + \frac{t}{2} \frac{1}{\kappa_{n,i} k_m} \int \frac{\partial \psi_{n,i}^j}{\partial t} \frac{\partial \chi_m}{\partial t} d\ell \quad (4.16) \]

\[ \int \left[ \tilde{e}_{n,j}^0 f_n(z) \right] \cdot \mathbf{E}_m^* dS = 0 + \frac{t}{2} \frac{1}{k_m} \int \frac{\partial \psi_{n,i}^0}{\partial t} \frac{\partial \chi_m}{\partial t} d\ell \quad (4.17) \]

\[ \int \left[ \tilde{e}_{n,j}^0 f_n(z) \right] \cdot \mathbf{E}_m^* dS = 0 + \frac{1}{k_m} \int \psi_{n,i}^* \frac{\partial \chi_m^*}{\partial t} d\ell - \frac{t}{2} \frac{1}{2(k_{n,i} k_m)} \int \frac{\partial \psi_{n,i}^0}{\partial t} \frac{\partial \chi_m}{\partial t} d\ell \quad (4.18) \]

\[ \int \left[ \tilde{e}_{n,j}^f f_n(z) \right] \cdot \mathbf{E}_m^0 dS = 0 + \frac{t}{2} \int \frac{\partial \psi_{n,i}^j}{\partial t} \frac{\partial \chi_m}{\partial t} d\ell \quad (4.19) \]

\[ \int \left[ \tilde{e}_{n,j}^0 f_n(z) \right] \cdot \mathbf{E}_m^0 dS = 0 + \frac{1}{k_m} \int \psi_{n,i}^* \frac{\partial \chi_m^0}{\partial t} d\ell - \frac{t}{2} \frac{1}{2(k_{n,i} k_m)} \int \frac{\partial \psi_{n,i}^0}{\partial t} \frac{\partial \chi_m}{\partial t} d\ell \quad (4.20) \]

\[ \int \left[ \tilde{e}_{n,j}^0 f_n(z) \right] \cdot \mathbf{E}_m^0 dS = 0 + \frac{t}{2} \int \frac{\partial \psi_{n,i}^0}{\partial t} \frac{\partial \chi_m}{\partial t} d\ell \quad (4.21) \]

Likewise, thanks to the orthonormal properties of the basis functions, the calculation of the losses in (4.14) reduces to the following general contour integrals

\[ \int \left[ \tilde{e}_{n,j}(x, y) f_n(z) \right] \cdot \left[ \tilde{e}_{n,j}(x, y) f_n(z) \right] dS_n = \delta_{ij} + \frac{1}{2} \int \tilde{e}_{n,j}(x, y) \tilde{e}_{n,j}(x, y) d\ell \quad (4.25) \]

Finally, considering all the combination of basis functions (superscripts \(^,\) and 0 indicate TM, TE, and TEM modes, respectively), the possible contour terms are

\[ \int \left[ \tilde{e}_{n,j} \cdot \tilde{e}_{n,j}^* \right] d\ell = \frac{1}{\kappa_{n,i} k_{n,j}} \int \frac{\partial \psi_{n,i}^j}{\partial t} \frac{\partial \psi_{n,i}^j}{\partial t} d\ell \quad (4.26) \]
As in the case of MoM/BI-RME modeling of thin capacitive FSS, all the contour integrals (4.16)-(4.24) and (4.26)-(4.34) involve quantities that are obtained analytically or as basic outputs of the BI-RME analysis.

## 4.3 Validation Example

The cross shaped capacitive FSS considered for the preliminary study (inset of Fig. 4.1) is analyzed in this section to validate the proposed MoM/BI-RME method. The structure is illuminated by a uniform plane wave incident from the normal direction. A perfect electric conductor models the metal patch.

The convergence of the MoM/BI-RME method was obtained with 68 basis analysis functions \((N = 34)\) and \(M = 1100\) Floquet modes. The computing time for the calculation of the transmission and reflection coefficients in 100 frequency points was 16 sec on a computer with Intel Core Duo processor 2 GHz and 2 GB RAM: it corresponds to 6 sec for computing the basis functions by the BI-RME method and 0.1 sec per frequency point for the solution of the MoM problem (for example, a convergent analysis of the same simple structure with HFSS requires simulation times of the order of minutes, instead of seconds, for less amounts of frequency points). Fig. 4.7a shows the frequency response of the FSS for values of the metal thickness ranging from 0 to 100 μm, while Fig. 4.7b reports the frequency shift of the stop band of the FSS versus the thickness \(t\) of the metal patch, with a comparison with HFSS values. From these graphs, it is possible to confirm that simulation results obtained with the extended MoM/BI-RME method are in good agreement with data from the...
commercial FEM code HFSS. In particular the frequency shifts computed with the two methods exhibit an identical trend.

![Graph (a)](image1)

**Fig. 4.7** – Validation of the extended MoM/BI-RME method, in the case of cross-shaped capacitive FSS (inset of Fig. 4.2): (a) frequency response for different thickness values; (b) comparison with FSS, with frequency shift versus metal thickness.

Finally, an estimation of ohmic losses is presented. The resistivity of the cross-shaped patches has been set equal to \(0.24 \times 10^{-5} \, \Omega \text{m}\). Fig. 4.8 reports a comparison between HFSS and MoM/BI-RME analysis in the case of a metal thickness of 100 \(\mu\text{m}\). As expected, the thin patch approximation leads
to an overestimate of the losses, because the computed electric current density on the single metal layer is too high. Again, MoM/BI-RME and HFSS curves follow the same trend. In particular, the small discrepancy among them in the thin case is most likely due to a convergence problem related to a stronger field singularity, as described in Chapter 3.

![Graph showing S11 vs. frequency for different methods](image)

**Fig. 4.8** – Validation of the extended MoM/BI-RME method, in the case of cross-shaped capacitive FSS (inset of Fig. 4.2): losses evaluation, for a finite metal conductivity equal to $0.24 \times 10^{-5}$ $\Omega$ m and 100 $\mu$m thickness.

### 4.4 FINAL REMARKS AND CONCLUSIONS

A novel surface integral equation technique for the modeling of capacitive frequency selective surfaces with thick metal patches, based on the MoM/BI-RME method, has been proposed. The final implementation results very efficient from the computational point of view. In fact, since there is no need to derive new basis functions, the entire-domain vector basis functions are computed by the BI-RME method only for one patch, and the extension to the new total domain is completely analytical. The only increment in the computing time is due to the number of elements of the MoM matrix, which is double in size compared to the case of infinitely thin metal patches. Nevertheless, since the size of the matrices obtained by using the MoM/BI-RME method is typically small (some tens of unknowns for convergent and accurate responses, see Chapter 3), the analysis time required for the solution of the matrix problem remains extremely short. Moreover, also in this case it is possible to transform all the coupling integrals between basis functions and Floquet modes and between two basis functions from surface to line integrals, which involves either analytical calculation or standard BI-RME outputs. As a result, the filling-matrix computational effort is extremely reduced.
PART TWO

Breaking the Periodicity
Photonic crystals are dielectric structure at optical frequency, defined by a periodic lattice that creates photonic band-gaps where the propagation of electromagnetic waves inside the crystal is inhibited.

Line defects permit to guide the light from one point to the other of a photonic crystal. The basic idea is to break the periodicity, in order to shape a waveguide out of an otherwise-perfect crystal [9]. Light that propagates in the waveguide with a frequency within the band-gap of the embedding structure is confined to, and can be directed along, it. At optical frequencies, visible light is commonly guided in fiber-optic cables, which rely on total internal reflection due to the step-index phenomenon between dielectric and air. However, if a fiber-optic cable is tightly bent, the angle of incidence is too high for total internal reflection to occur, so light escapes. PhC join together step-index and crystal band-gap behavior, therefore can confine light even around tight corners.

As already mentioned in Chapter 1, linear photonic crystal waveguides can be realized either in fully three-dimensional lattice or in PhC slabs. It is curious to see how many configurations of different lattices and materials can be considered, and at the same time only few arrangements are of practical interest. In a comparison with the consolidated field of solid semiconductors, photonic crystals could provide even a wider set of configurations. However, since three dimensional structures are more difficult to fabricate, only PhC slabs with triangular lattice of air holes proved to be suitable candidates for applications to integrated optics. In fact, in high-index membranes, truly guided-wave propagation can be obtained, where propagation losses are determined only by roughness or disorder-induced scattering [92]. In addition, PhC slabs can be easier integrated because they are defined on a plane.

At present, research in PhC subject regards simultaneously theoretical refinement and design of innovative components. In particular, though excellent results have been achieved both in Silicon and GaAs-based structures, it is still true that propagation losses in PhC systems are too large as compared with dielectric strip waveguides [93], [94]. It is then important to develop accurate models for the prediction of total losses, also due to fabrication imperfection [95] and coupling with input/output lines [96]-[98]. Another very important requirement consists in single-mode behavior in the proper frequency band. Indeed, when bent waveguides or more complex interconnections are realized, the existence of two or more modes at the same frequency can give rise to modal coupling: again, the performance deteriorates [99]. On the other side, several PhC components have been recently proposed. In fact, all-optical solution should be realized in order to support increased data-traffic networks, where photonic crystal circuits will substitute residual electronic circuits [28]-[31].

The work presented in this chapter deals with the design of a directional coupler, based on a travelling-wave configuration, which reproduces the well-known concepts developed at microwave frequencies. The results have been carried out throughout a master-science thesis project, in the frame of a collaboration with the Department of Physics “A. Volta” of the University of Pavia.
5.1 **ELECTROMAGNETIC FIELDS IN PHOTONIC CRYSTALS**

Photonic crystals are defined by a periodic pattern in a complete dielectric structure. Fig. 5.1 reports a two-dimensional structure with a triangular lattice of triangular holes. If the dielectric constants of the macroscopic lattice are different enough, and the absorption is minimal, the scattering at the interfaces can prevent light from propagating in certain direction, and create photonic band-gaps.

![Example of two-dimensional photonic crystals: triangular lattice of triangular air holes with lattice constant $a$ and triangle side $L$; the lattice is patterned in a high index suspended membrane of thickness $d$. The Brillouin zone with main symmetry directions in reciprocal space is also drawn.](image)

All of macroscopic electromagnetism is governed by the four macroscopic Maxwell equations, also the propagation of light in photonic crystals [9]. Without losing any interesting properties, the study of the electromagnetic fields in PhC can consider the following assumptions:

- the general mixed dielectric medium is free of charges or currents;
- field strengths are small enough so that they are in linear regime;
- the material is macroscopic and isotropic;
- at the frequencies of interest, the dielectric constant does not exhibit explicit frequency dependence;
- the dielectrics are lossless (i.e. the dielectric permittivity is purely real).

In such a way, since both the involved materials and the Maxwell equations are linear, the fields can be expanded in a set of harmonic modes. By means of some algebraic manipulations, the so-called *master equation* for the computation of electromagnetic field inside a PhC slab, in the phasor domain, results

$$\nabla \times \left( \frac{1}{\varepsilon(r)} \nabla \times \mathbf{H}(r) \right) = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(r)$$

(5.1)

where $\varepsilon(r)$ is the dielectric permittivity of the medium, $c$ is the speed of light, and $\omega$ is the angular frequency. After solving for the magnetic field, the expression for the calculation of the electric field is

$$\mathbf{E}(r) = \left( -\frac{jc}{\omega \varepsilon(r)} \right) \nabla \times \mathbf{H}(r)$$

(5.2)

where $j$ indicates the imaginary unit.
The equation (5.1) represents an eigenvalue problem, where each solution $H(r)$ is the eigenfunction and $(\omega/c)^2$ is the corresponding eigenvalue: the eigenfunctions correspond to magnetic modes of the PhC structure, at a given frequency $\omega$. Besides, the operator that on the left side of the master equation is applied to $H(r)$ is a Hermitian operator. Consequently, the eigenfunctions are orthogonal, they have real eigenvalues, they can be obtained by a variational principle, and they may be cataloged by means of their symmetries.

In particular, the electromagnetic variational principle determines the behavior of electromagnetic fields and the presence of band-gaps in PhC. It states that “a mode tends to concentrate its energy pattern in regions of high dielectric constants, while remaining orthogonal to the modes below it in frequency” [9]. This means that the lowest eigenmode, $H_0$, will minimize the energy functional; then, the upper next eigenmode will minimize the energy functional in a subspace orthogonal to $H_0$ and so on.

Finally, please note that all this formulation for electromagnetic fields in dielectric media does not require periodicity and there is no other fundamental length scale except for the hypothesis that the system is macroscopic.

### 5.1.1 Harmonic Modes

Photonic crystals present a discrete translational symmetry, because they are invariant only under distances that are a multiple of some fixed step lengths (for example, the lattice constant $a$ in Fig. 5.1). In fact, they are based on a periodic lattice, characterized by a generic translational vector $a$. Exploiting this symmetry, each single eigenvector $H_k$, solution of (5.1), can be written as

$$H_k(r) = e^{ikr}u_k(r)$$

where $u_k(r) = u_k(r+a)$ is a periodic function of the PhC lattice and $k$ is the wave vector. The eigenvector corresponds to a Bloch state (see Section 1.2.1) and it is associated to the proper eigenvalue $\omega(k)$. Usually the modes are classified by specifying the wave numbers $k_x, k_y, \text{and } k_z$.

The eigenvalue problem can be restricted to a single unit cell of the reciprocal lattice (i.e. the first Brillouin zone, Section 1.2.2): if the problem is defined on a finite volume, then a discrete spectrum of eigenvalue is expected. In particular, an infinite set of modes is associated to a single value of $k$. The modes, discretely spaced in frequency, are collected in the same “family” with name $\omega_n(k)$, where index $n$ indicate the band number in increasing order. The $\omega_n(k)$ functions carry all the information about the band structure of the crystal.

Finally, it is possible to consider all the other symmetry properties of the PhC geometry: rotation, mirror reflection, inversions. In these cases, the band structure itself presents the same properties. The analysis procedure exploits them, by reducing the investigation of the first Brillouin zone, to the irreducible Brillouin zone only. In addition, mirror reflection symmetry in a photonic crystal deserves special attention because under certain conditions it allows to separate the eigenvalue equation (5.1) into two separate equations, one for each held polarization. In particular, in one case $H_k$ is perpendicular to the mirror plane and $E_k$ is parallel; while in the other case, $H_k$, is in the plane and $E_k$ is perpendicular. These simplifications are convenient, because they provide immediate information about the mode symmetries and also facilitate the numerical calculation of their frequencies [9].
5.1.2 The Photonic Band Gap

The main feature of a photonic crystal is its periodic band diagram and the existence and width of one or more band gaps. Even a very simple structure, like the one-dimensional crystal in Fig. 5.2, shows the basic concept applicable to general two- or three-dimensional PhC.

![Fig. 5.2 – Example of one-dimensional PhC: the dielectric is periodic along x-direction. It consists of alternating layers of materials with different dielectric constant, spaced by a fixed distance a.](image)

As already mentioned, the electromagnetic modes supported by the crystal can be described by means of the symmetry properties. The material of the PhC in Fig. 5.2 is periodic in the x-direction and homogenous in the yz-plane, so that the modes can be indexed by \( k_y \), \( k_x \) and \( n \), i.e. the wave vector in the plane, the wave vector component in the x-direction and the band number. In fact, the crystal has continuous translational symmetry in the yz-plane, and \( k_y \) can assume any value. Then, because of discrete translational symmetry in the x-direction, \( k_x \) can be restricted to the first one-dimensional Brillouin zone, which is defined by \(-\pi/a < k_x < \pi/a\) (see section 1.2.2, with \( r=\alpha x \) and reciprocal vector \( R=(2\pi/a)k_x \)). The Bloch modes, following the expression (5.3), result

\[
H_{n,k_x,k_y} = e^{jk_y(y+a)} e^{jk_x x} u_{n,k_x,k_y}.
\]  

(5.4)

In order to introduce photonic band-gap effect, it is possible to consider light propagating entirely in the x-direction, crossing the dielectric layers with normal incidence. In this way, \( k_y \) results null and the analysis simplifies to the evaluation of \( k_x \) (hereafter called \( k \)).

![Fig. 5.3 – Band diagram of the one-dimensional photonic crystal in Fig. 5.2: (a) band folding effect for the case of homogeneous medium (\( \varepsilon_1 = \varepsilon_2 \)); (b) open band-gap for a small dielectric contrast.](image)

In Fig. 5.3 the band diagram for two different composition of the multilayer film in Fig. 5.2 is shown. First of all, a complete homogenous medium is considered, where \( \varepsilon_1 = \varepsilon_2 = \varepsilon \) and \( a \) becomes an artificially periodicity. It is well known that in a uniform dielectric medium the propagation of the electromagnetic field is governed by the index of refraction [9]. Thus, the frequency spectrum is just the light line.
\[ \omega(k) = \frac{ck}{\sqrt{\epsilon}} \]  

(5.5)

where \( c \) is the speed of light. At the edge of the Brillouin zone, the light line is “folded” back, because of the periodicity (Fig. 5.3a).

The plot in Fig. 5.3b refers to a nearly uniform medium, with alternate layers of small dielectric contrast. Since the structure is no longer homogeneous, a gap in frequency between the lower and upper folded lines appears. In this frequency gap no mode, for any \( k \), can propagate in the crystal. As the dielectric contrast increases, the photonic band-gap opens widely. The reason of the gap opening comes from the variational theorem, from which we know that low-frequency mode tends to concentrate its displacement of energy in high-dielectric permittivity regions, with respect to the mode immediately above in frequency. So the frequency of the high mode is raised a bit and the band-gap appears. In one-dimensional PhC a gap occurs between every set of bands, at the edge of the Brillouin zone or at its center. It opens for any dielectric contrast, but the smaller the contrast, the smaller the gaps. Moreover, the width of the gap depends also on the thickness of the dielectric layers.

The presence of band-gaps in PhC is of paramount importance, since almost all of the practical applications deal with the location of band gaps and their width. In particular, it is possible to localize modes at frequency inside the gap, by introducing defects to the periodic pattern. In fact, in the band-gap, only evanescent modes exist, with exponential amplitude decay. They are solutions of the eigenvalue problem, but they do not satisfy the translational symmetry boundary condition of the crystal, so it is impossible to excite them in a perfect infinite crystal. However, a defect or an edge in a photonic crystal might either excite or support the propagation of such a mode. This is the basic concept of PhC waveguides, where the periodicity is broken in order to allow and confine propagation of light inside the band-gap.

### 5.2 Waveguides in 2D Photonic Crystals

Two dimensional photonic crystals constituted of periodic lattices in \( x \)-and \( y \)-directions present mirror reflection symmetry with respect to the \( xy \)-plane. PhC slabs, defined on a dielectric membrane of finite thickness in the \( xy \)-plane, still present reflection symmetry with respect to the plane itself (Fig. 5.4), and their modes can be classified into two different groups:

1. Transverse Electric modes, TE-like, also termed \( \sigma_{xy} = +1 \) modes, where \( \mathbf{H} \) is normal to the plane (\( H_z \)), whereas \( \mathbf{E} \) is parallel to it (\( E_x \) and \( E_y \));

2. Transverse Magnetic modes, TM-like, also termed \( \sigma_{xy} = -1 \) modes, where \( \mathbf{E} \) is normal to the plane (\( E_z \)), whereas \( \mathbf{H} \) is parallel to it (\( H_x \) and \( H_y \)).

The band structures for TE and TM modes are usually completely different: in general, there can be band-gaps for one type of modes and not for the other. There are various possible configurations of planar crystals; however, only one kind of arrangement guarantee a complete bang-gap in the \( xy \)-plane, that is the triangular lattice of air holes in a dielectric substrate with high electric permittivity. In particular, a detailed study of these indefinite two-dimensional PhC (i.e. indefinite in the \( z \) direction) proved that TE and TM band-gaps overlap in some regions, for particular values of the \( r/a \) ratio, where \( r \) is the radius of the air holes, and \( a \) is the lattice constants [9]. Then, when PhC of finite extent are considered for practical applications, the band-gap is complete only for TE modes [102].
The radius of the holes, the lattice constant, the slab thickness and the dielectric material play the main role in determining the band structure of the crystal. Usually, Silicon or GaAs membrane are used, and all the geometrical dimensions are then normalized with respect to the lattice constant $a$. The relationship in the definition of the band diagram can be summarized as follows:

- The normalized slab thickness $d/a$ can shift the bands of the modes: the thicker the slab, the lower the frequency of the modes. For a small value of $d/a$ the slab is monomode over a wider frequency region (i.e. it presents a larger gap) and second-order modes cannot contribute to losses, by coupling with cavities or linear defects if present. So, for the dielectric slab, small values ($d/a < 0.6$) are more favorable.

- The band gap is a function of the slab thickness. A large slab thickness corresponds to little energy barrier to create higher-order modes, while, on the other hand, a small thickness does not provide a good light confinement in the plane. Considering a trade-off, thickness $d/a$ below 0.6 are most often employed.

- The variation of the radius of the holes yields a frequency shift of the band structure, when the lattice constant is fixed: if $r/a$ increases, the bands move towards high frequencies (i.e. blue shift).

After the choice of the pattern and of the proper slab, the design of a PhC waveguide in a two-dimensional slab starts by breaking the periodicity of the lattice and introduce linear defects that can guide the light. In Fig. 5.4, for example, a row of air holes is removed: this configuration is commonly named W1.0, because the width of its channel, $w$, is equal to $1.0 \times w_0 = \sqrt{3}a$. In PhC slabs (defined in the $xy$-plane), the TE-like mode previously defined are of major interest, because of the complete band-gap. Their classification can be completed by considering also the mirror symmetry with respect to the $xz$-plane, which is the plane bisecting the channel (Fig. 5.4):

1. even TE-like modes, $\sigma_{kz} = +1$, which present only the components that is parallel to the $xz$-plane, that are $H_z$ and $E_y$;
2. odd TE-like modes, $\sigma_{kz} = -1$, which present only the component normal to the $xz$-plane, that is $E_x$.

As an example, Fig. 5.5b shows the band diagram of the waveguide in Fig. 5.4. The three lower modes are bulky or substrate modes. The rest of the reported frequency range belongs to a band-gap region for TE-like modes. It is possible to observe that between 0.81 and 0.86 eV (i.e. 195.86 GHz - 209.16 THz) a single mode with symmetry $\sigma_{kz} = -1$ can propagate in the slab, along $x$-direction [99]. When the frequency exceeds 0.865 eV, the mode crosses the light line; therefore, it cannot be guided by the step-index in the vertical direction anymore, and it will become a radiative
mode. This waveguide 1.0 will be considered in the next section for the design of couplers in PhC technology.

![Manufactured sample and band diagram](image)

**Fig. 5.5** – Manufactured sample (a) and band diagram (b) of the PhC waveguide in Fig. 5.4, where \( a = 0.4 \) \( \mu \)m, \( r/a = 0.335 \), and \( d = 0.23 \) \( \mu \)m. The membrane has a dielectric permittivity of 12 (Si). The propagation along \( x \)-direction is considered. The energy is expressed in eV, where 1eV = 241.796 THz. The band diagram shows a comparison between simulated (continued and dashed lines) and measured (black and white spots) values. The dotted line represents the light line (5.5). The yellow-shaded region represents the single-mode frequency band [99].

### 5.3 Numerical Modeling

Two different numerical methods have been adopted for the analysis of PhC slabs throughout this work. The first one is the commercial software HFSS: it consists in a very common electromagnetic simulator based on the finite element method. It is mainly used in the microwave and mm-wave range, but it can be applied also at higher frequencies, thanks to its general approach that starts from Maxwell equation. The only limitation regards computation time and memory allocation when the dimensions of the structure to be characterized are electrically large. The second method is called GME, Guided-Mode Expansion. It has been developed by the Department of Physics of the University of Pavia. It is ad hoc devoted to the analysis of two-dimensional photonic crystals, which may also include line defects. It proved to be very fast and accurate, and it has been considered as a reference to “calibrate” HFSS results.

#### 5.3.1 Guided-Mode Expansion method: a Dedicated Approach

The guided-mode expansion method considers the Maxwell equations in a PhC slab by expanding the magnetic field into the basis of guided modes of an effective homogeneous dielectric waveguide, and then by numerically solving the eigenvalue equation. It can also compute out-of-plane diffraction losses with a perturbation theory [57].
Basically, the electromagnetic field is represented by a standard combination of two-dimensional plane waves in $xy$, in conjunction with guided modes along the $z$ direction. The set of modes does not include radiative modes of the guide; therefore the GME is an approximate method. However, it can account for out-of-plane scattering, because when a photonic mode in the PhC slab falls above the light line it is coupled to leaky modes of the slab by the dielectric modulation and it becomes quasi-guided, i.e. it is subject to intrinsic losses. Moreover, line and point defects can be treated by introducing a super-cell in one or two directions, respectively.

The two parameters that govern the convergence properties of the method are the number of plane waves and guided modes retained in the construction on the eigenvalue problem. Then, the overall dimension of the numerical problem is equal to the product of these two values. For PhC slabs defined on high-index membrane, a number of guided modes equal to 4 for each kind of mirror symmetries should be enough. Concerning the number of plane waves, about a hundred of terms are usually included in the calculation. In particular, the code provides some possible choices, which already account for the symmetries of the structure, in order to avoid non-physical splitting of the modes at the edges of the Brillouin zone. From the user’s point of view, with GME everything is ready for the analysis of two-dimensional PhC slabs. Band diagrams are the main output of the method (for example, a comparison with literature results is reported in Fig. 5.6), in addition with calculation of losses.

![Band diagram of a PhC slab](image)

**Fig. 5.6** – Band diagram of a PhC slab in air, defined by a triangular lattice with $\varepsilon=12$, $r/a=0.45$, and $d/a=0.6$: (a) numerical method developed Johnson et al., at MIT [103]; (b) GME method.

### 5.3.2 HFFS: The General Finite Element Approach

The main purpose of HFSS is the calculation of the modal scattering parameters of microwave and mm-wave structures, such as microstrips, waveguides, transmission lines, patch antennas, etc. However, it can also address eigenvalue problems, when no sources are defined at the ports of the devices. In this case, eigenmodes, or resonances, of a structure are computed, in terms of both frequency values and field distribution. Therefore, the characterization of a PhC slab can be
accomplished by an eigenmode analysis of the unit cell, and then by a full-wave simulation of the whole PhC structure, with the definition of the input and output transmission lines, usually constituted by classical dielectric waveguide. Please note that this kind of analysis cannot be undertaken by the GME method.

While the study of the whole structure represents a classic problem to be solved by HFSS, in the contest of a FEM simulation, eigenmode analysis of the PhC cell requires special care in the definition of the boundary condition. In the case of triangular lattice, both the geometric and the reciprocal unit cells are hexagonal (Fig. 1.11). HFSS model needs the drawing of the geometric model, then the scanning of the irreducible Brillouin zone for the calculation of the band diagram. In Fig. 5.7 an example of unit cell modelling is shown. The hexagon in the membrane is enclosed by two volumes that simulate the air above and below the substrate. These two volumes are terminated by PML boundary conditions, which absorb the incident field and simulate an open space (see section 1.2.3). The height of the air box strongly influences the final results. In fact, if the boxes are too small, HFSS can mix resonances inside and outside the dielectric substrate, proving spurious values, or it can absorb the physical resonating modes, making them undetectable. On the contrary, if the boxes are too high, the number of tetrahedra, and therefore the number of unknowns, grows and the computing time becomes unfeasible. During the study, comparisons with GME band diagrams have allowed to set a proper height of the air boxes, equal to about four times the slab thickness $d$.

In order to compute the resonance frequencies of the PhC modes, periodic boundaries on the side walls of the domains must be defined. These boundaries permit to scan the border of the irreducible Brillouin zone (as indicated in Fig. 5.8), for the determination of the band diagram. HFSS implements special conditions to be applied to this purpose on each opposite side walls: they
are called master and slave. In particular, on the slave boundary a phase delay referred to its master is forced: it will simulate a propagating wave in the $xy$-plane, and it can accomplish the scanning of the irreducible Brillouin zone (for further details about the periodic boundary definition, see [100]). Thanks to the simultaneous calculation of the field distribution, it is then possible to recognize TE and TM resonant modes.

Fig. 5.8 – Triangular lattice: (a) geometric unit cell; (b) scanning of the irreducible Brillouin zone in the reciprocal domain.

Fig. 5.9 shows a comparison of HFSS and GEM results. The agreement is good, but a constant shift between the two curves is observed: this discrepancy should be related to the mesh representation in the finite element method. The computation time with HFSS is in the order of hours on a standard PC, versus the few minutes that GME needs to compute hundreds of band modes. However, these studies permit to configure HFSS for the analysis of PhC slabs: its reliability can be used for the design of circuit component at optical frequencies, like the coupler presented in the next section.

Fig. 5.9 – Comparison of band diagram calculation with GME and HFSS of a triangular lattice PhC slab, with $a=0.42 \, \mu m$, $d=0.2 \, \mu m$, $r/a=0.4$, and $\varepsilon=12.11$. 
5.4 **DESIGN OF A TRAVELLING-WAVE PHC COUPLER**

Among the proposed coupler designs in PhC technology, cavity channel drop and single resonator couplers present disadvantages referred to fabrication and performances, because of strictly positioning requirements and high losses due to resonance operating mode [104]-[105]. An ideal device would not be so sensitive to fabrication parameters, and it should be modular and tunable over a range of frequencies. With regards to these characteristics, a new kind of coupler based on the properties of the PhC band-gap has been investigated.

The key feature of this device is therefore the exploitation of the relationship between the radius of the lattice and the frequency of the band-gap. In particular, it is possible to locally modify the periodic pattern of the slab in order to couple the guided mode of two separate waveguides, which represent two separate channels for the light propagation. In Fig. 5.10 a possible layout of the device is shown. The modified region, highlighted in green, is characterized by larger holes: since the radius increases, the band-gap frequency also increases; therefore, the mode travelling in one channel can penetrate the periodic pattern and reach the other channel. This is the reason why these couplers are usually termed *travelling-wave* coupler. Apart from the computation of band diagrams for the study of the proper hole dimension, all the design have been performed by using HFSS.

![Fig. 5.10 – Example of travelling-wave coupler implemented in HFSS: the highlighted region can couple the light between upper (port1-port2) and lower (port4-port3) channels.](image)

Waveguides W1.0 have been chosen, because they provide sufficiently low propagation losses and wide mono-modal band for an odd TE-like mode [99]. A self-standing membrane of Silicon, with \(a=0.42 \ \mu m\), \(d=0.2 \ \mu m\) and \(\varepsilon=12.11\), has been considered: from the calculation of the dispersion diagram, the working frequency has been set equal to \(0.288c/a\), or equivalently about 206 THz, which lies in the single-mode range.

The way of operation of the device seems extremely simple; indeed, the determination of the size of the coupling region and of the new value of the hole radius is not a trivial task. The main requirements for a travelling-wave coupler are:
1. the odd TE-like that propagates in the waveguide must couple with the mode that travels in the coupling region;

2. a coherent matching of the wavevectors in the waveguides and in the modified lattice is needed; basically, the most suitable wavevector for the coupling region to this aim would be the one with the highest \( k_x \) value, in order to have the maximum deviation of the propagating field in the \( x \)-direction.

Finally, the dimension of the coupling area must be evaluated. The basic idea is the use of the smallest possible size that does not modify the operation of the device, since a small device presents low losses and of course because it is compact. After comparisons of different HFSS analyses, the final optimal structure has the following characteristics: 5 rows of holes between upper and lower waveguides and 7 rows of holes of modified lattice along \( x \)-direction, to create the coupling region, as shown in Fig. 5.10 (input and output PhC waveguides will be longer in the real device; in the HFSS model they have been truncated in order to minimize the volume under investigation).

After performing an HFSS optimization, the radius of the holes has been set to the ultimate value of 0.158 \( \mu \)m. The performances of the travelling-wave coupler are reported in Fig. 5.11, in terms of scattering parameters. At the working frequency of 206.3 THz \((0.2888c/a)\), the transmission on the upper channel, \( S_{21} \), is less than -20dB, while the coupling between port 1 and port 3 (Fig. 5.10) is about -3dB; therefore, a half of the input power at port 1 moves to the lower channel. Furthermore, almost no travelling field reaches port 4, which is good isolated. This means that some losses occur during the coupling of the two channels, especially because the field tends to penetrate the periodic lattice (losses reduction is still a challenge topic in the design of PhC, as previously mentioned). However, the overall behavior of the coupler at its working frequency is very interesting, as it is clearly shown in Fig. 5.12, where the electric field on the surface of the dielectric is reported.

![Fig. 5.11 – Travelling wave coupler in PhC technology (Fig. 5.10), with a modified radius of 0.158 \( \mu \)m: scattering parameters.](image)
Fig. 5.12 – Travelling wave coupler in PhC technology (Fig. 5.10), with a modified radius of 0.158 μm: amplitude of the electric field on the surface of the substrate, at the working frequency of 206.3 THz.

5.5 FINAL REMARKS AND CONCLUSIONS

The presented work on two-dimensional photonic crystal, developed in collaboration with the Department of Physics of the University of Pavia, has permitted a fascinating integration between periodic structures at microwaves and at optical frequencies. In fact, both analysis issues and challenging design studies have been addressed.

In particular, the combined use of a generic electromagnetic simulator and an accurate numerical technique, devoted to the characterization of PhC slabs, has proved to be a key feature for the modeling of state-of-the-art components for optical communications. The possibility to investigate in detail the band diagrams of the modes as well as the overall performance of the complete device (with also PhC-to-dielectric waveguide transition) is very important in order to produce consistent designs. In conclusion, a new example of optical application of classic microwave circuits has been provided.
CHAPTER 6

Study on Aperiodic Phased Arrays for Space Applications

At present, most of space object information is collected and catalogued by USA (NORAD catalogue) and Russia. There is currently no dedicated European system able to catalog in-flight objects. In order to gain this information for the safety of its spacecrafts and astronauts, the European Space Agency (ESA) has launched in 2008 a program called Space Situational Awareness (SSA). One component of this program is the development of an autonomous system for the classification and the detecting of objects in space. The whole system will consist of a phased array radar and a network of optical telescopes. Due to the involved distances and technologies, the optical part will survey the Geostationary Earth Orbit (GEO) and the Medium Earth Orbit (MEO), whereas the radar will classify objects in Low Earth Orbit (LEO) [106].

The main purpose of LEO surveillance is the detection of objects with a diameter larger than 2 cm, which are orbiting between 200 and 2000 km above the Earth. For this reason, the receiver antenna of the new radar (either if the radar will operate in bistatic or monostatic mode) should be able to point with enough accuracy to the required directions in space, and it should be able to provide the necessary enhanced resolution with a very directive main beam. Moreover, it would need to observe and scan a wide region in LEO. These requirements will be fulfilled by phased array technology, which has proved to be well suited for survey observations. Among different configurations, aperiodic arrays are alternative solutions for the antenna geometry, where the break of periodicity offers incremental degrees of freedom throughout the design.

The topic of aperiodic arrays has recently gained renewed interest in the field of satellite applications. Above all, the Electromagnetics Division of ESA-ESTEC is currently working on aperiodic phased arrays for multiple beam antennas in high data rate, multimedia and mobile personal communications [106]-[110]. These arrays are designed for the transmitter (direct radiating arrays), and the number of single antenna elements results smaller than in the possible SSA radar (hundreds instead of thousands of elements). Nevertheless, the presented theoretical issues and design methodologies can be related to both satellite and space surveillance phased arrays.

In the chapter, the space surveillance program of the European Space Agency is described, then features and capabilities of aperiodic arrays are introduced, with a brief review of published works. Then a synthesis procedure is outlined, with a discussion about advantages and disadvantages of the considered layouts and methods. Finally, the most relevant results achieved throughout the study are reported and examined.
6.1 ESA’s Space Surveillance Program

In almost 50 years of space activities, since the Russian Sputnik in 1957, more than 4800 launches have placed some 6000 satellites into orbit, of which only a minor fraction - about 800 - are still operational today [111]. Besides this large amount of intact space hardware, several additional objects are known to orbit the Earth. Fig. 6.1 shows the evolution of the number of catalogued objects in orbit, from the regular data collected by the US Space Surveillance Network, until October 2008. When these objects cannot be considered as working devices, they are commonly termed space debris. In particular, the 38% of objects in space is attributed to decommissioned satellite, and 56% of them are due to in-orbit fragmentations (more than 200 have already been recorded since 1961).

Regarding the size of the objects, a current estimate expects about 20000 objects with a dimension larger than 10 cm, about 600000 objects larger than 1 cm and more than 3 millions smaller than 1 cm. The problem is that any of these fragments can cause harm to an operational spacecraft, where a collision with a 10-cm object would entail a catastrophic fragmentation, a 1-cm object will most likely disable a spacecraft and penetrate the ISS shields, and a 1-mm object could destroy sub-systems on board. Fortunately, the Earth’s atmosphere causes air drag that extracts orbital energy and leads to a contraction and final re-entry of a space object. Upper layers of the atmosphere are supported by lower layers, which are compressed under the weight of the air column above them. The air density increases, and hence the increase in air drag with decreasing altitude is progressive. After sufficient exposure to air drag the orbit decays, and the object re-enters into the denser Earth atmosphere, where the air drag converts orbital energy into heat. This heating process is normally sufficient to destroy an object. Therefore, only larger-size spacecraft or rocket bodies, approximately 20 to 40 percent of the mass, particularly high-melting steel or titanium alloys, may survive to ground impact.

![Catalogued Objects in Orbit as of October 2008](image)

Fig. 6.1 – Evolution of the number of debris objects tracked by the US Space Surveillance Network [111].
At any given altitude shell, debris generation-processes by normal launch operations, break-ups and other release events are counter-acted by natural cleansing mechanisms, such as air drag and luni-solar attraction. The result of these balancing effects is an altitude-and-latitude-dependent concentration (spatial density) of space debris objects. Maximum debris concentrations can be noted at altitudes of 800 to 1000 km, and near 1400 km, in LEO orbit, where spacecraft for Earth observation operate. Spatial densities in GEO (heavily used for telecommunications) and near the orbits of navigation satellite constellations are smaller by two to three orders of magnitude. Fig. 6.2 and Fig. 6.3 show artist’s impressions of the orbiting debris in LEO and GEO respectively [111].

Fig. 6.2 – ESA: debris objects in low-Earth orbit, which extends to 2000 km above the Earth’s surface (70% of all catalogued objects are in LEO) [111].

At present, NASA Orbital Debris Program Office, with US Space Surveillance Network, plays an international leading role in conducting measurements of the space environment. This means that, for example, databases of satellite orbits and debris objects are not fully disclosed to ESA due to the sensitivity of the information. The purpose of the European Space Situational Awareness (SSA) program, started in the last years, is to provide complete and accurate information on objects orbiting Earth. Moreover, the improvement of the quality of the available space surveillance data is addressed. The SSA system proposed by ESA will operate in two specific areas. The first is surveillance of objects orbiting the Earth in various orbits, to be achieved by detecting, tracking and imaging these objects. The second one regards space weather, addressing primarily the effect of solar activity on satellites and ground infrastructure. Due to the involved distances and technologies, in the new space surveillance network ground-based telescopes will detect GEO debris down to 10 cm in size, whereas ground-based radars are mainly suited for LEO observation, down to about 2 cm of debris size [106].
6.2 APERIODIC ARRAYS

Arrays with uniformly spaced elements have been widely studied and designed, due to the available analytic theory and the reliable simple technology in the fabrication process.

In 1960s the first papers covering the aperiodic arrays subject were published. The non uniform element spacing breaks the periodicity of the array layout and of the radiation pattern. It offers potential advantages with respect to periodic arrays [107]. Their most interesting characteristics are beamwidth narrowing and high resolution with relatively few elements, in conjunction with element interaction reduction and scan angle capability. For example, aperiodic arrays find applications in phased arrays for communication satellites, ground-based high-frequency radars, and interferometer arrays for radio astronomy. In the last years, the scientific research in this field is mainly focused on optimization methods for the array design and on new geometries for the antenna layout. However, due to the difficult of the subject, the synthesis of aperiodic arrays remains today an open issue.

6.2.1 Definition and Main Features

Aperiodic arrays offer to a designer more degrees of freedom than periodic arrays. Each element in an equally-spaced array has one degree of freedom: its complex amplitude. By taking the general case of arbitrarily distributed elements, the position of the elements represents the incremental degree of freedom. Therefore, even though there might be restrictions on the layout distribution, aperiodic arrays need, in most of the cases, fewer elements in order to achieve the required performance [3].
The synthesis of aperiodic layouts follows two different approaches:

1) **thinned arrays**: the initially uniform array antenna is thinned out, by switching off some elements (Fig. 1);

2) **sparse arrays**: the elements are placed in randomized locations, considering constraints on the minimum and maximum spacing (Fig. 2).

The main difference between the two approaches consists in the presence or the absence of a background periodic lattice. Totally random layout prevents any coherent interference in the side-lobe region; therefore it can suppress all grating lobes. However, it requires high computational costs in the synthesis process and specific manufacturing.

![Example of thinned array: geometric layout.](image)

Both thinned and sparse arrays present several interesting features [107]:

- the number of elements, and consequently the total costs, can be reduced without greatly broadening the beamwidth;

- SideLobe Level (SLL) may be increased to any desired value, whereas equally spaced linear and rectangular arrays with uniform excitation provide a maximum of 13.4 dB;

- possibility to avoid grating lobes, even if the average spacing between the elements is high in terms of wavelength, thanks to the aperiodic layout; moreover, this can improve the bandwidth of the overall antenna and its scanning capabilities;

- as the involved distances grow, the mutual coupling between the elements is weaker;

- reconfigurable antenna or cold redundancy options, because of the possibility of changing the elements to be switched on and off (in this case, the switched-off elements are connected to a matched load);

- spatial density tapering replaces amplitude tapering: uniform amplitude excitation leads to a simplification of the beam forming network, maximizing the power efficiency.
The limitations of aperiodic arrays are mostly related to the reduction in the number of elements and to the non-uniform layout:

- when the elements are not equally spaced, the aperture efficiency of the antenna decreases;
- the degrees of freedom in the control of the radiation pattern depend on the number of active elements;
- the maximum directivity is also proportional to the number of elements: it could be necessary to increase the power radiated by the active chain in order to achieve the required value;
- in general it is not possible to design the optimal pattern by analytical procedures, because the array factor becomes a nonlinear function of element spacing: analysis and synthesis of aperiodic array can be complex, costly and time-consuming;
- during the manufacturing process, the use of generic building blocks might not be possible.

Thinned and sparse arrays result very well suitable for both receiving and transmitting phased arrays, when, for example, narrow scanned beam and power efficiency of the feeding networks are the strictest requirements. In particular, aperiodic layouts have recently gained renewed interest especially for satellite applications.

6.2.1 **Analysis and Synthesis: an Overview**

The first works on aperiodic arrays, in the 60s and 70s, introduce the new subject with theoretical studies, comparisons and also numerical techniques based on deterministic and simple statistical approaches. While these former studies are mainly focused on particular configurations, in the last years procedures based on global optimization methods and especially on synthesis simplification become the dominant design choice [107].

Linear arrays with arbitrarily distributed elements have been introduced about 50 years ago by Unzip [113], who delineated a matrix relationship between the spatial distribution and the complex excitation of the elements and the far-field pattern of the array, highlighting the importance of the incremental degrees of freedom, i.e. the element positions, throughout the design.

In the following years, several studies have been carried out, with the aim of both outlining a general theory and developing synthesis methods. For example, in 1961 Harrington proposed one of the first procedures for the generation of linear sparse array by a perturbative approach, in order to
reduce the sidelobe level of uniform arrays [114]. He computed the fractional changes from the regular spacing via the formula for Fourier coefficients, with an argument that takes account for the desired radiation pattern; the method is simple, fast, and does not include simultaneous equations to be solved, but it is restricted to small variations from the uniform layout. In 1962 Ishimaru suggested an alternative analytical formulation, based on the Poisson's sum formula: the unequally-spaced array is transformed into an equivalent continuous source distribution [115]. He worked on sidelobe reduction and suppression of the secondary beam, presenting configurations with a main-lobe region very close to the desired pattern. Another group of papers introduced numerous techniques for matching a spatial taper to an amplitude one, for the reason that the standard method for designing directive antennas to achieve low sidelobes is to taper the amplitude of the aperture illumination. For example, in 1960 Sandler analyzed the equivalence between the amplitude taper and the variation of the individual element distances from the array centre, through the concept of spatial frequency [116]. Another interesting work on this topic appeared in 1964, when Skolnik described a statistical method to thin the equally-spaced array: the desired amplitude excitation corresponds to the probability density function, which is used for determining whether or not the element on the uniform grid should be switched on [117].

Some years later, critical reviews of the first works on aperiodic arrays and statistical comparisons of different array configurations have been published. For instance, in 1966 Lo and Lee approached the problem of the optimum solution [118]. They stated that there is no essential difference among many spacing functions of random arrays, where usually the positions of outer elements are more sensitive. Moreover, they showed that even though aperiodic arrays can yield very low sidelobes, there is almost no possibility to obtain by trial the optimum array, i.e. the array with the lowest sidelobe level.

The importance of all these studies consists in the definition of aperiodic array capabilities: although most of the examples present only one-dimensional arrays, various interesting features of unequally spaced arrays have been discussed. The studies have also made clear that comprehensive analytical theories are very difficult to provide as well as fast and general synthesis procedures. Given that, more recently several authors have been working on purely numerical or probabilistic approaches and, on the other side, on combinatorial solutions, for example alternative layouts which could simplify the optimization steps. In particular, in 1994 Haupt proposed his pioneering research on thinned array synthesis with Genetic Algorithm [119]. In the standard implementation, genetic algorithm deals with binary data, it has no theoretical limit to the number of variables and it is a global optimization method, so it can escape local minima thanks to the random components. Since the thinning problem is intrinsically binary and offers a vast number of possible outputs, genetic algorithm results well suited for their synthesis. Simulated Annealing is an alternative optimization method which shares with genetic algorithm a probabilistic nature; it is based on the behavior of the molecules of a pure substance during the slow cooling in the formation of a perfect crystal, which represents a minimum energy state. Simulation annealing can work in parallel with the number of elements and their position, with the spatial aperture of the array and eventually with weight coefficients, via the definition of an ad hoc cost function [120]. More recently, Ant Colony optimization, based on the behavior of ant colonies in obtaining food and carrying it back to the nest, and Particle Swarm optimization, derived from the movements and intelligence of swarms, have been proposed to deal with aperiodic array synthesis [121], [122]. All these methods perform a population-based search with probabilistic transition rules; they offer global solution and can handle numerous variables, but as a result they might suffer of slow convergence rate and high computational costs.

Alternative approaches are at the present the challenge in the field of aperiodic arrays; alternative means both specialized synthesis methods and innovative layouts. For example: isophoric...
arrays [123], where the element placement based on difference sets (a topic of combinatorial mathematics) forces uniformly weighted spatial coverage; sunflower antenna array [109], with a spatial density tapering derived from the typical locations of the sunflower seeds; sparse arrays based on the combination of random subarrays [112]. From the synthesis point of view, hybrid approaches based on genetic algorithm have been proposed [124], but also specific procedures which start form the one-dimensional design and permit to generate a planar layout [110],[125],[126].

6.3 **EXAMPLE OF THINNING PROCEDURE THROUGH NEAR FIELD COMPUTATION**

From a general point of view, synthesis is a crucial phase of a project, because it aims to provide the ultimate object or device, by starting from the desired final performances. In the case of arrays, the requirements are usually referred to the radiation pattern and, without any acceptable solution known a priori, the outputs of the procedure regard the geometry and the excitation of the antenna. Moreover, when studying aperiodic arrays for space applications, the large number of elements and the degree of freedom of the position increase the complexity of the synthesis.

6.3.1 **Outline of the Method**

The considered method is a modeling procedure devoted to the synthesis of an equivalent aperture, starting from a required far field distribution given on a hemispherical region, through the computation of the near field. In fact, the output of the method consists in the near field distribution all across the aperture in either the tangential components of the electric field, $E_x$ and $E_y$, or the tangential components of the magnetic field, $H_x$ and $H_y$. The equivalent tangential field on the aperture is able to radiate the required pattern, thanks to the field equivalence principle which relates also the near and far field of an aperture antenna.

In particular, the starting aperture must be defined: this represents a planar uniform array, which will be thinned out at the end of the procedure. Once the near field distribution able to radiate the required far field pattern over the starting aperture is known, the method can suggest a possible thinned layout. In fact, we can then place the isotropic point sources (in general, the elements) of the thinned array where the equivalent field is stronger, because in that position also the source contribution to the far field pattern is stronger.

The simulations have been performed with the Constrained Aperture Method of the synthetic tool of ADF [46]. The basic input data of the method are:

a) requirements, in terms of far field or radiation pattern, that will be indicated as a mask in either the $(\theta, \phi)$ space;

b) geometry of the aperture: minimum and maximum dimensions of the aperture and sample grid for the calculation of the equivalent field (throughout the work, all the dimensions are normalized with respect to the wavelength at the frequency of interest).

The sample grid indicates the positions in the aperture where it is possible to establish a relationship between the sources and the far field: the positions where we can place the elements. Therefore, the sample grid coincides with the layout of the uniform array that has to be thinned. After the near field distribution on the aperture has been computed, the layout of the thinned array, with some post-processing of the near field data.
6.3.2 Thinning and Threshold Considerations

A two-dimensional array in the $xy$ plane has been considered. The radiation pattern of a possible phased array antenna for the SSA radar shall be characterized by a narrow beamwidth, with a nominal broadside direction (pointing angles: $\theta_0 = 0^\circ$ and $\phi_0 = 0^\circ$), and relative low sidelobes. In terms of normalized array factor, the requirements are:

- SideLobe Level: $SLL < -20\text{dB}$
- BeamWidth: $BW_{3\text{dB}} = 1^\circ$

as shown in Fig. 6.4, for the cut $\phi=0^\circ$; such requirements are extended by symmetry to all other angles $\phi$.

![Pattern Requirement](image)

**Fig. 6.4** – Far field requirement: maximum value of the normalized array factor in the cut $\phi=0^\circ$.

![Near Field Distribution on the Aperture](image)

**Fig. 6.5** – Normalized tangential electric field on the equivalent aperture [dB].
Two hypotheses have been made during all the study: the single elements are modeled as isotropic point sources, and only uniform amplitude is applied, in order to maximize the efficiency of the power input network.

A uniform square array defines the initial layout. Since the dimensions of the aperture antenna are related to both the beamwidth and the directivity, the starting size of the array results equal to $51\lambda \times 51\lambda$ [127]. The considered inter-element spacing of the background lattice is $0.5\lambda$, for a maximum amount of 10609 elements. During this study, all the elements of the array are modeled as isotropic point sources. In Fig. 6.5, the computed near field distribution is shown: the total tangential electric field is expressed in dB, with a normalized scale, indicated by the colorbar on the right. As expected, the field is stronger at the centre of the aperture and it decreases when moving towards the edge.

In order to derive the thinned array layout, the field level, i.e. the threshold, which will represent the threshold for the source positioning must be chosen: where the near field is less that the threshold, the elements are switched off. This procedure is not deterministic: every project is characterized by different near field values (even though they are normalized) and the threshold has to be chosen by trials, with respect to the related performances of the array. For instance, Fig. 6.6, Fig. 6.7, and Fig. 6.8 show the thinned array layout and the corresponding normalized array factor when changing the threshold value from -70dB to -50dB. In the last two configurations, the sidelobe level satisfies the requirements: the example in Fig. 6.6 is very similar to a uniform array, with high sidelobes, but when the number of elements on the edges decreases, also the sidelobe level decreases, as expected. On the contrary, the harder the thinning, the broader the main beam: this behavior is due to the reduction in the overall dimension of the antenna. In fact, for the third example (Fig. 6.8), with less than a third of the elements of the initial uniform array, and consequently a significant potential diminution of the costs, the beamwidth value reaches 1.71°, instead of the requested 1°.

The second layout, which represents a kind of trade-off between beamwidth and sidelobes, has been further investigated. It is constituted by 6133 elements: this corresponds to 42% of removal from the initial square array. Since the scanning capability is another important feature of the aperiodic phased array, the following scanning range has been considered:

![Layout of the Thinned Array](image)

![Normalized Array Factor: $\varphi=0^\circ$ cut](image)

**Fig. 6.6 –** Example of synthesized thinned array: where the near field is less than -70dB, the elements are switched off. (a) array layout: 9769 elements, 8% of removal (in cyan color the switched-on elements); (b) normalized array factor in the cut $\varphi = 0^\circ$ (beamwidth: 1.167°; first sidelobe level: -13.98dB).
Fig. 6.7 – Example of synthesized thinned array: where the near field is less than -55dB, the elements are switched off. (a) array layout: 6133 elements, 42% of removal (in cyan color the switched-on elements); (b) normalized array factor in the cut $\phi = 0^\circ$ (beamwidth: 1.2$^\circ$; first sidelobe level: -20.23dB).

Fig. 6.8 – Example of synthesized thinned array: where the near field is less than -50dB, the elements are switched off. (a) array layout: 3009 elements, 72% of removal (in cyan color the switched-on elements); (b) normalized array factor in the cut $\phi = 0^\circ$ (beamwidth: 1.71$^\circ$; first sidelobe level: -20.77dB).

$\phi_0 = -20^\circ \div 20^\circ$

$\theta_0 = -22.5^\circ \div 22.5^\circ$

where $\theta_0$ and $\phi_0$ identify the pointing direction of the main beam.

The beamwidth values, when varying $\theta_0$ and $\phi_0$, are shown in the polar plot in Fig. 6.9, where the radius represents $\theta_0$, in degree, and the polar angle represents $\phi_0$, in radians. This graph clearly shows that the main beam dimension is almost independent on the azimuth angle, for the selected range. From theoretical considerations, we have expected a degradation of the array factor during the steering of the main beam; however, the degradation appears not too prohibitive in this case.
In general, the beamwidth is the most problematical requirement to satisfy with this type of synthesis procedure. A basic idea to reduce it from the first step of the design could be a strictest requirement for the main beam: the initial uniform array will be larger and this would lead to a small beamwidth after performing the thinning. Nevertheless, the obtained layouts with this simple computation method also represent good starting points for further refinements.

### Fig. 6.9 – Scanning analysis of the six-thousand-elements array in Fig. 6.7a: beamwidth contour plot, when varying $\theta_0$ and $\phi_0$, which are the angles that identify the pointing direction of the main beam. In the polar plot, the radius represents $\theta_0$ in degree, and the polar angle represents $\phi_0$, in radians.

#### 6.4 SKOLNIK’S METHOD FOR ADDITIONAL THINNING

An additional thinning has been applied to the synthesized thinned array, in order to further reduce the number of elements. In this design step, a random thinning has been selected, which can switch off some elements, by keeping the basic distribution suggested by the near field values, especially the overall antenna dimension. In fact, a comparison of the two thinned arrays in Fig. 6.7 and Fig. 6.8 shows that the configuration with more than six thousand elements is characterized by a beamwidth closer to the required value, because of a larger area of the overall antenna. On the other hand, the number of elements is doubled. Observing their layouts, an additional thinning to the six-thousand-elements array seems feasible. Since the only degree of freedom of the previous method is the threshold value on the near field data, a reduction of the number of elements cannot avoid a reduction of the antenna area. In fact, at the boundaries of the aperture the near field is weaker and, after performing an additional thinning, most of the elements closer to the edges would be switched off: the configuration will tend to the three-thousand-elements array with a particular care in trying to keep the same final dimension of the antenna. For these reasons, the random procedure proposed
by Skolnik in 1964 has been adopted. In particular, it relates a density (i.e. spatial) tapering to the desired amplitude one [117].

6.4.1 Outline of the Method

The random thinning method proposed by Skolnik is a simple procedure for the synthesis of thinned arrays. The procedure statistically creates a density tapering mask for a uniform array, by starting from an amplitude tapering distribution. Indeed, the theory of amplitude tapering presents several reliable methods for the beam-shaping, in particular for the reduction of the sidelobes. This is the reason behind the choice of relating the spatial distribution to the amplitude excitation.

The inputs of the method are a maximum number of array elements, which coincides with the number of elements of the original uniform array, and the coefficients of the excitation law that would have been applied as amplitude taper. In this example, a triangular distribution has been considered (Fig. 6.10).

\[ F_n = kA_n \]

where \( k \leq 1 \) is a factor that accounts for the degree of thinning: \( k = 1 \) means natural thinning; \( k < 1 \) means a harder thinning, with more switched-off elements. In this way, the amplitude distribution is used as a probability function for the design of the array layout. The approach is totally statistical and the final size of the density tapered array could not be known a priori.
6.4.2 Random Thinning

In Fig. 6.11 one of the possible output layouts is reported, in the case of pyramidal amplitude model. After the supplementary thinning, the element reduction reaches 66% with respect to the number of elements of the initial square array.

The radiation patterns before and after the additional thinning are shown in Fig. 6.12. We can observe that in the new configuration the beamwidth slightly increases, but it still results narrower than the beamwidth of the array in Fig. 6.8a, which has almost the same amount of elements. Moreover, after the additional thinning the number of the elements on the edge is reduced, and consequently also the sidelobe level decreases of more than 10 dB.

![Layout of the Thinned Array](image)

**Fig. 6.11** – Additional random thinning on the thinned array in Fig. 6.7a, from the pyramidal amplitude model in Fig. 6.10 (in cyan color the switched-on elements): 3644 elements, 66% of removal from the initial uniform array.

![Normalized Array Factor: $\phi=0^\circ$ cut](image)

**Fig. 6.12** – Normalized array factor: comparison between the six-thousand elements thinned array (layout of Fig. 6.7a) and the second thinned array, after the additional thinning (layout of Fig. 6.11).
6.5 **Final Remarks and Conclusions**

Aperiodic array capabilities for the application in the SSA phased array radar have been studied. From the literature, the possibility to break the periodicity and then obtain good performances with a less number of elements appears to be the first potential of aperiodic arrays. In fact, the presented examples show that by starting from a uniform array, a reduction in the number of elements of 40%, and even 70%, provides interesting outputs in terms of beamwidth, sidelobe level and steering of the main beam.

The adopted procedures for array thinning are very simple, but are able to provide very interesting layouts for the synthesis of aperiodic arrays with thousands of unknowns and thousands of possible configurations. In fact, since they are based on classic concepts of the antenna theory, they can be easily implemented and they are very fast in exploiting array feature in order to predict their radiation pattern. Of course, these methods should be then integrated with full-wave electromagnetic simulation (for example the ones mentioned in Chapter 1) and also powerful optimization tools, *ad-hoc* developed for aperiodic arrays with a large number of elements.
The PhD study presented here could glance at the fascinating world of electromagnetic periodic structures. Electromagnetism by itself is charming, with its waves, where electric and magnetic field run one after the other in order to propagate. Then periodicity and all its new applications: everything increases the potential of electromagnetism, and telecommunications above all.

Even though different topics have been investigated, the most theoretical and detailed activity regarded the modeling of frequency selective surfaces. The development of the MoM/BI-RME method for the analysis of capacitive FSS requested a specific insight into the integral equation approach, with the new representation of the unknowns, the analysis of the Green's function expression, and the filling of the system matrix with the calculation of coupling integrals. The derivation of the final formulation has been possible only after a preliminary study of the actual electromagnetic fields and currents, with the aim to provide both an accurate modeling and an efficient numerical computation. The presented extension of the theory would be implemented in the new version of the MoM/BI-RME code, which could address the precise analysis of capacitive FSS, by considering both the finite thickness and conductivity of the metallizations.

The new version of the MoM/BI-RME method would also include the results of the convergence study described in the third chapter. The application of the method of moments, in conjunction with the integral equation technique, to the analysis of FSS proved to be particularly reliable, when compared with measured characterizations. However, it may intrinsically suffer of relative convergence problems, which can be avoided by relating the representation of unknowns and kernel to the mathematical and physical properties of related modes and basis functions. The presented study has provided general rules that can be implemented for an automatic convergence procedure when modeling different kinds of FSS with the MoM/BI-RME method. In addition, since some structures present peculiar features, like elements with thin shapes, especially packed layouts, etc., the parameters to reach the convergence might be updated and modified, to account for the actual nature of the electromagnetic problem, as described in the chapter. In these cases, a manual but still guided convergence procedure could be the proper choice. The extended MoM/BI-RME code would then become a more powerful tool, first of all for the design of dichroic mirrors to be integrated in the beam-waveguides of ESA’s deep-space antennas.

The other activities have regarded quasi-periodic structures, which ask for a basic change in the modeling procedure: apart from the truncation of the periodicity, local defect are intentionally introduced to achieve the required performances. With respect to FSS, PhC slabs are all-dielectric devices, hence no conductors guide the light; moreover, at higher frequencies the light propagation is otherwise described with a different terminology too (different properties and classification of TE and TM modes, red and blue shifts to name frequency drifts, …). The characterization of PhC, however, starts from Maxwell equation, like in a standard electromagnetic problem. For this reason, the collaboration with the Department of Physics has been an opportunity to exchange knowledge, for a deeper understanding of periodic electromagnetism.
Lastly, the aperiodic array study for space application: the SSA program is a new project of the European Space Agency, and most of its requirements and characteristics still need to be defined. Nevertheless, the targets the new radar should be able to track and identify require large antennas, in phased array technology. The presented investigation provided interesting relationships between the antenna performances and all the available degrees of freedom in the determination the final layout. Concerning the modeling of the arrays, the importance of the application will need *ad hoc* full-wave simulators, which should handle large arrays. The simple numerical techniques adopted here are the basis for a proper electromagnetic insight.

Fine modeling is the starting point for a successful design. Modeling gives details about field distribution and material interaction. Modeling asks for robust and stable numerical mathematics. Definitely, it is one of the most motivating topics of scientific research.
... and at least I reached the acknowledgements: I’ll be delighted to spend my last thesis-night
with you!

This is the end; this is the page that states that definitely I would never be a student anymore.
You know me; I am that kind of romantic girl who could even move to tears at the idea of all these
years, with all the memories, the dreams... But no, I must say it’s time to apply what I learnt, 'cause
it goes on: time only can lead you on! (Elena, please...)

So, first of all, Maurizio, my super-super-Supervisor! The patches, the SIW, the lovely periodic
structures: I have the feeling you have always found the right steps to draw my microwave path: I
hope I could have followed at my best... but don’t you think now I speak too much?!? Yes, the
entire laboratory claims that I’m more like an engineer after about three years all alone with men!!!
But I love my lab: Luca, it’s has been a pleasure to work up there! Yes, of course thanks M&M! I
was the junior PhD student, and you taught me all the most important things, such as how to
simulate a dipole with AWAS, how to rename files and folders (for example the unforgettable bec-
up... or crick!), how to reach the furthest tree with hand-made airplanes, how to coordinate HFSS,
GRASP and YouTube (with the embarrassing song about the cobra!)... but then fortunately
Andrea and Francesco arrived, at least someone (pseudo-)normal! By the way Francesco, are all
your barbaapatà feeling good? Then the South-East corner (wonderful panorama of Pavia and
Oltrepò, just a bit too hot in summer... ), with Gaia and Wissam, while on the other side Mattia
used to generate thrilling - and quite absurd - discussions... don’t worry Marta, you’ll have fun (as
well as a full control of the situation!!!)! However, let me say that I really enjoyed these years,
because all the professors, Arcioni, Bressan and Conciauro, with Luca and Maurizio, always teach
electromagnetism with that mix of professionalism and passion that make the microwave lab so
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American week with Daniele and the Dutch group, Rome, ...), for the schools (where I met all my
lovely colleagues from Tuscany: Agnese, Giorgio, Cristian and Francesco!), for lunches and dinners
(Simone, please take care of the hairs on your shoulders!), and for the cheers (Pasiaaaaaan, water is
not allowed), ...

Then Maurizio and Luca proposed me to come to ESOC to spend the classical six months
abroad... here the second part of the PhD story begins... and, since I was again surrounded by
professionalism and passion, also my work experience begins here. The first day as trainee, the first
lunch with Piermario and the antenna section (and the first coffees to pay...), the first tour along
the corridors with Filippo and Guillaume, the first Spanish break in the afternoon... everything
started! Piermario, I’m sure you understood as soon as I showed you the first array factor that I was
not used to design antennas: thanks for your help and your patience. And how many times did I pass
by your office with strange questions, Guillaume? Yes, you’ve always answered, I know, I warmly
thank you for that. But you are not so normal too... I still have among the documents that picture
of your whiteboard with the statistical staircase comparison to be computed... And now Marco and
Gunther, with also Anouk, Alberto, Mattia, then Rebeca, Guillaume, Franck, ... No, no, my dear
office mate Magdalena and Gonzalo, I didn’t forget that you adopted me since the very beginning,
and, most of all, you’re sharing the pain of my thesis: I wonder what I should have done without you!

Don’t worry, I’ve almost finished…

Just a few words, tutti insieme appassionatamente! Some months ago you were all i meravigliosi del farewell, now you (You = Cri, Ele, Sil, Ilia, Luré, Otta, Nico, Clara, Deny, Cri, Deborah, Vale, Zar, Gabry, Cippo, Testo, Gigi, Giò, Pasquale, David, Stefano, and all the puzzle, the (three!) choirs, Vale-Cate-Kli, l’oratorio e il don, il piano H, il mio primo ed unico tesista e il Monty di sotto, gli ottici del piano F, Eleonora, all the profs, Stefania&Stefania, Andre e Fede, Marcello e Matteo, Paolo e il M° Torti, lo zio Giulio&Co., i cugini&Co., la mamma e il papà, la nonna Lena, l’Agne e il Giuse, …), you’re wonderful and that’s all.

Thanks my friends, my treasure,

Maria
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