Poverty between and within groups: a reformulation of the FGT class of index
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Abstract

In this paper we suggest a new approach to the identification and measurement of poverty in subgroups of population. Unlike the standard approach, in which ex-ante refers to a sole poverty line for the whole population and ex-post describes how poverty is distributed among different groups, the approach suggested here requires: i) the identification ex-ante of homogenous groups which relate to the purpose of analysis; ii) the definition of a specific (absolute or relative) poverty line for each subgroup; iii) the measurement of overall poverty by means of a reformulated FGT class index which is able to capture both between and within poverty components.

Keywords: poverty measurement, FGT index, group-specific poverty lines

JEL codes: I3, I32
1. Introduction

The aim of this paper is to formulate a different approach to identifying and measuring deprivation that is based on the conceptual and analytical distinction between poverty within and between different subgroups of population. The core of our proposal is that absolute and relative poverty can be better conceptualised and assessed if, instead of referring to a sole standard (i.e. the poverty line) for the entire population, homogenous subgroups are considered. As many Authors point out (see, among others, Sen and Atkinson), differences across households and individuals are so relevant and pervasive that they cannot be ignored: personal characteristics, household size and composition, and social and economic environment are all elements that affect the level of needs and, as a consequence, the achieved standard of living. The standard approach simply ignores these differences and adopts a unique, usually national, poverty line; it applies an equivalence scale when statistical data refer to households instead of individuals, but this “technical adjustment” can only partially capture the different elements that affect standard of living. Nevertheless, how poverty is distributed in society, seems to be a crucial topic of any poverty assessment as shown by the frequent use of additive poverty measures, like the Foster, Greer, Thorbecke class of index, which describes the level of deprivation for given socio-economic groups (i.e. rural and urban population, regional or sectoral poverty profile, race or household structures, etc.).

In this paper, we suggest a new approach for the identification and measurement of poverty that is based on two main features:

- Differentiated poverty lines for homogenous groups of population instead of a unique poverty line equal for all;
- A conceptual and analytical distinction between the degree of poverty within groups of population and that which exists between different groups.

As we will show, like in the case of inequality measurement, distinguishing what happens within a given group (the within component) and between different groups (the between component) has remarkable implications at a theoretical level as well as on the analytical structure of the synthetic index of poverty.

In the first part of the paper, our approach will be presented in detail, it will be compared to the standard approach (i.e. a unique poverty line) and some arguments for and against them will be discussed. In the second part, we will illustrate a new formulation of the Foster, Greer, Thorbecke class of poverty measurement that is able to quantify both within and between terms.
2. The standard approach to poverty measurement based on a national poverty line

The welfarist approach\(^1\) to poverty measurement traditionally focuses on a unique evaluative space within which a poverty line \(z\) is defined and used to identify who is poor and who is not. This threshold can be fixed in terms of the minimum amount of income required to satisfy a given set of basic needs (absolute poverty line \(z_a\)) or it can refer to some notion of the standard of living, such as the mean or median income or consumption (relative poverty line \(z_r\)); in this case, a cut-off level in terms of a given percentage of this standard will be chosen. An intermediate and alternative solution, suggested by Citro and Michael [13], is to adopt “hybrid” poverty lines that, unlike the absolute poverty lines, are sensitive to changes in the standard of living but are not as sensitive as a purely relative approach. Assuming a weighted geometric average of a relative threshold \(z_r\) and an absolute threshold \(z_a\), the hybrid poverty line \(z\) can be expressed in the following generalized way:

\[
\rho^{-1} = \rho z_r z_a
\]

where \(0 \leq \rho \leq 1\) represents the elasticity of the poverty line compared to the standard of living indicator \(^{2}\). A value of \(\rho\) equal to zero identifies an absolute poverty line whereas a value equal to one determines a fully relative threshold.

The choice of the \(\rho\) value is not simply a technical matter: it can be viewed as a purely normative question if we consider this parameter as a measure of how much poor people share economic growth. Assuming an absolute approach (\(\rho\) equals to zero) the standard of living of the poorest becomes totally independent on the economy’s growth dynamic while a value equal to one means that increases in general living standards will be fully reflected in terms of poverty line; intermediate values of \(\rho\) will reflect in-between positions. As Foster outlines, the key distinction between absolute and relative poverty lines is not in their specific value but in how this value changes when the economy changes\(^3\).

\(^1\) In a broad sense, we refer to all the approaches that conceive well-being in terms of utility and preferences. In contrast to these there is the category of non welfarist approaches that enlarges the analysis to other relevant aspects of the human life. On this latter see, in particular, Amartya Sen [28][29] and also Ravaillon [18].

\(^2\) Cfr. Fisher [17]. We will return to this formulation later when our proposal will be discussed in detail. On the hybrid approach to \(z\), see Citro, Michael [11], Foster [18] and Madden [22].

\(^3\) “The possibility of using a hybrid standard changes the question “absolute or relative?” to “exactly how relative?” with \(\rho\) as the relevant decision variable” Foster [18, p. 340]. General-
Following the choice of a poverty line and the identification of poor people, a poverty profile is drawn, for a number of reasons: in order to describe the pattern of poverty, to see how and why it varies across subgroups of population, and to understand what the relative risks of being poor are for specific groups. From the methodological point of view, this *modus operandi* can be seen, firstly as the attempt to identify ex-ante a common basis of comparison (a unique poverty line determined on a given evaluative space) for statistical units that are intrinsically different on a plurality of aspects and, secondly, as an attempt to evaluate ex-post what are the main variables that correlate with poverty or if certain population groups have a higher risk of being poor than others. In other words: homogeneity among individuals or households is postulated, then a comparison with a common reference point (the poverty line) is carried out and finally a poverty profile is constructed with the aim of finding out what the main differences between individuals or households (sex, race, age, geography or any other characteristics) are that determine different standards of living. If people really were identical in all aspects except for available income, then this procedure could be correct; unfortunately (or fortunately), as Amartya Sen always remarks, this is quite an unrealistic assumption which has a strong impact on the empirical results.

### 3. An alternative approach: different poverty lines for different groups

The content of our proposal is very simple. Let’s suppose that a population of *n* statistical units can be divided into *k* mutually exclusive and homogeneous groups with dimension *n* (i = 1, 2, ..., *k*) and let’s assume that individual income *y* (i = 1, 2, ..., *k*; j = 1, 2, ..., *n*) has been chosen as an adequate standard of living indicator. Instead of defining a unique threshold for all the *n* units, our proposal is to define *k* group-specific thresholds *z* > 0. As in the traditional approach to poverty measurement, they can be absolute, *z**, if they don’t depend on the standard of living of the community but instead refer to some exogenous criteria such as a minimum diet or food calorie intake, the cost of basic needs (nutrition, housing, shelter, etc.) or are arbitrarily fixed, for example like the symbolic value of “a dollar a day” that the World Bank recognizes as an extreme poverty line. In any case, whatever the absolute poverty line chosen, they will be independent

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*ally speaking, a one per cent increase in the mean or median income will determine a ρ per cent increase in the poverty line.*

*4 For simplicity’s sake, we assume individual income as the unique space of evaluation.*
from the current data and differentiated for different groups of population. Alternatively, relative group-specific poverty lines, $z_{ir}$ can be fixed which assume the standard of living of each subgroup as a reference point instead of the overall well-being. If, for example, this standard is expressed in terms of per capita income of the $i$ group, $\mu_i(y)$, the group-specific poverty line $z_{ir}$ will be:

$$z_{ir} = \gamma \mu_i(y)$$  \hspace{1cm} (2)

where $\gamma$ is the cut-off level expressed as some percentage of this standard. If the International Standard of Poverty Line, corresponding to 50 percent of the mean income, is chosen, then:

$$z_{ir} = \frac{1}{2} \mu_i(y)$$  \hspace{1cm} (3)

In this case too, we will have as many poverty lines $z_{ik}$ as reference groups $k$. Finally, for intermediate positions we can refer to a modified version of the hybrid approach suggested by Citro and Michael [13] which includes the presence of different subgroups:

$$z_i = z_{ir} z_{ir}^{-\rho}$$  \hspace{1cm} (4)

Excluding the two opposite values of zero and one, that lead respectively to purely absolute ($z_{ia}$) or purely relative ($z_{ir}$) poverty lines, the $\rho_i$ values can be arbitrarily chosen, as said above, or empirically determined as suggested and tested by Madden [29]. He estimates a $\rho$ value equal to 0.7 using expenditure data based on Irish households, adjusted by an appropriate equivalence scale to take into account the differences in household size and composition. However, if the focus is directed on the specific

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5 There are many definitions of the concept of absolute poverty. Ravaillon, for example, wrote that [23, p. 30] “...an absolute poverty line is best thought of as one which is fixed in terms of living standards and fixed over the entire domain of the poverty comparison; ... Thus absolute poverty comparisons will deem two people at the same standard of living to both be either “poor” or “not poor” irrespective of the time or place being considered, or without some policy change within the relevant domain”. However, a poverty line defined following an absolute method doesn’t imply that it must be constant over time or space: what differentiates the relative from the absolute threshold is how and if they are sensitive to what happens in society. For this reason, we prefer the definition given by Foster [18, p. 337] who wrote “the key distinction between absolute and relative thresholds is not seen in the specific values obtained from a given data but in how the values change as the distribution changes”.

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subgroups of households, as we suggest, the estimation of specific sub-
group parameters of elasticity, one for each of the $k$ groups considered, can
reflect the different expenditure behaviours of different households$^6$. We
will return to this topic later.

4. For and against the adoption of group-specific thresholds

In this section we want to show why our proposal can represent a possi-
bile solution to the conceptual and empirical problems briefly pointed out in
section 2. As already noted, the adoption of group-specific thresholds in-
stead of a unique poverty line equal for all can be justified by both concep-
tual and practical reasons. With reference to the former, the assumption of a
representative agent as the reference point in poverty and well-being analy-
sis ignores the fact that there are many differences, in needs, interpersonal
characteristics, household structure, socio-economic context, community,
region, race, etc. that reflect different deprivation risk. Moreover, the idea
of relative deprivation, as mainly formulated by the sociological literature,
is based on the assumption that individuals and households compare their
own conditions with the community to which they belong. However, given
the wide range of differences that characterizes the communities, it seems
more realistic to assume that each of us compares our own condition with
others that are not so far from our standard of living and mainly, from a
household structure, social, cultural, and demographic point of view$^7$.

This idea is not really new and we can find it partially reflected in Sen's
class of indexes that includes a Gini inequality measure of the income of
the poor, based on a "relative equity principle". Our proposal is to accept
this idea, which assumes that relative poverty can be viewed at two differ-
ent levels. Firstly, through a comparison of the individual position within a
homogeneous group which has been identified on the basis of some "objec-
tive" criteria (age, household structure, geographical area, race and so on)
instead of within the whole society or a generic group of "poor people".
And secondly, through a comparison of poor people belonging to different

$^6$ There are goods and services that are typically included in the expenditure patterns of the
household so that their absence could be viewed as a symptom of deprivation but this is not
always true. Depending on the geographical area where people live, for example, some
consumption (i.e. heating, car or public transport) can assume a different meaning or can
require a larger or smaller amount of expenditure and so cannot be automatically interpreted
as indicators of deprivation or well-being.

$^7$ For example, a young person living alone is more likely to match their own standard of
living to other analogous situations rather than to an elderly women or a family with young
children. Something like that could happen with reference to households who live in rural or
urban area or in different regions.
groups, which allows us to capture other important dimensions of poverty: those existing among different groups within the society\textsuperscript{8}.

The importance, but also the difficulty, of recognizing the within and between components as being conceptually and analytically different but at the same time as interrelated questions has also been outlined by Foster and Sen [26, p. 157]. In reference to inequality analysis, they wrote: “if there is even a modest amount of interdependence between groups in society, an exact separation into between-group and within-group terms may not be attainable. A residual term (as in the Gini breakdown) or some other modification to additivity may be needed to account for overflow or undercounting inherent in the problem”\textsuperscript{9}. Similarly, with reference to the poverty measurement, the same Authors outlined “even though a person’s deprivation has to be judged with respect to the poverty line \(z\), her sense of deprivation and the shortfall of her actual well-being vis-à-vis an acceptable standard may depend inter alia on influences other than her own income \(x\), and the poverty line \(z\). She could be influenced by the comparison of her own shortfall vis-à-vis the shortfall of others [...] These considerations suggest that the poverty line \(z\) may be drawn differently for different subgroups” [26, p. 185]. Our proposal takes this suggestion seriously.

Still with reference to conceptual matters, there are some fundamental aspects in well-being assessment that are usually recognized but sketchily included in the poverty analysis, issues like: consumption habits, self-consumption, the differing cost of life and, more generally, all these elements in relation to the informal economy. Whether or not these aspects are exchanged on the official or formal markets or are simply not easy to evaluate they are, however, absolutely crucial in defining people’s real standard of living. It is sufficient to think of the life conditions of people who belong to different religious, race or ethnic groups, who live in different geographical contexts characterised by significant differences in the cost of living or in the availability of goods due to the existence of imperfect markets\textsuperscript{10}. These important features are rarely considered and only partially captured by the equivalence scales which are empirically esti-

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\textsuperscript{8} By this approach is possible also to give an answer to the questions expressed by Sen [28, p. 106] who wrote: “Separability is certainly convenient property, and permit us to build up the overall poverty picture from the poverty measures applied to subgroups. [...] There remains a more general question as to whether it is sensible to assume that poverty indicators should be combinable in this way, which requires that the view of poverty for particular groups be, in some specific ways, insensitive to what happens to other groups and that the whole picture does not introduce anything other than what is already there is in the parts.”

\textsuperscript{9} On the decomposability of inequality and poverty measures see also Anand [1], Bottiroli Civardi [6], Foster, Sen [21], Foster, Shorrocks [25], Ravaillon [23].

\textsuperscript{10} The literature on the equivalence scales is very large: see, among the others, Deaton, Muehlbauer [13], Coulter, Cowell, Jenkins [12], Ruggles [26] and, with reference to Italy, Carbonaro [10] and Atella, Caiumi, Perali [2].
mated for households of different size and composition\textsuperscript{11}.

From an empirical perspective the importance of distinguishing what happens within and between subgroups is frequently and implicitly assumed. For instance at an international level, a double level of comparison is quite frequent, when empirical results on poverty extension based on the differentiated national poverty lines for each European country, are compared with the results deriving from a common poverty line corresponding to 50% of the European average income\textsuperscript{12}. In regards to this, Atkinson [3, 4] suggest using an adjusted pan-European standard which takes into account the different purchasing powers of European countries, or alternatively he suggests adopting an intermediate threshold between the poverty line which is calculated with reference to both the average income of the country ($Y_i$) and the European Union average income ($Y_{EU}$). If we follow the International Standard Poverty Line, which assumes a cut-off level corresponding to 50% of the standard of living indicator chosen, we will have:

$$z_i = \frac{Y_{EU}^{\theta} Y_i^{1-\theta}}{2}$$

(5)

where $\theta$ is a weighting parameter. A value of $\theta$ equal to one means that the poverty threshold will correspond to half the European average income; if $\theta$ is equal to zero, the national poverty line will be adopted for the country $i$; an intermediate value, for instance 0.5, will be associated to a poverty threshold for country $i$ which corresponds to half the geometric average between national income and European income.

At this point, it could be useful to mention several criticisms that could be made against the adoption of specific poverty lines for sub groups and propose some counter arguments.

A critique to our approach could refer to the elements of subjectivity that are implicitly introduced when homogeneous groups must be defined. This could certainly be a risk. However it can be significantly reduced if groups are chosen on the basis of "objective" criteria such as age, geographical area, race or generally speaking any of the traditional groups included in a poverty profile, the only constraint would be the statistical

\textsuperscript{11} On this topic, see Deaton and Muellbauer [13], Deaton, Ruiz Castillo and Thomas [14] and Browning [8]. See also Ravaillon [23]. Concerning developing countries, see Bidani and Ravaillon [5] and Ravaillon and van de Walle [24]. On the cost of living in Italy see Campiglio [9].

\textsuperscript{12} The World Bank Development Report [31] compares the results in terms of incidence of poverty deriving from the adoption of an absolute international poverty line (1 dollar a day) vis-à-vis to relative national poverty lines (usually fixed as 50% of mean income or expenditure) too.
significance of each group.

A second and more serious critique could refer to the fact that for a given group (let's suppose, rural population) who lives in conditions of endemic deprivation and in which all the people are equally poor, the poverty line and the number of poor people will be low in absolute terms. On the contrary, for relatively affluent groups (for instance people with a university degree, managers or other people with a high occupational status) who present a high level of income inequality, we will obtain a high absolute value of the poverty line and a large percentage of poor people.

This is an old problem that refers to the controversial and unsolved question between absolute and relative poverty and this latter and inequality. It is quite evident that when we refer to relative poverty lines, the incidence and the intensity of poverty are positively related to the degree of inequality existing in the society. However, it is also true that there could be a higher level of inequality without poverty if all people were able to achieve a given (absolute) threshold of needs. Our proposal could offer a solution to this dilemma because, like in the inequality measurement field, it makes a clear distinction between (absolute or relative) poverty within each group and relative poverty among different groups.

5. The FGT class of measure with group-specific thresholds $z_i$ : a reformulated version of $P_{\alpha}$

We will refer here to the class of decomposable measure formulated by Foster, Greer and Thorbecke [24] that includes the most famous poverty indexes such as the head count ratio ($H$), income gap ratio ($I$) and poverty gap index ($PG$).

This class of additive poverty measures is usually expressed in the following way:

$$P_{\alpha} (y, z) = \frac{1}{n} \sum_{j=1}^{q} \left[ p(y_j, z) \right]^\alpha$$

where $p(y_j, z)$ is an individual measure of poverty and $q$ is the number of the poor, that is those whose income is below the poverty line ($y_j < z$).
Assuming \( p(y, z) = (z - y_i)/z \), the value of the non-negative parameter \( \alpha \) identifies the most common poverty measures:

- \( P_\alpha \) corresponds to the head count ratio when \( \alpha = 0 \):
  \[
  H = \frac{q}{n}
  \]  
  \( (7) \)

- when \( \alpha \) takes on the value 1, \( P_\alpha \) is the poverty gap:
  \[
  PG = \frac{1}{n} \sum_{j=1}^{n} \frac{g_j}{z}
  \]  
  \( (8) \)

  and \( g_j = z - y_j \) is the gap between the income of individual \( j \) and the poverty line;

- finally, a squared poverty gap index, better known as the Foster, Greer and Torbecke index, will occur when \( \alpha = 2^{16} \):
  \[
  FGT = \frac{1}{nz^2} \sum_{j=1}^{n} g_j^2
  \]  
  \( (9) \)

This general class of measures \( P_\alpha(y, z) \) satisfies a basic axiomatic structure that includes focus axiom, symmetry, replication invariance, scale invariance and continuity. Other properties like monotonicity, transfer sensitivity, decomposability and subgroup consistency are satisfied only for several values of \( \alpha > 0 \)\(^{17}\).

As already stated, in the traditional approach to poverty measurement, not very relevant, can however affect the properties of the indexes. Generally, both definitions are equally adopted, even if some Authors suggested that only the former should be used because it is consistent with the notion of the poverty line. See, Foster, Sen [21] and Zheng [32].

\(^{16}\) The class of measure \( P_\alpha \) is defined for values of \( \alpha \) from zero to infinite. Higher values of this parameter assign a higher weight to the gap between personal income and the poverty line and the poverty measure will be more sensitive to the well-being of the poorest person. For instance, when \( \alpha = 1 \) the same value is assigned to a marginal increment of income lower than \( z \); the marginal evaluation is linearly decreasing when \( \alpha = 2 \). Finally, higher values of \( \alpha \) assign a higher weight the lower the income. As \( \alpha \) approaches to infinite it collapses to a measure that only reflects the poverty of the poorest person. Cfr. Ravaillon [23] and Atkinson [4].

\(^{17}\) About the axiomatic structure of the class of measure \( P_\alpha(y, z) \) and, in particular, the consequences on this structure that occur when group specific poverty lines are applied: see Bottioli Civardi and Chiappero Martinetti [7]).
the shortfall of each person \( j \) depends only on their own income \( y_j \) and the poverty line \( z \); there is no interdependence between individual deprivation measures and the \( P_\alpha \) class of poverty measures is, by construction, totally decomposable and subgroup consistent. In other terms: \( P_\alpha \) has no between-group term because the standard against which deprivation is evaluated (the poverty line) is equal for all groups. However, if we assume a group specific threshold then the between term exists and \( P_\alpha \) must be reformulated in order to quantify both between and within components.

Returning to the basic assumptions of section 3 (population of size \( n \) divided into \( k \) mutually exclusive groups according to sex, age, geographical region, race, ethnic community or any other relevant criteria; poverty measured only on the basis of personal income \( y_{ij} \)), we will now reformulate this class of decomposable measures with reference to purely absolute, relative and hybrid poverty lines:

- **1st case: purely absolute poverty lines**

  Let’s assume that group specific absolute poverty lines \( z_i \) are defined, for instance in terms of a minimum basket of goods. Differences among threshold can refer to a different kind and amount of goods and services included in this basket (for example, an elderly person living alone will have a different basket of goods compared to a young person or a single adult, different consumption habits for different ethnic groups will result in different poverty lines, etc.) or to the different cost of this basket (urban and rural populations face different prices and cost of living) or both.

  For each \( k \) group, poor people are identified in the traditional way (the \( j \) person belonging to the \( i \) group is poor if \( y_{ij} \leq z_i \)) and poverty is measured for each group. To measure the overall poverty when group specific poverty lines are used, we define a new class of poverty index \( P_{WB_\alpha} \) as the sum of a within term \( P_W \) and a between term \( P_B \):

  \[
P_{WB_\alpha} = P_W + P_B
  \]

  where

  \[
P_w = \frac{1}{nz^\alpha} \sum_{i=1}^{k} n_i z_i^\alpha P_\alpha
  \]

  If no difference among groups exists \( P_B = 0 \) and \( z_1 = z_2 = ... = z_k \), while in any other case \( z_1 \neq z_2 \neq ... \neq z_k \). Without losing generalization, we assume that groups are arranged in a non decreasing order of \( z_i \). To quantify the between term it seems reasonable to assume that each group is compared to the group which is better off, in other words to the group with poverty line equal to \( z_k \).
If $\alpha = 0$, a sort of *head-count ratio*, called $H_{WB}$, can be defined as follows:

$$H_{WB} = \sum_{i=1}^{k} H_i \frac{n_i}{n} + \sum_{i=1}^{k} [H_i(z_k) - H_i(z_i)] \frac{n_i}{n}$$  \hspace{1cm} (12)$$

The first term identifies the within term $H_W$ as the weighted average of the head-count ratios $H_i$ referring to the $k$ groups, with the weights being given by the ratio of the population of each group to the total population $n$. The between component (second term included in the square brackets), can be viewed as the poverty level in the first $k-1$ groups if $z_k$ were assumed as a standard to measure poverty (positive term) instead of $z_i$ minus the part already included in the within component (negative term).

If $\alpha = 1$, $P_{z} = PG$ and the reformulated version of the index $PG_{WB}$ is:

$$PG_{WB} = \sum_{i=1}^{k} PG_i \frac{n_i z_k}{nz_k} + \sum_{i=1}^{k-1} [PG_i(z_k) - PG_i(z_i) \frac{z_i}{z_k}] \frac{n_i}{n}$$  \hspace{1cm} (13)$$

In this case too the first term reflects the within component calculated as the weighted average, where the weights are expressed in terms of population shares and relative gap between $z_i$ and $z_k$. The second term of this sum is the between component and has the same meaning as discussed above with reference to the $H_{WB}$ index.

Finally, for $\alpha = 2$ the corresponding index $FGT_{WB}$ can be expressed as:

$$FGT_{WB} = \sum_{i=1}^{k} FGT_i \frac{n_i z_k^2}{nz_k^2} + \sum_{i=1}^{k-1} [FGT_i(z_k^2) - FGT_i(z_i^2) \frac{z_i^2}{z_k^2}] \frac{n_i}{n}$$  \hspace{1cm} (14)$$

Once again, the meaning of the two poverty components doesn’t change.

- **2nd case: purely relative poverty lines**

  From the methodological point of view, things are not substantially different when relative poverty lines, such as those described in section 3, are applied. In this case too, a common term of comparison is needed to capture the between component. It could be the higher poverty line $z_k$ again or, as in the standard approach to poverty analysis, a national poverty line $z_n$. In this latter case, if the cutoff parameter $\gamma$ is equal for all $k$ groups, $z_n$ will simply be a weighted average of all $z_i$ with weights corresponding to the population share of each group.
When \( z_k \) is chosen as a comparison criterion, the decomposability of the indexes and their interpretation is the same as in the case of absolute poverty line and the class of poverty measure (10) will be:

\[
P_{WBA} = \sum_{i=1}^{k} P_{\alpha i} \frac{n_i z_i^\alpha}{nz_k^\alpha} + \sum_{i=1}^{k-1} \left[ P_{\alpha i} (z_k^\alpha) - P_{\alpha i} (z_i^\alpha) \right] \frac{n_i}{n} (15)
\]

If, instead of \( z_k \), we refer to a national poverty line \( z_n \) the (15) becomes:

\[
P_{WBA} = \sum_{i=1}^{k} P_{\alpha i} \frac{n_i z_i^\alpha}{nz_n^\alpha} + \sum_{i=1}^{k} \left[ P_{\alpha i} (z_n^\alpha) - P_{\alpha i} (z_i^\alpha) \right] \frac{n_i}{n} (16)
\]

In this last case, however, the addends of the between terms will be positive when \( z_i < z_n \) while they will be negative if \( z_i > z_n \). This circumstance can determine a between component equal to zero or negative with obvious difficulties both at a conceptual level - there are differences in the standard of living between groups that, however, cancel each other out or reduce the global index - and from an empirical point of view. If one of the aims of poverty analysis is to evaluate the efficiency of public policies, the positive effects on a group must be identified and quantified independently from the negative effects on other groups. For this reason, it is important to distinguish the positive component corresponding to the \( k' \) groups with poverty lines lower than \( z_n \) from the negative component associated to the \( k - k' \) groups with poverty lines higher than \( z_n \). So we can write:

\[
P_{WBA} = \sum_{i=1}^{k'} P_{\alpha i} \frac{n_i z_i^\alpha}{nz_n^\alpha} + \sum_{i=1}^{k} \left[ P_{\alpha i} (z_n^\alpha) - P_{\alpha i} (z_i^\alpha) \right] \frac{n_i}{n} +
\]

\[
- \sum_{i=k'+1}^{k} \left[ P_{\alpha i} (z_n^\alpha) - P_{\alpha i} (z_i^\alpha) \right] \frac{n_i}{n} (17)
\]

- **3rd case: hybrid poverty lines**

With reference to the (4) general poverty line formulation it is quite evident that when a \( \rho \) value equal to zero or one is chosen for all groups, the formulation and interpretation of the poverty indexes are identical to the cases discussed above. If different \( \rho_i \) values are chosen, and this could occur if, for example, they were empirically estimated on the basis of group specific expenditure behaviors, then the common standard necessary to quan-
tify the between term could be, as before, the weighted average of the hybrid poverty lines $z_i$. The $\rho$ value, which refers to the whole population, can be derived ex-post on the basis of the expression (4).

In order to clarify the features of our proposal and how it differs from the traditional approach, some simple numerical examples are included in the appendix.

6. Conclusions

The aim of this paper was to suggest a new approach to poverty analysis based on group specific poverty lines for homogenous subgroups of population. This approach allowed us to make the conceptual and analytical distinction between two important dimensions which are implicitly included in the idea of poverty clearer: the amount of deprivation existing within a given group and the differences in the standard of living between different groups. Justifying the pros and cons of our proposal from the conceptual point of view, at a methodological level we have shown that our proposal can be applied by means of absolute, relative or hybrid methods for setting poverty thresholds. Finally, with reference to the most common class of poverty measure $P_\alpha$, the main poverty indexes (head-count, poverty gap and FGT) have been reformulated and the between and within components quantified.
Appendix

A numerical example of the decomposability of $P_\alpha$ class measure with unique and group-specific poverty lines

Let’s suppose that income data are available which refer to a hypothetical population of $n=12$ individuals, are ordered by non-decreasing values of income and are distributed into $k=3$ groups. In Table A.1 absolute poverty lines are arbitrarily chosen and applied, the only constraint is that, for simplicity’s sake, $q_i$ is greater than zero for all groups. Table A.2 refers to a relative poverty approach and uses the same data set. Both tables compare the traditional approach with the results obtained when group-specific poverty lines are used.

Table A.1 – Absolute poverty lines and decomposability of $P_\alpha$

<table>
<thead>
<tr>
<th>$n_i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_i$</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>30</td>
<td>50</td>
<td>60</td>
<td>110</td>
<td>200</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>$n_i$</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{y}_i$</td>
<td>18.33</td>
<td>55</td>
<td>91</td>
<td>60.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_{ai}$</td>
<td>18</td>
<td>50</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_{ki}$</td>
<td>0.22</td>
<td>0.62</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_i(z_{ai})$</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_i(z_{ki})$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{aligned}
\alpha &= 0 \\
&\text{decomposability} \\
&\text{%} \\
&H_1 = 0.67 \\
&P_{W} = 0.67 \\
&H_2 = 0.75 \\
&P_{B+} = 0.50 \\
&H_3 = 0.60 \\
&P_{B-} = 0.42 \\
&H_{WB} = 0.75 \\
&100
\end{aligned}
\]

\[
\begin{aligned}
\alpha &= 1 \\
&\text{decomposability} \\
&\text{%} \\
&PG_1 = 0.20 \\
&P_{W} = 0.19 \\
&PG_2 = 0.20 \\
&P_{B+} = 0.33 \\
&PG_3 = 0.34 \\
&P_{B-} = 0.05 \\
&PG_{WB} = 0.47 \\
&100
\end{aligned}
\]

\[
\begin{aligned}
\alpha &= 2 \\
&\text{decomposability} \\
&\text{%} \\
&FGT_1 = 0.07 \\
&P_{W} = 0.11 \\
&FGT_2 = 0.1 \\
&P_{B+} = 0.23 \\
&FGT_3 = 0.22 \\
&P_{B-} = 0.01 \\
&FGT_{WB} = 0.32 \\
&100
\end{aligned}
\]
Table A.2 – Relative poverty lines and decomposability of $P_\alpha$

<table>
<thead>
<tr>
<th>$n_i$</th>
<th>$i = 1$</th>
<th>$i = 2$</th>
<th>$i = 3$</th>
<th>$y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
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<td>15</td>
</tr>
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<td>4</td>
<td>110</td>
<td>150</td>
<td>20</td>
<td></td>
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<td>5</td>
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</tr>
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<td></td>
</tr>
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<td>60</td>
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<tr>
<td>10</td>
<td></td>
<td></td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>11</td>
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<td>150</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

$n_i$ | $\frac{n_i}{N}$ | $\bar{y}_i$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.25</td>
<td>18.33</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
<td>55</td>
</tr>
<tr>
<td>5</td>
<td>0.42</td>
<td>91</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>60.83</td>
</tr>
</tbody>
</table>

$z_{ri} = \frac{1}{2} \bar{y}_i$

<table>
<thead>
<tr>
<th>$z_{ri}$</th>
<th>$q_i(z_{ri})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.1</td>
</tr>
<tr>
<td>0.90</td>
<td>1.0</td>
</tr>
<tr>
<td>1.50</td>
<td>1.0</td>
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</tbody>
</table>

$\alpha = 0$

<table>
<thead>
<tr>
<th>decomposability</th>
<th>$H_1 = 0$</th>
<th>$H_2 = 0.25$</th>
<th>$H_3 = 0.4$</th>
<th>$H = 0.5$</th>
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<tbody>
<tr>
<td>$%$</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

$\alpha = 1$

<table>
<thead>
<tr>
<th>decomposability</th>
<th>$PG_1 = 0$</th>
<th>$PG_2 = 0.07$</th>
<th>$PG_3 = 0.20$</th>
<th>$PG = 0.17$</th>
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<tbody>
<tr>
<td>$%$</td>
<td>50</td>
<td>50</td>
<td>100</td>
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</table>

$\alpha = 2$

<table>
<thead>
<tr>
<th>decomposability</th>
<th>$FGT_1 = 0$</th>
<th>$FGT_2 = 0.02$</th>
<th>$FGT_3 = 0.11$</th>
<th>$FGT = 0.09$</th>
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<tbody>
<tr>
<td>$%$</td>
<td>86.0</td>
<td>14.0</td>
<td>100</td>
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</table>

As table A.1 underlines, using differentiated absolute poverty lines for each group, the value of the overall index doesn’t change when compared to the traditional method based on a sole threshold but in the former case it is possible to separate the within and between components. In particular, the weight of the between component (absent in the traditional approach) grows for increasing values of $\alpha$; it is equal to ten per cent when the head count index ($\alpha = 0$) is considered, the between term is higher than 60 per cent for $\alpha = 2$. We tested
the sensitivity of our results for different arbitrarily chosen amounts of $z_a$, and this relationship was always confirmed.

Similar results, shown in table A.2, are obtained by adopting a relative approach. This numerical example clarifies the meaning of the between term: given that $z_r$ is the weighted average of specific poverty lines $z_{ri}$, the positive and negative components of the between terms encompass the different relative positions of each group as regards the standard as well as the degree of asymmetry existing among them. In this case too the sensitivity of the between component is directly related to the $\alpha$ value, in other words to the kind of index used. In our example, the between component is negative and as such is able to reduce the burden of poverty within the group.
Riferimenti Bibliografici


ELENCO DEI QUADERNI DEL DIPARTIMENTO DI ECONOMIA PUBBLICA E TERRITORIALE PUBBLICATI

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<td>La gestione delle aree protette: il finanziamento dei parchi regionali</td>
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<td>Marco Missaglia e Paul de Boer,</td>
<td>Employment programs in Palesatine: food-for-work or cash-for-work?</td>
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<td>n. 3/2004</td>
<td>Marisa Bottirolì Civardi e Enrica Chiapperò Martinetti,</td>
<td>Povertà between and within groups: a reformulation of the FGT class of index</td>
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Maggio, 2004