Dipartimento di
Economia, Statistica e Diritto
Università di Pavia

Serie Statistica

n. 2/2011

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QUADERNI DEL DIPARTIMENTO DI ECONOMIA, STATISTICA E DIRITTO
UNIVERSITÀ DI PAVIA

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Bayesian efficient capital at risk estimation

Silvia Figini*Lijun Gao†and Paolo Giudici‡

May 11, 2011

Abstract: Operational risk is hard to quantify for the deficiency of loss data and the presence of fat-tailed distribution. Extreme value distributions are conventionally used in such context. However, such distributions are very sensitive to the data and this may be a problem when data are scarce, as it occurs in operational risk management. To overcome this problem, in this paper we propose Bayesian extreme value models for operational risk management, using both non-informative and informative prior distributions.
We test the proposed models on two real databases. One is an external database that contains loss data of Chinese commercial banks, the other is an internal loss database of an anonymous European bank. For the Chinese database, there is no prior information available and, on the basis of an extensive sensitivity analysis, we provide appropriate prior specification for both frequency and severity distributions.
For the European database, instead, we have prior information available from internal self assessment and, therefore, we can compare uninformative with informative prior Bayesian models.
To assess our models, and compare it with classical ones, we also derive posterior predictive distributions for both loss frequency and severity and, therefore, we backtest the estimate of capital at risk obtained under different models. The obtained results show that, for both applications, Bayesian models perform better with respect to classical extreme value models, as they lead to a smaller quantification of capital at risk required to cover losses.

Keywords: Bayesian risk models, Extreme value distributions, Operational risk management, Self assessment prior distributions.

1 Introduction

In this paper we improve the state of the art on statistical models for risk measurement (see e.g. Behrens et al. 2006). Recently there has been a rapid and

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widespread development of models for a new category of financial risks: operational risks. This is due not only to regulatory compliance, but also to the recognition of the fact that business complexity and sophistication need a correct evaluation for this type of risk as well.
According to regulatory terminology (see e.g. www.bis.org), we focus our attention on the advanced measurement approach (AMA) and in particular, on the loss distribution approach (LDA). These approaches can give greater flexibility in comparison with traditional accounting approaches, as they take into account the particular characteristics of banking institutions: for example, they measure the capital at risk on the basis of the classification of operational losses in business line/event type (BL/ET, see e.g. Dalla Valle and Giudici, 2008, Figini et al. 2010, Figini et al. 2010, Figini et al. 2007).
The objective of the present work is to introduce a novel LDA method to estimate the capital at risk required to cover operational losses. Our proposed model is derived under a Bayesian paradigm. More precisely, we extend Behrens et al. (2006) considering a convolution between the loss frequency and severity distributions. Prior distributions are elicited for the parameters of both distributions, under two different settings: an uninformative prior setting, when expert opinions are not available, and an informative prior setting, build exploiting expert opinions specified by means of a self assessment process (see e.g. Bonafede and Giudici, 2007, Bilotta and Giudici, 2004).
Bayesian estimation of the parameters is provided through Markov Chain Monte Carlo (see e.g. Gamerman, 1997). We then obtain the predicted total loss distribution and therefore compute a capital at risk measure (see e.g. Artzner et al., 1999), such as the Value at Risk (VaR) and the Expected Shortfall (ES).
We shall compare the results achieved in our Bayesian framework with those obtained under a classical extreme value model based on a Poisson distribution for the frequency and a generalized Pareto distribution for the severity (see e.g. Demoulin et al., 2006).
The paper is organized as follows: Section 2 reports the classical extreme value model; Section 3 introduces our proposed Bayesian model; Section 4 describes the real data at hand and underlines the empirical evidences obtained using both classical and Bayesian approaches. Section 5 ends with concluding remarks and further ideas of research.

2 Classical Extreme Value models for operational risk

Extreme Value Theory (EVT) is considered a useful statistical tool for analyzing rare events: operational risk data exhibit properties, such as heavy tails, which, in natural way, call for EVT analysis. In EVT models, the peak over threshold procedure is a method which can solve data the parameter estimation problem. The procedure builds upon results of Balkema and de Haan (1974) and Pickands (1975) who show that, for a broad class of distributions, the distribution values above a sufficiently high threshold u will follow a generalized Pareto distribution (GPD) with three parameters: the location index μ , the scale index σ and the shape index or tail parameter, ξ , which indicates the thickness of the tail of the distribution. Its cumulative distribution function can be expressed as:

$$G_{\xi,\mu,\sigma} = 1-(1+\xi(x-\mu)/\sigma)^{-1/\xi}, \quad \xi \neq 0 \quad ; \quad 1-exp(-(x-\mu)/\sigma), \quad \xi = 0. \tag{1}$$

A GPD can be thought of as the conditional distribution of the loss $X = x$ such that $x > u$. Let $F_x(x)$ be the (unknown) distribution function of a random variable X (with right-end point x_F) and let $F_u(y)$ be its excess distribution at the threshold u. The excess distribution can be shown to be:

$$F_u(y) = P(X - u \leq y | X > u) = \frac{F_x(x) - F_x(u)}{1 - F_x(u)}, y = x - u \tag{2}$$

When $X > u$ let $\lambda_u = 1 - P(X \leq u)$, be the frequency of the exceedances, supplied with a subscript to stress its dependence on the threshold u. Assuming that $F_u(y)$ exists only for $x \geq u$, the exceedance distribution can be written equivalently as:

$$\hat{F(x)} = \lambda_u G_{\xi,\mu,\sigma}(x) + 1 - \lambda_u. \tag{3}$$

When a random variable X follows a generalized Pareto distribution, one can easily obtain that:

$$\lambda_u = \left(1 + \xi \frac{u - \mu}{\sigma}\right)^{-1/\xi} \tag{4}$$

When the threshold u is given, with the maximum likelihood method, we can get the parameter estimates of μ, σ, ξ. When the parameters have been estimated, we can obtain different p percentiles of the fitted generalized Pareto distribution. Let p_q be the percentile that corresponds to a probability equal to q that X is greater than p_q : in other words, the upper $q - th$ confidence level. In risk management, p_q is known as the VaR, that corresponds to a given risk tolerance levels (such as p=0.01 or p=0.001). The VaR is a risk indicator that measures the worst expected loss over a specific time interval T (usually one year) at a given confidence level $1 - p$. Under the GPD distribution assumed here, the estimated VaR can be shown equal to:

$$VaR_p(x) = u + \frac{\hat{\sigma}}{\hat{\xi}} \left\{ \left[\frac{n}{n_u}(1 - p) \right]^{-\hat{\xi}} - 1 \right\} \tag{5}$$

The weaknesses of the VaR have been generally accepted in the academic literature. It was shown that the VaR is not a coherent risk measure, because it could underestimate risks when dealing with leptokurtic variables, with potential great losses (Yamai and Yoshiba, 2002).
An alternative risk measure, which has recently received great attention, is the Expected Shortfall (ES) (Acerbi and Tasche, 2002). The ES at the $1 - p$ confidence level for a given time horizon represents the expected value of the losses that have exceeded the percentile given by the VaR. The ES estimates

the potential size of the loss exceeding a selected level L of the distribution. In the problem at hand, the expression for the ES can be shown to be (see e.g. Demoulin and Neslehova, 2006):

$$ES_u = L + \frac{\hat{\sigma} + \hat{\xi}(u - \mu) + \hat{\xi}u}{1 - \hat{\xi}}, L \geq u \tag{6}$$

3 Our proposal

There are various reasons for preferring a Bayesian analysis of extremes over the more traditional likelihood approach. Since extreme data are (by their very nature) quite scarce, the ability to incorporate other sources of information through a prior distribution has an obvious appeal (see e.g. Figini and Giudici, 2011). Recently, some Bayesian extreme value models (BEVT) have been proposed in the literature (see e.g. Behrens et. al. (2006), Bermudez and Turkman(2003), Diebolt et.al. (2005), Pandey and Rao (2009)). Such models consider the case of developing risk measures for a loss distribution that is assumed to be a Generalized Pareto.

We extend these models in two ways: on one hand we consider a convolution between a GPD distribution for the mean loss (severity), with a Poisson distribution, necessary when the number of loss events (frequency) is unknown; on the other hand, we consider the case of operational risk management that is more complex than the financial risk management problems considered in the available literature (see e.g. Alexander, 2003; Cruz, 2002; Figini et al., 2007).

In terms of the operational risk management terminology introduced in Section 1, the model applied in the present work belongs to the Advanced Measurement Approach and we call it the BLDA (Bayesian Loss Distribution Approach).

Our model can be described by expressing of the losses in terms of Frequency (the number of loss events during a certain time period) and Severity (the mean impact of the event in terms of financial loss). Such expression will be applied to a matrix with Business Lines (BL) on the rows and Event types (ET) on the columns. Formally, for each intersection i (where $i = 1, ..., r$) in the matrix M and for a given time period t, the total operational losses could be defined as the sum of a random number n_t (frequency) of losses:

$$L_{it} = X_{i1}, ..., X_{in_t}$$

where for $t = 1, ..., T$, (T representing the number of time periods available), L_{it} denotes the total operational loss, $X_{i1}, ..., X_{in_t}$ denote individual loss severities and n_t denotes the unknown frequency.

Note that, for each intersection and for each time period, the total loss can be expressed as $L_t = s_t \times n_t$, where n_t is the frequency, defined as before, and s_t (commonly referred to as the severity) is the mean loss for that period.

We proceed with the assumptions usually adopted in the LDA approach, described in section 2. Specifically, we assume that, for each time period: (1) the individual losses (X_{ij}), where $j = 1, ..., n_t$, are independent and identically distributed random variables; (2) the distribution of the frequency n_t is independent of the distribution of the severities (X_{ij}), for $j = 1, ..., n_t$; this implies that n_t is independent of s_t; (3) the losses L_{it}, for $t = 1, ..., T$, are independent and identically distributed random variables.

For a given intersection BL/ET in the matrix M, we assume a discrete probability density for the number of loss events n_t during the considered time period and s_t continuous probability densities for the loss severities (see e.g. Alexander, 2003). Following Dalla Valle and Giudici (2008), we can express the likelihood function for each intersection in a general way, that depends on some unknown parameters. Indicating the severity distribution with $f(x_j|\theta)$ and the frequency distribution with $f(n_t|\eta)$, where θ denotes the parameter vector of the severity distribution and η denotes the parameter vector of the frequency distribution, we obtain the following form for the likelihood function:

$$L(x, n|\theta, \eta) = \prod_{t=1}^{T}[\prod_{j=1}^{n_t} f(x_j|\theta)]f(n_t|\eta). \quad (7)$$

According to the Basel II requirements, each financial institution may choose to use different functional forms for the frequency and severity distributions for each ET and for each BL. Our first problem is to estimate, on the basis of the data, the parameters of the frequency and of the severity distributions, denoted by η and θ , in Eq.(7).

Here we propose an approach, based on Bayesian methods, which allows the combination of quantitative data, coming from the time series of operational losses collected by the financial institution, and prior information, represented by expert opinions, see e.g. Bernardo and Smith (1994).

We now proceed with prior specification, first of the frequency and then of the severity. As a loss frequency distribution we assume a Poisson with parameter η. In order to maintain a clear interpretation of our results we choose independent conjugate Bayesian priors (see e.g. Bernardo and Smith, 1994) for the parameter η, $\eta \propto Ga(\alpha, \beta)$. In order to set the values of the hyperparameters α and β we can use expert opinions. When we do not have such opinions, we can use an "uninformative" prior distribution, characterised by a very large variance (see e.g. Beherens et al. (2006)).

Alternatively, we can compare different prior specifications and perform sensitivity analysis to select one of them. According to Bayes Theorem, the posterior distribution of the parameter η is then computed by multiplying the likelihood function with the prior distribution and normalizing such product. It can be shown that the distribution we get is a Gamma as follows:

$$\pi(\eta|n) \sim Ga(\sum_{t=1}^{T} n_t + \alpha, T + \beta). \quad (8)$$

We now move to the issue of estimation of the predictive distribution from which to extract risk measures. Let y denote a frequency observation, in a future time

period, with density function $f(y|\eta)$, where $\eta \in H$. The posterior predictive density of such future frequency y, given the observed data, $n = (n_t, t = 1, ..., T)$ is equal to $f(y|n) = \int_H f(y|\eta)\pi(\eta|n)d\eta$.
Concerning the specification of a prior for the severity, we have decided to employ Extreme Value Theory distributions because in operational risk problems, data are usually sparse. It is reasonable to hope that experts should provide relevant prior information about extreme behavior, since they have specific knowledge of the characteristics of the data under study. Nonetheless, expressing prior beliefs directly in terms of the three GPD parameters $\theta = (\mu, \sigma, \xi)$ is not an easy task. Here we follow Coles and Tawn (1996), who suggest the elicitation of information within a parameterization on which experts are familiar. More precisely, by the inversion of the cumulative distribution function expression in equation (1), we can obtain the $(1-p)$ quantile of the distribution, indicated with q, $q = u + \frac{\sigma}{\xi}\left\{\left[\frac{n}{n_u}(1-p)\right]^{-\xi} - 1\right\}$.
Note that q can be viewed as the return level associated with a return period of $1/p$ time units. The elicitation of the prior information can thus be done in terms of a triple of quantiles, ordered as $(q_1 < q_2 < q_3)$ that are obviously function of the θ vector. For sake of interpretability, we have chosen to consider gamma priors for each of the three quantiles, functions of the three unknown parameters. Note that, for $c > 1$, q_c does not depend on μ. However (q_2, q_3) depend on the scale and shape parameters (σ, ξ).
Therefore we set $q_c \sim Ga(a_c, b_c), \quad c = 1, 2, 3$. Let $\pi(\theta)$ denote the prior distribution for θ. Such density can then be expressed as:

$$\pi(\theta) \propto J \prod_{c=1}^{3} q_c^{a_c - 1} exp(-q_c/b_c)$$

where J is the Jacobian of the transformation from (q_1, q_2, q_3) to $\theta = (\mu, \sigma, \xi)$.
In order to derive the posterior distribution, we first recall the expression of the likelihood under the GPD model. For convenience we use the log-likelihood, as follows:

$$logL(\theta; x) = -nlog\sigma - (1 + 1/\xi) \sum_{j=1}^{n_t} log[1 + \xi(x_j - \mu)/\sigma], \tag{9}$$

provided that $1 + \xi(x_j - \mu)/\sigma$ is positive for each $j = 1, ..., n_t$. Given the prior density $\pi(\theta)$ and the likelihood $L(\theta; x)$, we can get the posterior density $\pi(\theta|x)$. Computing $\pi(\theta|x)$ directly is problematic because it requires the computation of the integral $\int_\Theta \pi(\theta)L(\theta; x)d\theta$. Markov Chain Monte Carlo (MCMC) (see e.g. Gamerman, 1997 or Robert and Casella, 1999) approximate the calculation producing stationary sequences of simulated values with marginal density $\pi(\theta|x)$ (see e.g. Bernardo and Smith, 1994).
We now consider estimation of the predictive severity. Once we have got the estimated marginal posterior distribution, we can derive the posterior predictive distribution of the severity.

Let z denote a future observation of the severity with density function $f(z|\theta)$, where $\theta \in \Theta$. The posterior predictive density of z , given the observed data x ,is $f(z|x) = \int_\Theta f(z|\theta)\pi(\theta|x)d\theta$.
Using the MCMC approach, the predictive distribution can be estimated using $\frac{1}{n-b+1}\sum_{i=b}^{n} P(Z \leq z|\theta^{(k)})$, where n is the length of the chain and b is an appropriately chosen burn-in parameter.
In order to derive a measure at risk we need to merge the predictive severity with the predictive frequency obtained before. This can be done by convoluting the predictive frequency with the predictive severity via a Monte Carlo simulation (see e.g. Dalla Valle and Giudici 2008 and Gao et al. 2006, Kenett et al. 2010). Computationally this can be done along the following steps:
Step1. For each time period to be predicted, generate n random observations from the predictive frequency distribution;
Step 2. For each period, generate a number of losses from the predictive severity distribution equal to the corresponding frequency observation drawn in step 1 (that is, if the simulated frequency of events for period k is n_k, we simulate n_k severity losses from the predictive severity marginal distribution) .
Step 3. For each period, sum the losses obtained in step 2, obtaining a loss observation for the period, drawn from the convoluted marginal distribution as described, thereby obtaining one loss observations for each period.
Step 4. Using the loss observations obtained in step 3, estimate the predictive loss distribution and obtain the VaR and ES, the risk measures that establish how much capital is at risk.

4 Application

In this section we describe how our proposal works on two different sets of data. The first data set we use in our analysis contains the external operational losses of Chinese commercial banks, ranging from 1993 to 2006. The number of loss data collected is equal to 860. In this analysis we concentrate on 331 loss events extracted for a specific Business Line and a given Event Type. Table 1 reports the frequency and the severity (million yuan) for each year.

Table 1 about here

From the data in Table 1, the overall average yearly loss is equal to 1535.81 million Yuan, the minimum is equal to 0.95 million Yuan and the maximum is equal to 9220.22 million Yuan.
For the data at hand, we have first considered classical EVT models, as described in Section 2. In order to select the threshold, and consequently estimate the parameters, we have employed the mean residual life plot and the threshold choice plot (see e.g. Coles, et. al. 1999). We have found that the threshold can be taken equal to 43. In correspondence to this settings, the frequency of the exceedances is equal to 60.45%, which suggests the employment of an EVT model. On the basis of the estimated parameters, under the EVT model, we can derive the combined distribution of frequency and severity via a Monte Carlo

simulation. Table 2 reports the VaR and the ES estimated under the classical approach.

Table 2 about here

Since the estimated shape parameter of the GPD distribution is greater than 1 (is equal to 1.1), in Table 2 we obtain that all expected shortfall become negative. This means that the classical model is not adequate to obtain a coherent measure of risk.
We move now to the application of our Bayesian model, using 50000 iterations in the MCMC algortihm. First we should choose the right priors for the distribution. Since we have no actual expert opinion, we follow a sensitivity analysis approach, specifing a number of alternative priors under different combinations of hyperparameters settings. In particular for the frequency parameter prior distribution, we consider two prior settings: low=(1,20) and high=(40,1). On the other hand, for all of the three severity quantiles prior distributions we consider two prior specifications: low=(1,1) and high=(30,30). This leads to a total of 16 alternative prior specifications that will be compared in terms of backtesting: choosing the prior that leads to the smallest difference in absolute value between the VaR and the observed losses. For sake of precision we have run the backtesting using the first 9 years as a training period and the subsequent 5 years as test period on a moving window basis. Table 3 reports the results of the sensitivity analysis: for each specified prior we report the loss exceedances for periods 10 trough 14.

Table 3 about here

From Table 3 note that the HLLH combination is the best one: with it the VaR covers all the real total losses and the mean difference between the VaR and the observed losses is the smallest one. Table 4 shows the posterior summaries for all parameters under the chosen prior specification.

Table 4 about here

From Table 4, the posterior mean frequency is equal to 24.74 and the median is equal to 25. The results seem coherent with the observed sample mean, which is equal to 23.64. Using the results described before, we are thus able to obtain the predictive distribution for the severity and to combine it with the predictive frequency distribution via Monte Carlo simulation, as explained in Section 3. The combined distribution allows us to derive the risk measures, as in Table 5.

Table 5 about here

We can compare the results in Table 5 with those in Table 2, relative to the classical model. Note that, using our BLDA model, we can obtain a remarkable reduction in terms of VaR (always lower in the BLDA case) and, therefore, a lower capital charge.
The second data set we use in our analysis contains one specific business line (retail banking) and a given event type (external fraud) of the internal database

of operational losses of an anonymous European bank, ranging from October 2006 to December 2010. The number of loss data collected is equal to 1855. In order to use the self-assessment data, in this analysis we concentrate on the loss of 2009, composed of 396 loss events. The overall average month loss is equal to 38777.41 Euro, while the minimum and the maximum are equal to 17298.68 and 115662.5 Euro respectively. Considering retail banking and external fraud as business line and event type, Table 6 reports for each month the frequency and severity distributions.

Table 6 about here

As we can observe from Table 6, external fraud events are generally stable across the time period considered, except for April and August, while the average for the severity shows an increasing in March and October.
We now apply a classical model. In order to estimate the threshold and consequently the scale and the shape parameters, we have employed the mean residual life plot and the threshold choice plot (see e.g. Coles et al. 1999). We have found that the threshold can be fixed equal to 1230. In correspondence to this setting, the frequency of the exceedances is equal to 18.45%, which confirms the employment of an EVT model. In the classical perspective, on the basis of the estimated parameters, we can compute the combined distribution of frequency and severity via a Monte Carlo simulation. Table 7 reports the VaR and the ES estimated for the integrated distribution at hand using the classical model.

Table 7 about here

From Table 7, note that the ES is positive, differently from what happened in the chinese data.
In order to build our Bayesian model, we now construct a prior distribution for the frequency using qualitative data provided by the bank. The information contained in the qualitative data is based on prior opinions obtained from a self assessment questionnaire submitted to the process owners of the bank.
In order to evaluate the impact of actual prior opinions we first specify an uninformative prior without self-assessment characterised by a very high variance. Using such prior, the posterior mean is equal to 32.98 and the posterior median is equal to 33. This result seems coherent with the observed sample mean, which is equal to 33. The number of iterations of the MCMC algorithm was taken equal to 35000.
We now consider an informative prior that uses self assessment opinions. The summary of such opinions, on frequencies, are shown in Table 8.

Table 8 about here

We have decided to use an approximate confidence interval for binomial proportions to specify the prior hyperparameters, for example under a 95% confidence level. Under this framework we get two reference prior distributions: one that we call lower informative, with hyperparameters set at the lower bound of the confidence interval, and one, conversely, upper informative. More precisely, the lower informative prior turns out to be Gamma (7.3748, 0.2951) and the upper informative prior Gamma (19.5329, 0.4803). Table 9 shows the summaries of the posterior distribution of the frequency parameter under, respectively, an uninformative prior, a lower informative and an upper informative priors.

Table 9 about here

Table 9 shows that the insertion of a self-assessment prior in the model does not change substantially the posterior estimates obtained with an uninformative prior.

We now move to the task of specifying prior distribution for the severity. We first consider uninformative Gamma prior characterised by a large variance. Table 10 reports summary measures for the corresponding posterior distribution. In the MCMC algorithm the number of iterations is equal to 35000.

Table 10 about here

We now consider an informative prior, based on self-assessment data, for the severity distribution. Following a procedure similar to what exposed for the frequency prior, as opinions on severity are expressed on an ordinal scale as frequency ones, we get lower informative priors for the quantile parameters as Gamma (13.5540, 91.7646), Gamma (0.1982, 26486.183) and Gamma (0.1532, 14396.1), upper informative priors as Gamma (35.899, 56.386), Gamma (0.525, 16274.64) and Gamma (0.406, 8845.817). Table 11 shows the posterior distribution summaries.

Table 11 about here

Using the results described before, we are thus able to obtain the predictive distribution for the severity and combine it with the predictive frequency distribution via Monte Carlo simulation, as explained in Section 3. The combined distribution allows us to derive the risk measures, as shown in Table 12.

Table 12 about here

We can compare the VaRs estimated in the above table, based on the 2009 monthly data, with the actual losses occurred in the year 2010. Concerning the 99.9% VaR, which is the most used reference in operational risk management, we have obtained that all VaRs, except that calculated with the LIFUNS model cover all actual 2010 losses.

For all model we also derived the Expected Shortfall. In order to chose among all remaining models we have computed in Table 13 the sum of the differences between each VaR and all 2010 observed losses.

Table 13 about here

From Table 13 the best models, in terms of minimisation of the differences with the VaR and the observed losses are : LIFUNS, UIFLIS, UNFUNS, UIFUNS.

However, LIFUNS can be excluded as the corresponding VaR does not cover all losses. Among the remaining three models, the model with the lowest Expected Shortfall is UIFUNS which therefore will be chosen. UIFLIS has a high Expected Shortfall and, therefore, will also be excluded, being incoherent. In conclusion, we support choosing the UNFUNS or the UIFUNS model, the former being totally uninformative and the second partly informative.
We remark that the latter has indeed a cover expected shortfall. In order to compare our BLDA model with the classical one, we can use the VaR, as reported in Table 7 and Table 13. Note that, using the BLDA model, we can obtain a remarkable reduction in terms of VaR (always lower, especially with self-assessment prior, in the BLDA case) and, therefore, a lower capital charge.

5 Concluding remarks

The main purpose of this paper is to introduce a new methodology for estimating the loss distributions in operational risk management in a predictive framework. Our main outcome is that the application of Bayesian methodology causes a reduction of value at risks and, therefore, of the capital charge compared to the classical extreme value analysis method. This is a very important result in terms of money saved by the financial institution adopting this approach.
We also remark that the two examined cases are based on very different, yet complementary, datasets: the Chinese data are basically uninformative data, with dynamic nature; the European data considers a short time horizon and include self-assessment data, and consequently, the BLDA model is based on informative data to derive the prior knowledge. In both cases, our BLDA shows a loss reduction compared with the classical extreme model and therefore a great reduction in terms of capital charged.
Our results finally show that using a self-assessment data to specify an informative prior, leads to results that are little better than with an uninformative prior model, so it is helpful that banks collect and analyse self-assessment expert opinions.
An interesting development of our research could be the extension to other risk types, and a multivariate analysis of all event type/business lines combinations, although this latter would involve multivariate self-assesment (see e.g. Bonafede and Giudici, 2007).

6 Figures and Tables

Year	Frequency	Severity
1993	2	3.5
1994	4	0.95
1995	17	4099
1996	18	991
1997	10	336
1998	13	358
1999	24	966
2000	33	713
2001	27	9220
2002	31	337
2003	33	166
2004	51	919
2005	39	2205
2006	29	1188

Table 1: Summary of Chinese loss data

Percentile	VaR	ES
99%	5194.8	-101255.7
99.90%	57707.2	-1138423
99.97%	204282.5	-4033423
99.99%	647761	-12792539

Table 2: VaR and ES under the classical model (Chinese data)

Frequency	Location	Scale	Shape	Diff10	Diff11	Diff12	Diff13	Diff14
Low	Low	Low	Low	116	458	123	-1484	-526
Low	Low	Low	High	727	549	-7	-828	260
Low	Low	High	Low	251	427	-143	-1558	-385
Low	Low	High	High	1918	5310	2033	1351	2049
Low	High	Low	Low	346	397	-303	3926	273
Low	High	Low	High	663	942	409	-267	-9
Low	High	High	Low	205	379	-229	-1488	3990
Low	High	High	High	4691	5014	4825	-1477	-324
High	Low	Low	Low	977	1047	693	-311	277
High	Low	Low	High	1458	1658	1588	450	1236
High	Low	High	Low	887	1032	574	-650	469
High	Low	High	High	10462	5310	12604	17223	8524
High	High	Low	Low	1095	1262	295	-416	390
High	High	Low	High	2210	2295	1772	1495	1127
High	High	High	Low	912	976	345	-546	725
High	High	High	High	10185	10256	8501	5913	12781

Table 3: Prior specification: hyperparameters combinations and corresponding absolute distance from the VaR (Chinese data)

Parameters	Min	Median	Mean	Max	Std
η	8	25	24.74	50	5.145
μ	0.13	1.05	1.01	1.13	0.11
σ	6.72	10.61	10.66	16.31	1.2
ξ	0.58	0.67	0.67	0.78	0.03

Table 4: Summary measures for the posterior distributions of the parameters (Chinese data)

Percentile	VaR	ES
99%	3828.6	9315
99.90%	13195.6	38037.7
99.97%	26637.1	81501.5
99.99%	48619.2	165566.3

Table 5: VaR and ES under the BLDA model (Chinese data)

Month	Frequency	Severity
Jan	24	35364
Feb	24	22494
Mar	36	115663
Apr	53	50360
May	32	21797
Jun	29	26793
Jul	25	26531
Aug	49	34500
Sep	26	26430
Oct	44	58889
Nov	23	17299
Dec	31	28930

Table 6: Summary of European loss data

Percentile	VaR	ES
99%	124374.4	162184.3
99.90%	210456.5	304489.8
99.97%	304337.2	420995.3
99.99%	392317.6	568834.4

Table 7: VaR and ES under the classical model (European data)

Type	Yearly or rarely	Monthly	Weekly	Daily	Total
Frequency	249	200	33	8	490

Table 8: Self assessment distribution of the frequency: observed counts of beliefs on yearly occurrence of losses (European data)

Model	Min	Max	Mean	Median	Std
Un Freq	12	63	32.98	33	5.95
Low Inf Freq	12	60	32.79	33	5.94
Upp Inf Freq	14	60	33.27	33	5.98

Table 9: Summary measures for the frequency parameter posterior means under the three different prior specifications (European data)

Parameters	Min	Median	Mean	Max	Std
μ	445.8	684.1	673.5	703.1	32
σ	49.2	312.8	333.9	1201.3	132.4
ξ	-0.41	0.29	0.29	1	0.2

Table 10: Summary measures for the posterior severity parameters under an uninformative prior (European data)

Low	Min	Median	Mean	Max	Std
μ	129.5	543	526	568.8	51.2
σ	76	373.2	397.9	1189.5	148
ξ	-0.25	0.28	0.29	0.94	0.19
Upper	Min	Median	Mean	Max	Std
μ	223.8	540.1	525.8	568.8	45.4
σ	122.3	515.2	541.9	1477.7	137.9
ξ	-0.39	0.28	0.28	0.87	0.19

Table 11: Summary measures for the posterior parameters under the two informative priors (European data)

Model	VaR 99%	VaR 99.9%	VaR 99.97%	VaR 99.99%
LIFUNS	67519	111208	192350	413386
UIFUNS	68244	125752	203349	426953
LIFLIS	65986	139424	248751	643641
LIFUIS	81073	141538	219601	495748
UIFLIS	67830	119599	183164	257015
UIFUIS	82424	154822	264339	354807
UNFLIS	66048	139779	242925	490587
UNFUIS	79916	153766	266226	626557
UNFUNS	67568	122634	236121	359226
Model	ES 99%	ES 99.9%	ES 99.97%	ES 99.99%
LIFUNS	96630	257281	513272	961494
UIFUNS	99756	268337	524409	930257
LIFLIS	133452	606196	1545844	3547730
LIFUIS	113138	274391	511727	901157
UIFLIS	112641	408922	992471	2378685
UIFUIS	113669	244735	354755	460954
UNFLIS	101528	292657	564637	994957
UNFUIS	118552	318824	595871	946322
UNFUNS	110495	384555	875528	1859656

Table 12: VaR and ES under the BLDA models (European data)
Legend:
1) UNF means uninformative frequency prior.
2) UNS means uninformative severity prior;
3) LIF means lower bound informative frequency prior;
4) UIF means upper bound informative frequency prior;
5) LIS means lower bound informative severity prior;
6) UIS means upper bound informative severity prior.

Model	Sum of difference with estimated VaR (99.9%)
LIFUNS	72430
UIFUNS	86974
LIFLIS	100647
LIFUIS	102761
UIFLIS	80822
UIFUIS	116044
UNFLIS	101002
UNFUIS	114989
UNFUNS	83857

Table 13: Sum of the differences between the observed (future) data and the estimated VaR (99.9%) (European data)

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