A CONCEPTUAL MODEL FOR THE ESTIMATION OF LOADOGRAPHS IN SEWER NETWORKS DURING METEOROLOGICAL EVENTS

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SUMMARY

With reference to the phenomena ruling the complex process of erosion and solution of pollutants from a catchment surface during a runoff period, a simple global model is presented in order to analyse the transport of contaminants due to drainage flows.

The model, based on the instantaneous unit hydrograph of a single linear reservoir system, appears to be suitable whenever convection phenomena are predominant during the transport, because of the small size of the pollutant particles, or whenever, because of the hydrodynamic situation during the meteorological event, sedimentation and resuspension phenomena can be considered negligible (separate sewer systems or high slope combined sewer systems).

Empirical relationships are used to establish the mass rate of pollutants which enters the urban drainage system during a meteorological event.

Numerical simulations have been carried out for some events recorded in the experimental catchment of Cascina Scala (Pavia, Italy), for which both quantity and quality data are available.

Satisfactory results have been obtained.
INTRODUCTION

Several studies have given evidence that urban storm waters must be considered a major source of pollution for receiving waters (Weibel et al., 1964; De Filippi and Shih, 1971; Sartor et al., 1974; Wanielista et al., 1977; Lindholm and Balmér, 1978; Randal et al., 1982; Reinertsen, 1982; Novotny et al., 1985; Artina et al., 1997).

Since it is a widespread opinion that depuration of dry weather flows is not sufficient to obtain effective improvements in the stream water quality, the development of control strategies aiming to minimise storm water discharges of both separate and combined sewer systems has become a problem of major interest.

In order to analyse the effectiveness of different techniques, both hydrologic and hydraulic aspects involved in the transformation of rainfall into runoff and the phenomena influencing the pollutant dynamics must be simulated properly. Obviously adequate mathematical models are required.

The phenomena of formation and propagation of pollutants are very complex processes; however, most of the commercial codes used in engineering practice address the problem with a deterministic approach, so that quality simulations are usually referred to suspended solids, since it has been proven that most of the other pollutant factors (BOD, COD, nitrogen forms, etc.) are strictly correlated with them (Artina et al., 1997).

In these models, the dynamics of pollutants is simulated analysing separately the following processes:

- the accumulation of pollutants during dry weather on catchment surfaces;
- the removal of accumulated pollutants by runoff;
- the transport of pollutants due to drainage flows.

The buildup of contaminants on the catchment surface is usually represented by means of simple empirical relationships which involve a limited number of parameters; these formulations, even though the underlying physics of the generation of pollutants from sources such as street pavement, vehicles, atmospheric fallout, vegetation etc. is essentially ignored, provide an estimation for the mass available at the beginning of the storm as a function of the time elapsed since the previous event.

The removal of pollutants by means of urban runoff is also generally simulated in a quite simple way with empirical exponential relationships which are deduced making the assumption that the mass washed off by the rainfall is proportional to the available mass and to the rainfall intensity. These relationships depend just upon 1 or 2 parameters.

The values of the parameters which influence the modelling of the processes previously described can be established with calibration procedures whenever the availability of quality data makes it possible (a rare situation indeed); otherwise they are usually defined by resorting to the mean calibration values which are reported in the scientific literature for similar urban areas.

Modelling the transport of contaminants due to drainage flows appears to be a much more complex problem. In fact, it may be an impossible task to provide an accurate theoretical
description of the propagation of the highly heterogeneous solid material in real sewer systems.

Depending on the size of the contaminant particles and on the hydraulic conditions, solids can move either as suspended or bed load, can settle in different parts of the drainage system (gutters, catchbasins, conduits) or be carried again by the current.

A physically based model requires therefore the knowledge of particle size distribution and specific gravity for each pollutant and an accurate simulation of the hydraulic transients which occur in the drainage system.

This approach is commonly adopted in commercial codes (SWMM of the Environmental Protection Agency, HYDROWORKS of the Wallingford Software Society, MOUSE of the Danish Hydraulic Institute) where relationships and criteria for the analysis of scour and deposition processes are developed as a function of particle size distribution and flow characteristics.

The model, however, being a distributed one, raises a lot of practical problems since an extremely detailed information is required about the basin characteristics and the network geometry.

In addition, the adoption of a physically based and therefore complex scheme to represent the propagation of pollutants appears to be inconsistent with the great empiricism of the formulations utilised to evaluate for each pollutant the mass rate of each pollutant washed off the surfaces into the sewer network.

Considering also that, for engineering applications, the loadograph associated to a certain meteorological event is required only at a certain section, it seems reasonable to evaluate the possibility of using global models to represent the average behaviour of the system for what concerns both the quantitative and qualitative aspects.

In the following pages a global simplified model is proposed and validated with some experimental data. The model appears to maintain, at least when pollutant transport is dominated by convection, a close link with the essential features of the phenomena to be simulated.

As a matter of fact this approximation may be considered reasonably acceptable whenever passive pollutants are considered or whenever the hydrodynamic situations, peculiar to separate sewer systems or combined sewer systems with medium to high slopes, make particulate sedimentation and resuspension phenomena negligible.
THE GLOBAL MODEL SUITABLE FOR THE SIMULATION OF POLLUTANT TRANSPORT DUE TO DRAINAGE FLOWS

The proposed model is based on the conceptual scheme of a linear reservoir which is one of simplest model used to analyse the liquid flows.

As it is well known, this model hypothesises that the flow formation is mainly due to retention phenomena and that the basin and the drainage network behave as a single linear reservoir subjected to the inflow \( p(t) \) and to the outflow \( q(t) \) which are assumed proportional to the stored volume \( w(t) \).

Under these hypotheses the phenomenon of the flow formation can be simulated through the continuity equation:

\[
\frac{dw(t)}{dt} = p(t) - q(t)
\]

(1)

and through the outflow relationship which rules the reservoir behaviour:

\[
w(t) = k \ q(t)
\]

(2)

where \( k \) is a proportionality constant which ideally schematises and simplifies the complex phenomena of lamination and transport characterising the particular system.

Linearily combining equations (1) and (2) a linear differential equation with constant coefficients is obtained:

\[
k \ \frac{dq(t)}{dt} = p(t) - q(t)
\]

(3)

Integrating equation (3) with the initial condition that at the beginning of the effective precipitation there is no flow rate leaving the system, the well known expression for the convolution integral is obtained:

\[
q(t) = \int_0^t p(\tau) \ h(t - \tau) \ d\tau
\]

(4)

where the term:

\[
h(t) = \frac{1}{k} \ exp\left(-\frac{t}{k}\right)
\]

(5)

represents the instantaneous unit hydrograph for a single linear reservoir system.

The convolution integral (4) allows to obtain for each hyetograph \( p(t) \) the corresponding hydrograph \( q(t) \), once the retention constant \( k \) is defined. The value for the retention constant is generally influenced by the morphology of the drainage system and the precipitation and it can be estimated by means of some relationships which have been proposed in several studies (Desbordes, 1975; Ciaponi and Papiri, 1992).
The conceptual scheme of a linear reservoir model can be extended to the analysis of water quality aspects. Supposing that the inflow \( p(t) \) is characterised by a pollutant concentration \( c_p(t) \) and that the outflow \( q(t) \) is characterised by a pollutant concentration \( c_q(t) \) which is the concentration of the storage volume as well, assuming moreover that there is complete instantaneous mixing between the flows entering and leaving the drainage system, the continuity equation for the mass of pollutant appears in the form:

\[
\frac{dm(t)}{dt} = c_p(t) \cdot p(t) - c_q(t) \cdot q(t) \tag{6}
\]

where \( m(t) \) represents the mass of pollutant stored in the system. It is quite easy to recognise that the linear relationship (2) which relates the flow leaving the system to the stored volume can be rearranged in terms of mass

\[
m(t) = c_q(t) \cdot w(t) = k \cdot c_q(t) \cdot q(t) \tag{7}
\]

Denoting with \( x(t) = c_p(t) \cdot p(t) \) the mass rate entering the system (input loadograph) and denoting with \( y(t) = c_q(t) \cdot q(t) \) the mass rate leaving the system (output loadograph), from the linear combination of equations (6) and (7) a linear differential equation with constant coefficients is obtained:

\[
k \cdot \frac{dy(t)}{dt} = x(t) - y(t) \tag{8}
\]

Integrating equation (8) with the initial condition that at the beginning of the effective precipitation there is no mass rate leaving the system, the well known expression for the convolution integral is obtained:

\[
y(t) = \int_0^t x(\tau) \frac{1}{k} \exp \left( -\frac{t - \tau}{k} \right) d\tau \tag{9}
\]

which expresses that the mass rate leaving the drainage system depends on the mass rate removed from the catchment surface by the rainfall and on the retention constant which characterises the rainfall-runoff process.

Coupling equation (9) with a model suitable for the estimation of \( x(t) \), which depends on the mass accumulated on the impervious area and the rainfall intensity, it is then possible to get the loadographs relative to the considered section once it is known the event representative hyetograph.
THE MODEL FOR THE SIMULATION OF THE POLLUTANT BUILDUP ON THE CATCHMENT SURFACE AND THE WASHOFF OPERATED BY RAINFALL

To evaluate the pollutant mass rate \( x(t) \) which goes into the sewer network during the meteorological event, the model previously described employs empirical relationships available in the scientific literature, which are also adopted in many commercial codes.

The buildup of solids on the catchment surface is simulated through an exponential relationship adopted by the SWMM model of the EPA (Alley and Smith, 1981):

\[
m_a = \frac{B}{D} [1 - \exp(-D T_d)] A \text{IMP} \phi_{imp} + m_r \exp(-D T_d)
\]  

(10)

where the symbols assume the following meaning:
- \( m_a \) is the mass of solids accumulated on the catchment surface [M];
- \( B \) is the buildup rate of solids due to the different phenomena which influence the accumulation on the catchment impervious area [M L^{-2} T^{-1}];
- \( D \) is a spreading coefficient which represents the dispersion of the particles due to the wind, traffic and biological and biochemical degradation [T^{-1}];
- \( A \) is the catchment area [L^2];
- \( IMP \) is the ratio between the impervious area and the total area;
- \( \phi_{imp} \) is the ratio between the contributing impervious area and the total impervious area;
- \( T_d \) is the dry time elapsed since last cleaning by rain [T];
- \( m_r \) is the mass remaining on the catchment at the end of the previous meteorological event [M].

The formulation of equation (10) assumes that only a part of the impervious area is contributing to runoff.

For what concerns the surfaces washed off by rain, assuming that the removed material is proportional to the accumulation of mass on the catchment, a linear differential equation with constant coefficients is obtained (Sartor et al., 1974):

\[
\frac{dm_a(t)}{dt} = -k_w m_a(t)
\]

(11)

which integrated appears in the form:

\[
m_a(t) = \bar{m}_a \exp(-k_w t)
\]

(12)

where \( \bar{m}_a \) is the mass available at the beginning of the storm.

Therefore the cumulative amount washed off by rain is:

\[
m_a(t) = m_a(t) - \bar{m}_a \exp(-k_w t)
\]

(13)

The parameter \( k_w \) is a proportionality coefficient which depends on the physical and dimensional characteristics of the particles and on the rainfall intensity. It can be defined
referring to one of the many formulations which have been proposed in the scientific literature. In this paper $k_w$ has been assumed proportional to the precipitation intensity generalising the formulation used in the SWMM model (Huber and Dickinson, 1988) in the following way:

$$k_w = r_{coef} \left( \frac{rr}{rrr} \right)^{washpo}$$

(14)

where:
- $r_{coef}$ is a calibration coefficient $[T^{-1}]$;
- $rr$ is the runoff rate over the catchment $[L T^{-1}]$;
- $rrr$ is a reference runoff rate $[L T^{-1}]$;
- $washpo$ is a calibration coefficient which is usually assumed to be greater than 1.

The particular form of equation (14) allows the adoption of the numerical values of the coefficients $r_{coef}$ and $washpo$ which are reported in the American literature simply assuming for the reference runoff rate $rr$ the value of 25.4 $[mm h^{-1}]$.

However, it must be considered that, since usually $washpo$ is greater than 1, the washoff velocity increases, keeps constant or decreases in relation to the fact that the ratio between the runoff rate and the reference runoff rate is respectively greater, equal or smaller than 1.

The reference runoff rate may also be considered as a calibration parameter through which the runoff rate can influence the washoff velocity.

Moreover it must be kept in mind that the concentration $c_p(t)$ of the meteorological flow $p(t)$ which washes the catchment surface before getting into the sewer network can be calculated with the following expression:

$$c_p(t) = \frac{r_{coef} \left( \frac{rr}{rrr} \right)^{washpo} m_o(t)}{A IMP \phi_{imp} \left( \frac{rr}{rr} \right)^r}$$

(15)

**EXPERIMENTAL VALIDATION OF THE MODEL**

The global model described before has been implemented to simulate both the quantitative and qualitative features (hydrographs and loadographs) of some meteorological events recorded in the experimental catchment of Cascina Scala (Pavia, Italy). For these events, besides the hydrologic data, flow rates and concentrations of some pollutants are available with reference to the final reach of the sewer network (Ciaponi et al., 2002).

The physical characteristics of the experimental catchment, the structure of the drainage system and the experimental equipment needed to measure and record hydrologic data, flow rates and concentrations are reported in other papers (Ciaponi and Papiri, 1994; Ciaponi et al., 2002).
The effective hyetographs have been defined ignoring the contribute of the pervious area and assuming a constant percentage $\phi_{imp}$ of the impervious area hydraulically connected to the drainage system. In order to get a perfect agreement between the recorded inflow and the measured outflow, hydrologic losses have then been determined assuming an initial depression storage $istore$ that has to be filled prior to the occurrence of runoff. The value of $istore$ is therefore variable for each event.

The results of the simulations are reported in Figures 1-8, where for each of the 8 selected events, the recorded hyetograph, the simulated and measured hydrograph, the simulated and measured loadograph inherent to suspended solids have been represented.

The selected values of the calibration parameters are reported in Table 1. They correspond to the average of the values which are associated with the best numerical simulation for each event. There is large evidence that these values are consistent with the scientific literature.

Table 1. Selected values of the calibration parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit of Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{imp}$</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>8</td>
<td>[min]</td>
</tr>
<tr>
<td>$B$</td>
<td>18</td>
<td>[kg ha$^{-1}$ d$^{-1}$]</td>
</tr>
<tr>
<td>$D$</td>
<td>0.3</td>
<td>[d$^{-1}$]</td>
</tr>
<tr>
<td>$rcoeff$</td>
<td>0.2</td>
<td>[min$^{-1}$]</td>
</tr>
<tr>
<td>washpo</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>$rr$</td>
<td>0.42</td>
<td>[mm min$^{-1}$]</td>
</tr>
</tbody>
</table>

An examination of the results shows clearly that, even if the simulations have been carried out adopting the constant values for the calibration parameters, the model can satisfactorily reproduce the experimental measurements. It must be pointed out moreover that the anticipation, noticeable between the numerical simulations and the experimental data of both hydrographs and loadographs, is a typical characteristic of the instantaneous unit hydrograph of a linear reservoir system. This anticipation has nonetheless little relevance for engineering purposes, since it may be easily eliminated introducing a delay factor in the model.

The results evidence that the accuracy of the simulations of both the flow rate and the mass rate are strictly correlated; whenever the model is unable to simulate correctly the hydrograph even the loadograph cannot be reproduced precisely.

The numerical simulation brings out furthermore that the final results depend strongly on the parameters which influence the buildup and the washoff of the pollutants. In particular it has been observed that the value of the total mass washed off is conditioned by the total mass present on the catchment surface at the beginning of the storm.
The total mass itself is mainly influenced by the time elapsed since the last meteorological event, but sometimes, even the mass $m_r$ left on the catchment at the end of the previous storm plays a significant role, especially when the dry time preceding the event is short.

This is the reason why the total mass available for the washoff has been calculated, for each selected event, considering also the effect of all the previous recorded events.

CONCLUSIONS

The proposed model is based on the hypothesis that the pollutant transport in an urban drainage system during storm events is essentially influenced by convection and that other phenomena like scour, sediment deposition, biological and biochemical degradation of contaminants are negligible.

An application of the model to a combined sewer system characterised by low to medium slopes (average value 0.0042) has given satisfactory results as far as the simulation of the measured loadographs is concerned.

The simplifying assumptions, which the model is based on, make it reasonable to assume that even better results could be obtained whenever the model should be applied to separate sewer system or to high slope sewer systems, where particulate sedimentation during dry weather and scour during storms are practically negligible.

In addition, it must be underlined that all the simulations have been carried out adopting constant values for the calibration parameters. This proves the good stability of the model and suggests the possibility of extending the results to similar urban realities.

Moreover numerical simulations have shown that, in order to get a good reproduction of the loadographs, it is crucially important to choose the values of the parameters ruling the pollutant buildup and washoff accurately. It is also evident that, since these processes, which define the mass rate going into the sewer network, are evaluated with quite rough formulations, a detailed description of the propagation phenomena in the drainage conduits seems scarcely meaningful as compared with the effort required for determining the network geometry.

The proposed model, instead, offers the advantage that the behaviour of the drainage system can be described with just few parameters (the same parameters needed for the definition of the instantaneous unit hydrograph) and therefore it seems to be an helpful tool for technical considerations whenever the information needed for using a distributed model is not available.

Finally, the proposed model seems suitable for evaluating the influence of the global characteristics of different basin and sewer networks on the design and construction of urban storm water management systems planned to control the discharges to receiving waters.

ACKNOWLEDGEMENTS

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Figure 1. Quantity and quality simulations for event 1.
Figure 2. Quantity and quality simulations for event 2.
Figure 3. Quantity and quality simulations for event 3.
Figure 4. Quantity and quality simulations for event 4.
Figure 5. Quantity and quality simulations for event 5.
Figure 6. Quantity and quality simulations for event 6.
Figure 7. Quantity and quality simulations for event 7.
Figure 8. Quantity and quality simulations for event 8.
REFERENCES


